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Group

Date

Exploration 8: Continuous and Discontinuous Functions

Objective: Given a function specified by two different rules, make the function continuous at the boundary between the two branches.

Let f be the function defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x < 2 \\ k(x - 5)^2, & \text{if } x \ge 2 \end{cases}$$

where k stands for a constant.

1. Plot the graph of f for k = 1. Sketch the result.

2. Function f is **discontinuous** at x = 2. Tell what it means for a function to be discontinuous.

3. Find $\lim_{x\to 2^{-}} f(x)$ and $\lim_{x\to 2^{+}} f(x)$. (The second limit will be in terms of k.) What must be true of these two limits for f to be **continuous** at x = 2?

4. Find the value of k that makes f continuous at x = 2. Sketch the graph of f for this value of k.

5. The graph in Problem 4 has a **cusp** at x = 2. What is the origin of the word *cusp*, and why is it appropriate to use in this context?

6. Suppose someone asks, "Is f(x) increasing or decreasing at x = 2 with k as in Problem 4?" How would you have to answer that question? What, then, can you conclude about the derivative of a function at a point where the graph has a cusp?

7. What did you learn as a result of doing this Exploration that you did not know before?