

AB Calculus
Chapter 6 Review

1) If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

$$f'(x) = \int (2x - \cos x) dx$$

$$= \int 2x dx - \int \cos x dx$$

$$= \frac{2x^2}{2} - \sin x + C$$

$$= x^2 - \sin x + C$$

$$f(x) = \int (x^2 - \sin x + C) dx$$

$$= \frac{x^3}{3} + \cos x + Cx + C_1$$

1993 AB 38
Calc

2) $\int \frac{5}{1+x^2} dx =$ 1973 AB 32

(A) $\frac{-10x}{(1+x^2)^2} + C$

(B) $\frac{5}{2x} \ln(1+x^2) + C$

(C) $5x - \frac{5}{x} + C$

(D) $5 \arctan x + C$

(E) $5 \ln(1+x^2) + C$

$$5 \int \frac{1}{1+x^2} dx$$

$$= 5 \tan^{-1} x + C$$

3)

$$\int_1^2 \frac{1}{x^2} dx =$$

1998 AB 3

- (A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

$$\begin{aligned} & \int_1^2 x^{-2} dx \\ &= -x^{-1} \Big|_1^2 \\ &= -\frac{1}{2} + 1 \\ &= \frac{1}{2} \end{aligned}$$

4)

$$\int_1^{500} (13^x - 11^x) dx + \int_2^{500} (11^x - 13^x) dx =$$

1993 AB 28
Calc

- (A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

5)

A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

1988
AB 3

- (A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

$$\int_0^2 e^t dt$$

$$= e^t \Big|_0^2$$

$$= e^2 - e^0$$

$$= e^2 - 1$$

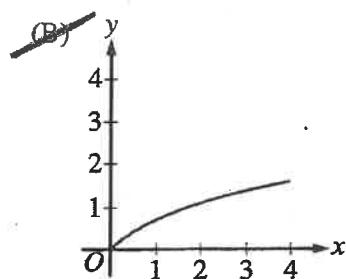
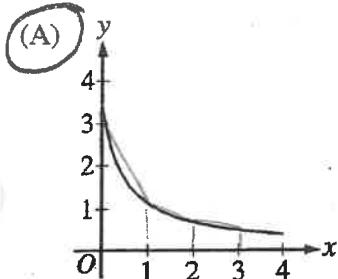
6) The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$? 1985 AB 14

- (A) 32 (B) 40 (C) 64 (D) 80 (E) 184

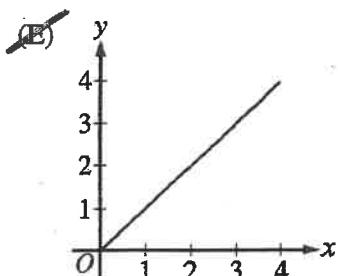
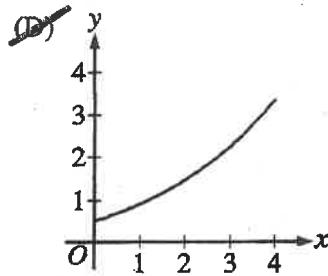
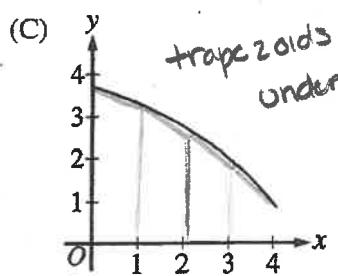
$$\begin{aligned} & \int_0^4 (3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}) dt \\ &= \frac{2}{3} 3t^{\frac{3}{2}} + \frac{2}{5} 5t^{\frac{5}{2}} \Big|_0^4 \\ &= 2\sqrt{4}^3 + 2\sqrt{4}^5 - (0) \\ &= 2(8) + 2(32) \\ &= 16 + 64 \end{aligned}$$

2003 AB 85 Calc

→ If a trapezoidal sum overapproximates $\int_0^4 f(x) dx$, and a right Riemann sum underapproximates $\int_0^4 f(x) dx$, which of the following could be the graph of $y = f(x)$?



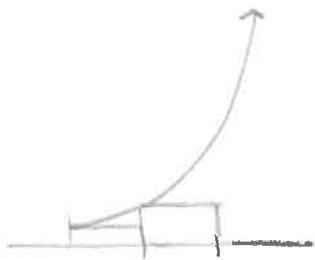
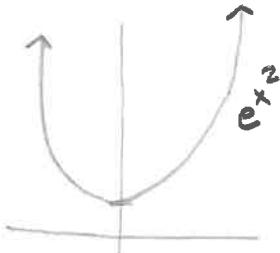
concave up
decreasing
or
concave down
decreasing



exaggerate w/ concave
down & large
width to see
trapezoidal underapproximation

inside \Rightarrow under approximate

- 8) If the definite integral $\int_0^2 e^{x^2} dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is
- (A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78



$$\frac{2-0}{2} = 1$$

$$1(e^{0^2} + e^{1^2}) \\ = 1(1 + e)$$

$$= 3.71828182846\dots$$

$$\text{Trapez} = \frac{1}{2}(e^{0^2} + e^{1^2}) + \frac{1}{2}(e^{1^2} + e^{2^2}) \\ = \frac{1}{2}(1 + e + e + e^4) = 36.517356845$$

Calc
2003 AB 83

- 9) The velocity, in ft/sec, of a particle moving along the x -axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

- (A) 20.086 ft/sec
(B) 26.447 ft/sec
(C) 32.809 ft/sec
(D) 40.671 ft/sec
(E) 79.342 ft/sec

$$\frac{1}{3-0} \int_0^3 (e^t + te^t) dt$$

- 10) What is the average value of $y = \frac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

2008 AB 91

Calc

- (A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

$$\frac{1}{3-(-1)} \int_{-1}^3 \frac{\cos x}{x^2 + x + 2} dx$$

x	2	5	7	8
$f(x)$	10	30	40	20

1998 AB 85 Calc

The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of

$$\int_2^8 f(x) dx?$$

- (A) 110 (B) 130 (C) 160 (D) 190 (E) 210

$$\frac{3}{2}(10+30) + \frac{2}{2}(30+40) + \frac{1}{2}(40+20)$$

$$= 60 + 70 + 30$$

$$= 160$$

- 12) Let f be a continuous function on the closed interval $[0, 2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x) dx$ is

1985
AB 40

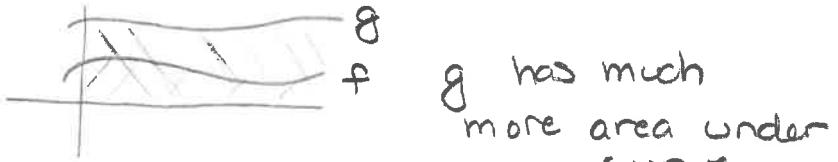
- (A) 0 (B) 2 (C) 4 (D) 8 (E) 16



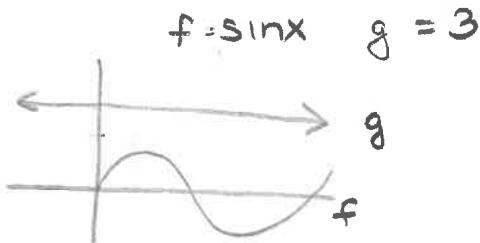
- 3) Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x , which of the following must be true?

1985 AB 38

- I. $f'(x) \leq g'(x)$ for all real x
- II. $f''(x) \leq g''(x)$ for all real x
- III. $\int_0^1 f(x) dx \leq \int_0^1 g(x) dx$ ✓



- (A) None (B) I only (C) III only (D) I and II only (E) I, II, and III



$$f' = \cos x \quad g' = 0$$

$f < g$
 $f' > g'$

14) If $\int_a^b f(x) dx = a + 2b$, then $\int_a^b (f(x) + 5) dx =$

1997 AB 3

- (A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

$$\begin{aligned}\int_a^b (f(x) + 5) dx &= \int_a^b f(x) dx + \int_a^b 5 dx \\&= a + 2b + 5(b - a) \\&= a + 2b + 5b - 5a \\&= 7b - 4a\end{aligned}$$

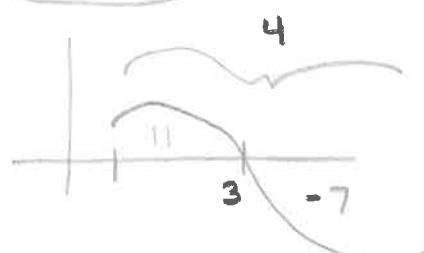
15) If $\int_1^{10} f(x) dx = 4$ and $\int_{10}^3 f(x) dx = 7$, then $\int_1^3 f(x) dx =$

1988 AB 39

- (A) -3 (B) 0 (C) 3 (D) 10 (E) 11

$$\int_1^{10} f(x) dx = 4$$

$$\int_3^{10} f(x) dx = -7$$



$$\int_1^3 f(x) dx = \int_1^{10} f(x) dx - \int_3^{10} f(x) dx$$

$$= 4 - (-7)$$

$$= 11$$

1) $\int_1^4 |x-3| dx =$

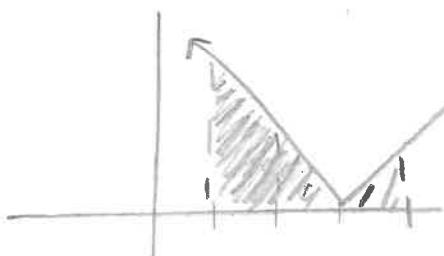
1988 AB 28

- (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{9}{2}$

(E) 5



$$\begin{aligned} \int_{-1}^4 |x-3| dx &= \frac{1}{2} (2)(2) + \frac{1}{2} (1)(1) \\ &= 2 + \frac{1}{2} \\ &= \frac{5}{2} \end{aligned}$$

If $\int_{-1}^1 e^{-x^2} dx = k$, then $\int_{-1}^0 e^{-x^2} dx =$

1985 AB 9

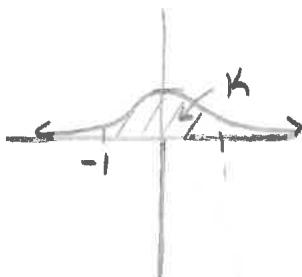
(A) $-2k$

(B) $-k$

(C) $-\frac{k}{2}$

(D) $\frac{k}{2}$

(E) $2k$



even function

$$f(x) = e^{-x^2}$$

$$\begin{aligned} f(-x) &= e^{-(-x)^2} \\ &= e^{-x^2} \end{aligned}$$

$$\int_{-1}^0 e^{-x^2} dx = \int_0^1 e^{-x^2} dx$$

18) $\frac{d}{dx} \int_0^x \cos(2\pi u) du$ is

1993 AB 41 calc

- (A) 0 (B) $\frac{1}{2\pi} \sin x$ (C) $\frac{1}{2\pi} \cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi \cos(2\pi x)$

19) If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

1998 AB 15

- (A) -3 (B) -2 (C) 2 (D) 3 (E) 18

$$F'(x) = \sqrt{x^3 + 1}$$

$$F'(2) = \sqrt{2^3 + 1}$$

$$= \sqrt{9}$$

$$= 3$$

20) $\frac{d}{dx} \left(\int_0^{x^2} \sin(t^3) dt \right) =$

2003 AB 23

- (A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x \sin(x^3)$ (E) $2x \sin(x^6)$

Solutions

I. Find the following integrals.

$$1. \int (5x^2 - 8x + 5)dx = \boxed{\frac{5x^3}{3} - 4x^2 + 5x + C}$$

$$2. \int (-6x^3 + 9x^2 + 4x - 3)dx = \boxed{\frac{-3x^4}{2} + 3x^3 + 2x^2 - 3x + C}$$

$$3. \int (x^{\frac{3}{2}} + 2x + 3)dx = \boxed{\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x + C}$$

$$4. \int \left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3} \right) dx = \int \left(\frac{8}{x} - 5x^{-2} + 6x^{-3} \right) dx \\ = 8\ln(x) - \frac{5x^{-1}}{-1} + \frac{6x^{-2}}{-2} = \boxed{8\ln(x) + \frac{5}{x} - \frac{3}{x^2} + C}$$

$$5. \int (\sqrt{x} + \frac{1}{3\sqrt{x}})dx = \int \left(x^{\frac{1}{2}} + \frac{1}{3}x^{-\frac{1}{2}} \right) dx \\ = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{3} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}x^{\frac{1}{2}} + C}$$

$$6. \int (12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx = \boxed{\frac{48x^{\frac{7}{4}}}{7} - \frac{27x^{\frac{8}{3}}}{8} + C}$$

$$7. \int \frac{x^2 + 4}{x^2} dx = \int 1 + 4x^{-2} dx = \boxed{x - \frac{4}{x} + C}$$

$$8. \int \frac{1}{x\sqrt{x}} dx = \int x^{-\frac{3}{2}} dx = \boxed{-\frac{2}{\sqrt{x}} + C}$$

$$9. \int (1 + 3t)t^2 dt = \int t^2 + 3t^3 dt = \boxed{\frac{t^3}{3} + \frac{3t^4}{4} + C}$$

$$10. \int (2t^2 - 1)^2 dt = \int 4t^4 - 4t^2 + 1 dt = \boxed{\frac{4t^5}{5} - \frac{4t^3}{3} + t + C}$$

$$11. \int y^2 \sqrt[3]{y} dy = \int y^{\frac{7}{3}} dy = \boxed{\frac{3y^{\frac{10}{3}}}{10} + C}$$

$$12. \int d\theta = \boxed{\theta + C}$$

$$13. \int 7 \sin(x) dx = \boxed{-7 \cos(x) + C}$$

$$14. \int 5 \cos(\theta) d\theta = \boxed{5 \sin(\theta) + C}$$

$$15. \int 9 \sin(3x) dx = \boxed{-3 \cos(3x) + C}$$

$$16. \int 12 \cos(4\theta) d\theta = \boxed{3 \sin 4\theta + C}$$

$$17. \int 7 \cos(5x) dx = \boxed{\frac{7 \sin(5x)}{5} + C}$$

$$18. \int 4 \sin\left(\frac{x}{3}\right) dx = \boxed{-12 \cos\left(\frac{x}{3}\right) + C}$$

$$19. \int 4e^{-7x} dx = \boxed{-\frac{4e^{-7x}}{7} + C}$$

$$20. \int 9e^{\frac{x}{4}} dx = \boxed{36e^{\frac{x}{4}} + C}$$

$$21. \int -5 \cos \pi x dx = \boxed{-\frac{5 \sin(\pi x)}{\pi} + C}$$

$$22. \int -13e^{6t} dt = \boxed{-\frac{13e^{6t}}{6} + C}$$

II. Evaluate the following definite integrals.

$$1. \int_1^4 (5x^2 - 8x + 5) dx = \left(\frac{5x^3}{3} - 4x^2 + 5x \right) \Big|_1^4 = \frac{188}{3} - \frac{8}{3} = \boxed{60}$$

$$2. \int_1^9 (x^{\frac{5}{2}} + 2x + 3) dx = \left(\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x \right) \Big|_1^9 = \frac{1026}{5} - \frac{22}{5} = \boxed{\frac{1001}{5}} = 200.2$$

$$3. \int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}}) dx = \left(\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}x^{\frac{1}{2}} \right) \Big|_4^9 = 20 - \frac{20}{3} = \boxed{\frac{40}{3}} = 13.333$$

$$4. \int_1^4 \frac{5}{x^3} dx = -\frac{5}{2x^2} \Big|_1^4 = -\frac{5}{32} + \frac{5}{2} = \boxed{\frac{75}{32}} = 2.344$$

$$5. \int_{-1}^2 (1+3t)t^2 dt = \left(\frac{t^3}{3} + \frac{3t^4}{4} \right) \Big|_{-1}^2 = \frac{44}{3} - \frac{5}{12} = \boxed{\frac{57}{4}} = 14.25$$

$$6. \int_{-2}^1 (2t^2 - 1)^2 dt = \left(\frac{4t^5}{5} - \frac{4t^3}{3} + t \right) \Big|_{-2}^1 = \frac{7}{15} + \frac{254}{15} = \boxed{\frac{87}{5}} = 17.4$$

Riemann Sum Extra Practice

The rate that people are entering a local office is given below in people/hour. Use the table to answer questions 1-3.

Time (hours)	0	1	3	4	7
r'(t) ppl/hr	12	7	3	5	8

1. Use a left Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \leq t \leq 7$.

$$1(12) + 2(7) + 1(3) + 3(5)$$

$$= 12 + 14 + 3 + 15$$

$$= 44$$

2. Use a right Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \leq t \leq 7$.

$$1(7) + 2(3) + 1(5) + 3(8)$$

$$= 7 + 6 + 5 + 24$$

$$= 42$$

3. Use a trapezoidal approximation with 4 subintervals to approximate the total number of people entering the office over the interval $0 \leq t \leq 7$.

$$\frac{1}{2}(12+7) + \frac{2}{2}(7+3) + \frac{1}{2}(5+3) + \frac{3}{2}(5+8)$$

$$= \frac{19}{2} + 10 + 4 + \frac{39}{2}$$

$$= 43$$

Gasoline is being pumped into a car. The rate that the gas is being pumped is given in the table below at selected times (seconds). Use the table to answer questions 4-6.

Time (sec)	0	4	8	12	16	20	24
$g'(t)$ gal/sec	0	.34	.42	.56	.45	.34	.22

↑

midpoint of intervals

[0, 8]

[8, 16]

[16, 24]

4. Use a right Riemann Sum with 3 subintervals to approximate the total gallons of gasoline pumped in the car over the 24 seconds.

$$\frac{24-0}{3} = 8$$

$$8(0.42) + 8(0.45) + 8(0.22) \\ = 8.72$$

5. Use a midpoint Riemann Sum with 3 subintervals to approximate the total gallons of gasoline pumped in the car over the 24 seconds.

$$8(0.34) + 8(0.56) + 8(0.34) \\ = 9.92$$

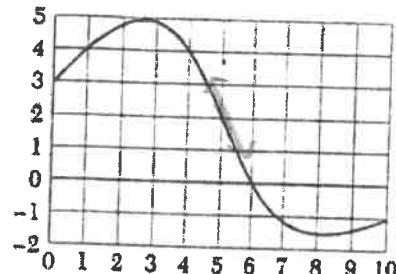
6. Use a left Riemann Sum approximation with 3 subintervals to approximate the total gallons of gasoline pumped in the car over the 24 seconds.

$$8[0 + 0.42 + 0.45] \\ = 6.96$$

7.

List, from smallest to largest:

Graph of h



iii) $\int_0^{10} h(x)dx$

iv) $\int_0^5 h(x)dx$

v) $\int_6^{10} h(x)dx$

vi) $\int_5^6 h(x)dx$

$$\int_6^{10} h(x)dx$$

$$\int_5^6 h(x)dx$$

$$\int_0^{10} h(x)dx$$

$$\int_0^5 h(x)dx$$

