$(-\sqrt{1/2})^4 = (-\sqrt{1/2})^2 - x^2$ $\quad$ $1/4 = x^2$ $\quad$ $\pm 1/2 = x$

$(\sqrt{1/2})^4 = (\sqrt{1/2})^2 - x^2$ $\quad$ $1/4 = 1/2 - x^2$ $\quad$ $1/4 = x^2$ $\quad$ $\pm 1/2$

4. Determine the point(s) at which the graph of $y^4 = y^2 - x^2$ has either a horizontal or vertical tangent. Be sure to label which is which, if either exist.

$4y^3\, dy/dx = 2y\, dy/dx - 2x$

$\frac{dy}{dx} = \frac{2x}{2y-4y^3}$

$= \frac{x}{y-2y^3}$

horizontal → $m=0$

$\boxed{(0,0)\ (0,1)\ (0,-1)}$

$0 = \frac{dy}{dx} = \frac{x}{y-2y^3}$

$x=0$ $\quad$ $y^4 = y^2 - 0$ $\quad$ $y^2(y^2-1)=0$ $\quad$ $y=0,\pm 1$

$\boxed{(0,0)\ (\pm 1/2, \pm\sqrt{1/2})}$ vertical → den $=0$

$0 = y - 2y^3$ $\quad$ $y = 0, \pm\sqrt{1/2}$

$0 = y(1-2y^2)$

5. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

$x^2 + x(0) + 0^2 = 7$

$x = \pm\sqrt{7}$

$x=\sqrt{7}$

$(\sqrt{7})^2 + \sqrt{7}y + y^2 = 7$

$y(y+\sqrt{7}) = 0$

$y=0,\ y=-\sqrt{7}$

$(\sqrt{7},0)\quad(\sqrt{7},-\sqrt{7})$

$x=-\sqrt{7}$

$(-\sqrt{7})^2 - \sqrt{7}y + y^2 = 7$

$-\sqrt{7}y + y^2 = 0$

$y(y-\sqrt{7}) = 0$

$y=0\quad y=\sqrt{7}$

$(-\sqrt{7},0)\quad(-\sqrt{7},\sqrt{7})$

$2x + y + x\,dy/dx + 2y\,dy/dx = 0$

$dy/dx = \frac{-2x-y}{x+2y}$

$dy/dx\Big|_{(\sqrt{7},0)} = \frac{-2\sqrt{7}-0}{\sqrt{7}+2(0)}$

$= -2$

$dy/dx\Big|_{(-\sqrt{7},0)} = \frac{2\sqrt{7}-0}{-\sqrt{7}-0}$

$= -2$

$\boxed{\text{The x-int are } (\sqrt{7},0) \text{ and } (-\sqrt{7},0) \text{ w/ a common slope of } -2}$

6. Find the equations of the normal lines to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$

① $2x + y = 0$

$2 + dy/dx = 0$

$dy/dx = -2$

* for normal lines to be parallel, slope $=-2$

Find points for line w/ $1/2$

② $xy + 2x - y = 0$

$y + x\,dy/dx + 2 - dy/dx = 0$

$\frac{dy}{dx} = \frac{-y-2}{x-1}$

$\frac{1}{2} = \frac{-y-2}{x-1}$

$x - 1 = -2y - 4$

$x = -2y - 3$

③ when $x = -2y-3$

$(-2y-3)y + 2(-2y-3) - y = 0$

$-2y^2 - 3y - 4y - 6 - y = 0$

$2y^2 + 8y + 6 = 0$

$y^2 + 4y + 3 = 0$

$(y+3)(y+1) = 0$

$y = -3, -1$

$y=-3$ $\quad$ $-3x + 2x + 3 = 0$ $\quad$ $x = 3$

$y=-1$ $\quad$ $-x + 2x + 1 = 0$ $\quad$ $x = 1$

④ $(3,-3)$ $\quad$ $m=-2$

$\boxed{y + 3 = -2(x-3)}$

$(1,-1)$ $\quad$ $m=-2$

$\boxed{y + 1 = -2(x-1)}$

7. If $y^2 + \cos xy - 4x = 5$, find $\frac{dx}{dy}$, yes, that's $\frac{dx}{dy}$.

$$2y\,\frac{dy}{dy} - \sin(xy)\left(\frac{dx}{dy}\cdot y + \frac{dy}{dy}\,x\right) - 4\,\frac{dx}{dy} = 0$$

$$2y - y\sin xy\,\frac{dx}{dy} - x\sin xy - 4\,\frac{dx}{dy} = 0$$

$$\frac{dx}{dy} = \frac{x\sin xy - 2y}{-4 - y\sin xy}$$

or

$$\boxed{\frac{dx}{dy} = \frac{2y - x\sin xy}{4 + y\sin xy}}$$

8. The slope of the tangent is -1 at the point $(0,1)$ on $x^3 - 6xy - ky^3 = a$, where k and a are constants. The values of the constants a and k are what?

$(0,1)$

$0^3 - 6(0)(1) - k(1)^3 = a$

$-k = a$

$-2 = a$

$$3x^2 - 6\left(y + x\,\frac{dy}{dx}\right) - 3ky^2\,\frac{dy}{dx} = 0$$

$$3x^2 - 6y - 6x\,\frac{dy}{dx} - 3ky^2\,\frac{dy}{dx} = 0$$

$(0,1)$ slope $= -1$

$$3(0)^2 - 6(1) - 6(0)(-1) - 3k(1)^2(-1) = 0$$

$-6 + 3k = 0$

$3k = 6$

$k = 2$

$$\boxed{\begin{array}{l} k = 2 \\ a = -2 \end{array}}$$