Let's See What We Remember!

1. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

$f'(x) = \begin{cases} 2x, & x<3 \\ 6, & x\geq 3 \end{cases}$

a. Undefined
b. Continuous but not differentiable
c. Differentiable but not continuous
d. Neither continuous nor differentiable
e. Both continuous and differentiable

$\lim_{x\to 3^-} f(x) = 9 \quad \lim_{x\to 3^+} f(x) = 9$

$f(3) = 9$

continuous ✓

$\lim_{x\to 3^-} f'(x) = 6 \quad \lim_{x\to 3^+} f'(x) = 6$

diff ✓

2. For what value of k, if any, is f continuous at $x = 3$? Justify your answer.

$$f(x) = \begin{cases} \frac{2x^2+5x-3}{x^2+4x+3} & \text{for } x < -3 \\ kx + \frac{1}{2} & \text{for } -3 \leq x \leq 0 \\ \frac{2^x}{3^x-1} & \text{for } x > 0 \end{cases}$$

$\frac{(2x-1)(x+3)}{(x+3)(x+1)}$

$\frac{2x-1}{x+1}$

$\lim_{x\to -3^-} f(x) = \frac{2(-3)-1}{-3+1} = \frac{-7}{-2} = \frac{7}{2}$

$\lim_{x\to -3^+} f(x) = k(-3) + 1/2$

$\frac{7}{2} = -3k + \frac{1}{2}$

$3 = -3k$

$\boxed{k = -1}$

3. Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true?

I. f is continuous at x=0. ✓

II. f is differentiable at x=0. no corner

III. f has an absolute minimum at x=0. ✓

a. I only
b. II only
c. III only
d. I and III
e. II and III

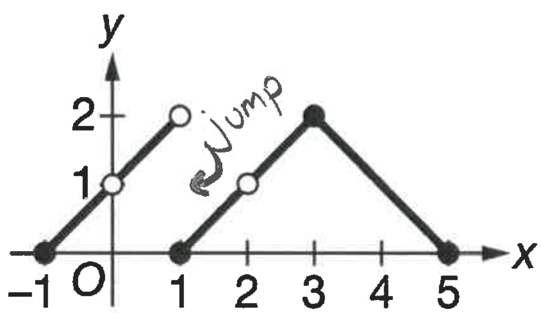
4. The function f has the properties indicated in the table below. Which of the following must be true.

a	$\lim_{x \to a^-} f(x)$	$\lim_{x \to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

~~a.~~ f is continuous at $x = -1$
~~b.~~ f is continuous at $x = 0$
c. f is continuous at $x = 1$ ✓
~~d.~~ f is differentiable at $x = 0$ no not cont
e. f is differentiable at $x = 1$

cant guarantee not a corner, cusp, vertical tangent

5. The graph of the function f is shown below. What are all values of x for which f has a removable discontinuity?



Graph of f

a. 0 only
b. 1 only
c. 0 and 2 only
d. 0, 1, and 2 only

6. Let f be the function defined by $f(x) = \frac{x^4-4x^2}{x^2-4x}$. Which of the following statements are true?

~~a.~~ f has a discontinuity due to a vertical asymptote at $x = 0$ and $x = 4$.
b. f has a removable discontinuity at $x = 0$ and ~~a jump discontinuity~~ at $x = 4$
c. f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 4$
d. f is ~~continuous~~ at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 4$.

$$f(x) = \frac{x^2(x^2-4)}{x(x-4)} = \frac{x^2(x-2)(x+2)}{x(x-4)} = \frac{x(x-2)(x+2)}{x-4}$$

$x \neq 0$

7. A student attempted to confirm that the function f defined by $f(x) = \frac{x^2+x-6}{x^2-7x+10}$ is continuous at $x = 2$. In which step, if any does an error first appear?

Step 1: $f(x) = \frac{x^2+x-6}{x^2-7x+10} = \frac{(x-2)(x+3)}{(x-2)(x-5)}$ ✓

Step 2: $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \frac{x+3}{x-5} = \frac{2+3}{2-5} = -\frac{5}{3}$

Step 3: $\cancel{f(2) = \frac{2+3}{2-5} = -\frac{5}{3}}$ $f(2) = \text{DNE}$ removable discontinuity

Step 4: $\lim\limits_{x\to 2} f(x) = f(2)$, so f is continuous at $x = 2$.

a. Step 2
b. Step 3
c. Step 4
d. There is no error