## February Break HWK

Name

1. If $b$ and $t$ are real numbers such that $0 < |t| < |b|$, which of the following infinite series has sum $\frac{1}{b^2+t^2}$ ?

(A) $\frac{1}{b^2}\sum_{k=0}^{\infty}\left(\frac{t^2}{b^2}\right)^k$

(B) $\frac{1}{b^2}\sum_{k=0}^{\infty}(-1)^k\left(\frac{t^2}{b^2}\right)^k$

(C) $b^2\sum_{k=0}^{\infty}\left(\frac{t^2}{b^2}\right)^k$

(D) $b^2\sum_{k=0}^{\infty}(-1)^k\left(\frac{t^2}{b^2}\right)^k$

2.

| $x$ | $g(x)$ | $g'(x)$ | $g''(x)$ | $g'''(x)$ | $g^{(4)}(x)$ |
|---|---|---|---|---|---|
| $-3$ | 1 | $-2$ | $-4$ | 2 | 16 |

Selected values of a function $g$ and its first four derivatives are shown in the table above. What is the approximation for the value of $g(-2)$ obtained by using the third-degree Taylor polynomial for $g$ about $x = -3$ ?

(A) $-\frac{8}{3}$

(B) $-\frac{7}{3}$

(C) $-2$

(D) $-3$

3. Let $T_3(x)$ be the third-degree Taylor polynomial for $f(x) = x^3$ about $x = 2$. Which of the following statements is true?

## February Break HWK

(A) $T_3(x) = 8 + 12(x-2) + 12(x-2)^2 + 6(x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ only for values of $x$ that are close to $x = 2$.

(B) $T_3(x) = 8 + 12(x-2) + 12(x-2)^2 + 6(x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ for all real numbers $x$.

(C) $T_3(x) = 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ only for values of $x$ that are close to $x = 2$.

(D) $T_3(x) = 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ for all real numbers $x$.

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4. Let $f$ be the function defined by $f(x) = \sqrt{x}$. What is the approximation for the value of $\sqrt{3}$ obtained by using the second-degree Taylor polynomial for $f$ about $x = 4$?

(A) $\frac{55}{32}$

(B) $\frac{111}{64}$

(C) $\frac{143}{64}$

(D) $\frac{167}{64}$

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5. What is the value of $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$?

(A) $-2$

(B) $-\frac{2}{5}$

(C) $\frac{3}{5}$

(D) 3

(E) The series diverges.

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6. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

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calc 7. If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$ , then $f(1)$ is

## February Break HWK

(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426

(E) 3.426

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**8.** To what number does the series $\sum_{k=0}^{\infty}\left(\frac{-e}{\pi}\right)^{k}$ converge?

(A) 0

(B) $\frac{-e}{\pi+e}$

(C) $\frac{\pi}{\pi+e}$

(D) The series does not converge.

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**9.** Consider the series $\sum_{n=1}^{\infty} a_n$ . If $a_1$ = 16 and $(a_{n+1}/a_n) = 1/2$ for all integers n ≥ 1, then $\sum_{n=1}^{\infty} a_n$ is

(A) 0

(B) 2

(C) 17

(D) 32

(E) divergent

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**10.** Let $f$ be a function with $f(3) = 2$, and $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for $f$ about $x = 3$?

(A) $2 - (x - 3) + 3(x - 3)^2 + 2(x - 3)^3$

(B) $2 - (x - 3) + 3(x - 3)^2 + 4(x - 3)^3$

(C) $2 - (x - 3) + 6(x - 3)^2 + 12(x - 3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

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**11.** Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function $f$ about $x = 0$. What is the value of $f^{(4)}(0)$ ?

## February Break HWK

(A) 0

(B) $\frac{1}{4}$

(C) 6

(D) 24

(E) 144

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calc

12. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function *f* about *x* = 0. What is the value of $f'''(0)$?

(A) -30

(B) -15

(C) -5

(D) $-\frac{5}{6}$

(E) $-\frac{1}{6}$

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13. The third-degree Taylor polynomial for a function *f* about $x = 4$ is $\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2$. What is the value of $f'''(4)$ ?

(A) $-\frac{1}{64}$

(B) $-\frac{1}{32}$

(C) $\frac{1}{512}$

(D) $\frac{3}{256}$

(E) $\frac{81}{256}$

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calc 14. The $n$th derivative of a function $f$ at $x=0$ is given by $f^{(n)}(0) = (-1)^n \frac{n+1}{(n+2)2^n}$ for all $n \geq 0$ . Which of the following is the Maclaurin series for $f$ ?

(A) $-\frac{1}{2} + \frac{1}{3}x - \frac{3}{32}x^2 + \frac{1}{60}x^3 - \ldots$

(B) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{16}x^2 + \frac{1}{10}x^3 + \ldots$

(C) $\frac{1}{2} + \frac{1}{3}x + \frac{3}{32}x^2 + \frac{1}{60}x^3 + \ldots$

(D) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2 - \frac{1}{60}x^3 + \ldots$

(E) $\frac{1}{2} - 3x + \frac{32}{3}x^2 - 60x^3 + \ldots$

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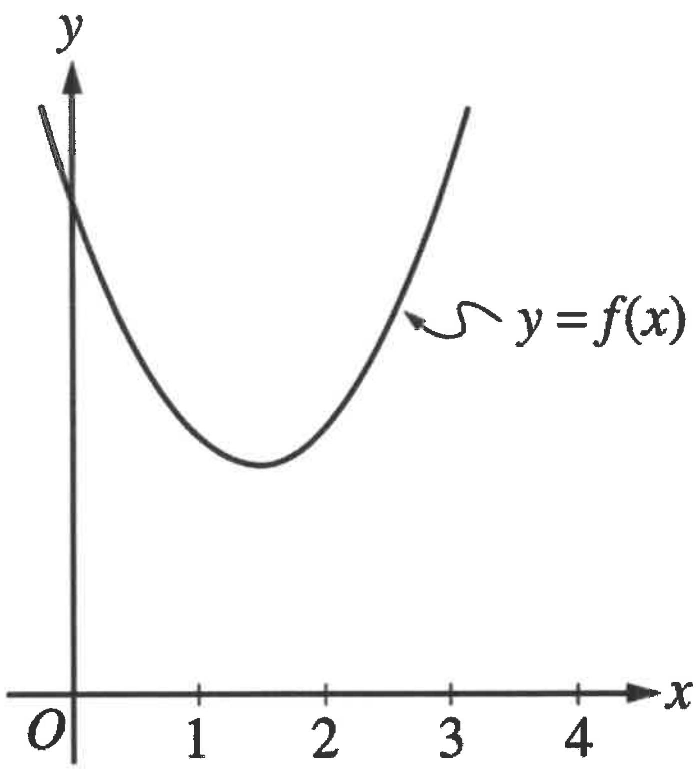
## February Break HWK

**15.**




The figure above shows the graph of a function f. Which of the following could be the second-degree Taylor polynomial for f about x = 2 ?

(A) $2-x-x^2$

(B) $2+x-x^2$

(C) $2-(x-2)+(x-2)^2$

(D) $2+(x-2)-(x-2)^2$

(E) $2+(x-2)+(x-2)^2$

## February Break HWK

calc **16.** Let $f$ be a function with $f(0) = 1$, $f'(0) = 2$, and $f''(0) = -2$. Which of the following could be the graph of the second-degree Taylor polynomial for $f$ about $x = 0$?

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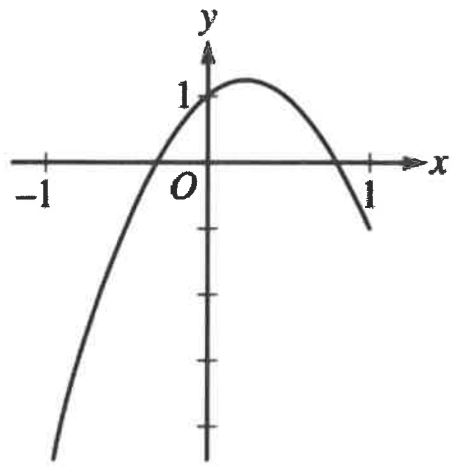


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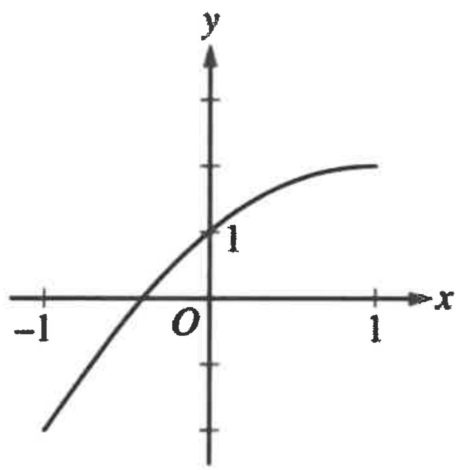
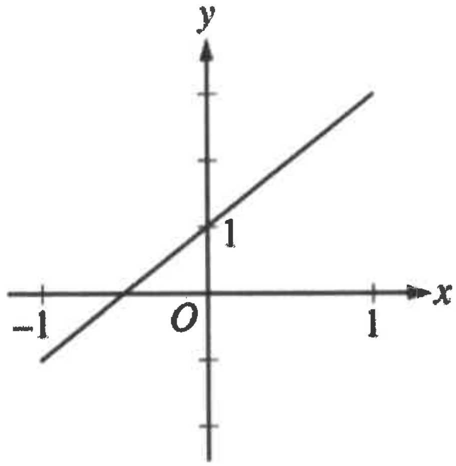
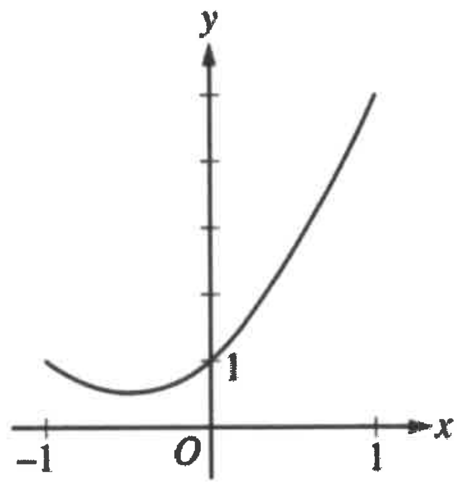


(C)




(D)




calc 17. Let $f$ be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for $f$ about $x = 3$ ?

(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(E) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

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calc 18. The function f has derivatives of all orders for all real numbers with *f(0) = 3, f'(0) = -4, f''(0) = 2*, and *f'''(0) = 1*. Let *g* be the function given by $g(x) = \int_0^x f(t)\,dt$. What is the third-degree Taylor polynomial for *g* about *x = 0* ?

(A) $-4x + 2x^2 + \frac{1}{3}x^3$

(B) $-4x + x^2 + \frac{1}{6}x^3$

(C) $3x - 2x^2 + \frac{1}{3}x^3$

(D) $3x - 2x^2 + \frac{2}{3}x^3$

(E) $3 - 4x + x^2 + \frac{1}{6}x^3$

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19. What is the radius of convergence of the Maclaurin series for $\frac{2x}{1+x^2}$?

(A) 1/2

(B) 1

(C) 2

(D) infinite

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**20.** What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$?

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) 4

(E) 6

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**21.** If the power series $\sum_{n=0}^{\infty} a_n (x-4)^n$ converges at $x = 7$ and diverges at $x = 9$, which of the following must be true?

I. The series converges at $x = 1$.

II. The series converges at $x = 2$.

III. The series diverges at $x = -1$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

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22. A function $f$ has Maclaurin series given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$ .Which of the following is an expression forf(x) ?

(A) $\cos x$

(B) $e^x - \sin x$

(C) $e^x + \sin x$

(D) $\frac{1}{2}(e^x + e^{-x})$

(E) $e^{x^2}$

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23. Which of the following is the Maclaurin series for the function $f$ defined by $f(x) = 1 + x^2 + \cos x$ ?

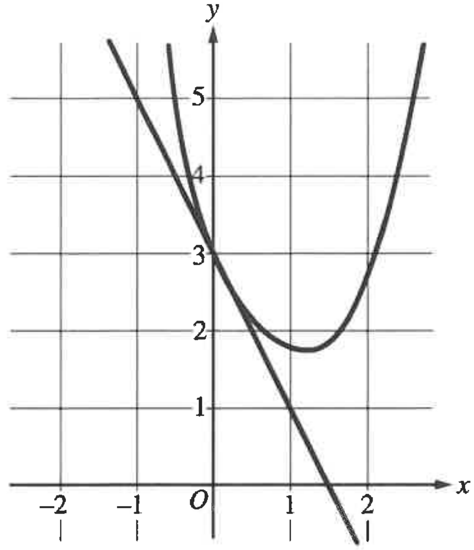
## February Break HWK

(A) $2+\frac{x^2}{2}+\frac{x^4}{24}+\cdots$

(B) $2+\frac{3x^2}{2}+\frac{x^4}{24}+\cdots$

(C) $1+x+x^2-\frac{x^3}{6}+\cdots$

(D) $2+x+\frac{3x^2}{2}+\frac{x^3}{6}+\cdots$




| $n$ | $f^{(n)}(0)$ |
|---|---|
| 2 | 3 |
| 3 | $-\frac{23}{2}$ |
| 4 | 54 |

6. A function $f$ has derivatives of all orders for all real numbers $x$. A portion of the graph of $f$ is shown above, along with the line tangent to the graph of $f$ at $x = 0$. Selected derivatives of $f$ at $x = 0$ are given in the table above.

(a) Write the third-degree Taylor polynomial for $f$ about $x = 0$.

(b) Write the first three nonzero terms of the Maclaurin series for $e^x$. Write the second-degree Taylor polynomial for $e^x f(x)$ about $x = 0$.

(c) Let $h$ be the function defined by $h(x) = \int_0^x f(t)\,dt$. Use the Taylor polynomial found in part (a) to find an approximation for $h(1)$.

(d) It is known that the Maclaurin series for $h$ converges to $h(x)$ for all real numbers $x$. It is also known that the individual terms of the series for $h(1)$ alternate in sign and decrease in absolute value to 0. Use the alternating series error bound to show that the approximation found in part (c) differs from $h(1)$ by at most 0.45.

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**STOP**

**END OF EXAM**

6. The Maclaurin series for $\ln(1 + x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1 + x)$. Let $f$ be the function defined by $f(x) = x\ln\left(1 + \frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for $f$.

(b) Determine the interval of convergence of the Maclaurin series for $f$. Show the work that leads to your answer.

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for $f$ about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

**STOP**

**END OF EXAM**

$$f(0) = 0$$
$$f'(0) = 1$$
$$f^{(n+1)}(0) = -n \cdot f^{(n)}(0) \text{ for all } n \geq 1$$

6. A function $f$ has derivatives of all orders for $-1 < x < 1$. The derivatives of $f$ satisfy the conditions above. The Maclaurin series for $f$ converges to $f(x)$ for $|x| < 1$.

(a) Show that the first four nonzero terms of the Maclaurin series for $f$ are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for $f$.

(b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

(c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)\, dt$.

(d) Let $P_n\left(\frac{1}{2}\right)$ represent the $n$th-degree Taylor polynomial for $g$ about $x = 0$ evaluated at $x = \frac{1}{2}$, where $g$ is the function defined in part (c). Use the alternating series error bound to show that

$$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}.$$

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**STOP**

**END OF EXAM**