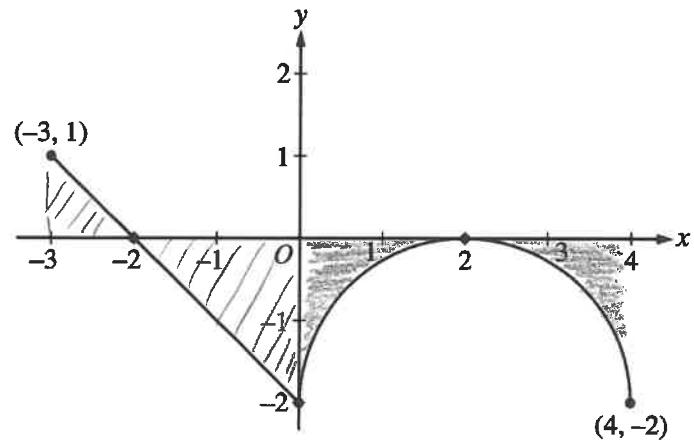
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

initial condition
* if they give you a point
in directions circle
and note
FTC

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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a) If $f' > 0$ than f inc

$+ \quad - \quad -$ f'
$-3 \quad -2 \quad 2 \quad 4$

f is increasing from $[-3,-2]$
b/c $f' > 0$ when $-3 \le x < -2$

b) f' slope $= 0$ or und. and changes sign

$- \quad + \quad -$ f''
$0 \quad 2$

f' changes from decreasing to increasing at $x=0$ and increasing to decreasing at $x=2$ so $x=0$ and $x=2$ are point of inflections

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c) $(0,3)$ $f'(0) = -2$

$$y - 3 = -2(x - 0)$$

$$y = -2x + 3$$

d) $$\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$$

$$\int_{-3}^{0} f'(x)\,dx = 3 - f(-3)$$ * initial condition *

$$f(-3) = 3 - \int_{-3}^{0} f'(x)\,dx$$

Geo for area

$$= 3 - \left[\tfrac{1}{2}(1)(1) - \tfrac{1}{2}(2)(2)\right]$$

$$= 3 - \tfrac{1}{2} + 2$$

$$= \frac{9}{2}$$

$$\boxed{f(-3) = 9/2}$$

$$\int_{0}^{4} f'(x)\,dx = f(4) - f(0)$$

$$\int_{0}^{4} f'(x)\,dx = f(4) - 3$$

$$3 + \int_{0}^{4} f'(x)\,dx = f(4)$$

rectangle-semicircle

change to neg b/c area under x-axis

$$3 - \left[4(2) - \tfrac{1}{2}\pi(2)^2\right] = f(4)$$

$$3 - (8 - 2\pi) = f(4)$$

$$\boxed{-5 + 2\pi = f(4)}$$

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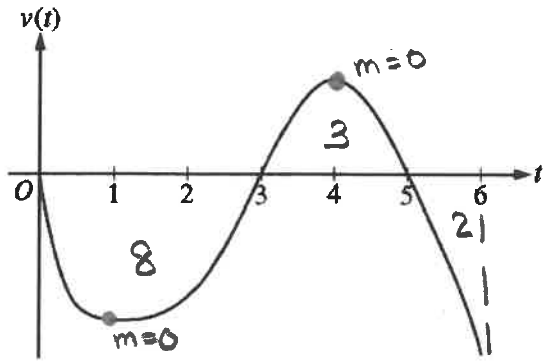

Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$. initial condition

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

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a) $\underset{3}{-} \;\; \underset{5}{+} \;\; - \quad v(t)$

$\int_0^3 v(t)\,dt = x(3) - x(0)$

$-8 = x(3) - (-2)$

$-10 = x(3)$

$\int_0^5 v(t)\,dt = x(5) - x(0)$

$-8+3 = x(5) - (-2)$

$-7 = x(5)$

$\int_0^6 v(t)\,dt = x(6) - x(0)$

$-8+3-2 = x(6) - (-2)$

$-9 = x(6)$

The particle is farthest to the left at time $t=3$, the particle is at $x=-10$

back →

b) IVT applies b/c $v(t)$ and $x(t)$ are continuous.

Since $x(0) = -2$ and $x(3) = -10$ there exists a value c, where $0 < c < 3$ such that $x(c) = -8$.

Since $x(3) = -10$ and $x(5) = -7$ there exists a value c, where $3 < c < 5$ such that $x(c) = -8$

Since $x(5) = -7$ and $x(6) = -9$ there exists a value c, where $5 < c < 6$ such that $x(c) = -8$

c) $2 < t < 3$

$v(t) < 0$

$a(t) > 0$

Since velocity is negative and acceleration (velocity is increasing) is positive when time is $2 < t < 3$ the speed is decreasing on the interval $(2, 3)$.

*vel & acc have opposite signs

d) vel slope neg → acc neg

$$\begin{array}{cccc} - & + & - & a(t) \\ & 1 & 4 & \end{array}$$

the acceleration of the particle is negative on the intervals $(0, 1)$ and $(4, 6)$ b/c velocity is decreasing (slope of velocity negative) on these intervals

x	2	3	5	8	13
$f(x)$	1	4	−2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

   (a) Estimate $f'(4)$. Show the work that leads to your answer.

   (b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

   (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

   (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

a) $f'(4) \approx \frac{-2-4}{5-3}$

$= \frac{-6}{2}$

$f'(4) \approx -3$

b) $\int_2^{13} (3-5f'(x))dx = \int_2^{13} 3 - 5\int_2^{13} f'(x)dx$ * FTC

$= 3(11) - 5[f(13) - f(2)]$

$= 33 - 5[6-1]$

$= 33 - 25$

$= 8$

c) $\int_2^{13} f(x)dx \approx 1(1) + 2(4) + 3(-2) + 5(3)$

$= 1 + 8 - 6 + 15$

$= 18$

d) $f'(5) = 3$ $(5,-2)$

$y + 2 = 3(x-5)$

$y = 3x - 17$

Since $f''(x) < 0$ the function is concave down so the tangent line at $x=5$ lies above the graph of $y=f(x)$ thus $f(7) \le 4$.

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continued)

secant line:

$(5, -2)$ $(8, 3)$

$$m = \frac{3+2}{8-5} = \frac{5}{3}$$

$$y + 2 = 5/3\,(x-5)$$

$$y = -2 + 5/3(x-5)$$

Since $f''(x) < 0$, $f(x)$ is concave down on the interval $5 \le x \le 8$ so the secant line lies below the function $y = f(x)$.

$$y(7) = -2 + 5/3\,(7-5)$$

$$y(7) = -2 + \frac{10}{3}$$

$$= \frac{4}{3}$$

The secant line approximate $f(7)$ to be $4/3$. The secant line lies below the function since the secant line

$f(7) \ge 4/3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

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a) $h(1) = f(g(1)) - 6$
$= f(2) - 6$
$= 9 - 6$
$= 3$

$h(3) = f(g(3)) - 6$
$= f(4) - 6$
$= -1 - 6$
$= -7$

IVT applies
since $h(1) = 3$ and $h(3) = -7$, there exists a value r where $1 < r < 3$ such that $h(r) = -5$

b) $m_{secant} = \frac{-7 - 3}{3 - 1}$
$= \frac{-10}{2}$
$= -5$

Since the slope of the secant line from $(1, 3)$ to $(3, -7)$ is -5 by Mean Value Theorem there exists a value c where $1 < c < 3$ such that $h'(c) = -5$

c) $w(x) = \int_1^{g(x)} f(t)\,dt$

$w'(x) = f(g(x))\, g'(x)$

$w'(3) = f(g(3))\, g'(3)$

$= f(4)\,(2)$

$= (-1)(2)$

$= -2$

d) $(g^{-1}(x))' = \dfrac{1}{g'(g^{-1}(2))}$ $\quad g(1) = 2 \quad g'(2) = 1$

$= \dfrac{1}{g'(1)}$

$= \dfrac{1}{5}$

$y - 1 = 1/5\,(x - 2)$