1. Determine the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n}$

* remember n is your "variable"

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(x-2)^{n+1}}{n+1}}{\frac{(-1)^n(x-2)^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{n+1} \cdot \frac{n}{(x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{(x-2)n}{n+1} \right|$$

$$= |x-2|$$

$|x-2|<1$

$-1<x-2<1$

$1<x<3$

* check endpoints

when $x=1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(1-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

diverges by p-series

when $x=3$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(3-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n(1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

converges by alt. series

$\boxed{1<x\leq 3}$

2. Determine the interval of convergence for $\sum_{n=1}^{\infty} \frac{n!x^n}{n^{10}}$

$$\lim_{n\to\infty} \left| \frac{(n+1)!\,x^{n+1}}{(n+1)^{10}} \cdot \frac{n^{10}}{n!\,x^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)x\,n^{10}}{(n+1)^{10}} \right| = \infty$$

* n greater degree in numerator

∞ never <1

converges only where centered

@ $\boxed{x=0}$

3. Consider the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n n}$

a. At which x-value is the interval of convergence centered for the power series above?

$x = -2$

b. The radius of convergence for the series above is 3. Find the interval of convergence for the power series.

$-5 < x < 1$ check endpoints

when $x = -5$

$$\sum_{n=1}^{\infty} \frac{(-5+2)^n}{3^n n}$$

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{3^n n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

converges by alt series

when $x = 1$

$$\sum_{n=1}^{\infty} \frac{(1+2)^n}{3^n n}$$

$$\sum_{n=1}^{\infty} \frac{3^n}{3^n n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges by p-series

$\boxed{-5 \le x < 1}$

4. Consider

$$\sum_{n=1}^{\infty} \frac{n}{4^n(n+1)} x^{2n}$$

a. Does the series converge for $x = 2$? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{n}{4^n(n+1)} (2)^{2n} = \sum_{n=1}^{\infty} \frac{n}{4^n(n+1)} 4^n = \sum_{n=1}^{\infty} \frac{n}{n+1}$$

diverges by n^{th} term test

$$\lim_{n\to\infty} \frac{n}{n+1} = 1$$

series $\boxed{\text{diverges}}$ @ $x = 2$

b. Based only on your answer from part a, what can you say about R, the radius of convergence of the series?

centered @ $x = 0$ and diverges @ $x = 2$

$\boxed{R \le 2}$

5. Consider

$$\sum_{n=1}^{\infty} \frac{1}{n5^n}(x-4)^{n+1}$$

a. Find the radius of convergence of the power series.

$$\lim_{n\to\infty} \left| \frac{(x-4)^{n+2}}{(n+1)\,5^{n+1}} \cdot \frac{n\,5^n}{(x-4)^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{(x-4)\,n}{(n+1)\,5} \right|$$

$$= \left| \frac{1}{5}(x-4) \right|$$

$$\left| \frac{1}{5}(x-4) \right| < 1$$

$$-1 < \frac{1}{5}(x-4) < 1$$

$$-5 < x-4 < 5$$

$$-1 < x < 9$$

$\boxed{R = 5}$

OR could find R by

$$\frac{1}{5}|x-4| < 1$$

$$|x-4| < 5$$

centered radius

b. For which values of x does the series converge absolutely? For which values of x does it converge conditionally?

$-1 < x < 9$ check endpoints

when $x = -1$

$$\sum_{n=1}^{\infty} \frac{(-5)^{n+1}}{n\,5^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n\,5}{n}$$

converges by alt. series

converges conditionally b/c $\sum_{n=1}^{\infty} \frac{5}{n}$ diverges by p-series

when $x = 9$

$$\sum_{n=1}^{\infty} \frac{5^{n+1}}{n\,5^n}$$

$$\sum_{n=1}^{\infty} \frac{5}{n}$$

diverges by p-series

interval of convergence

$-1 \le x < 9$

converges absolutely for $-1 < x < 9$
converges conditionally for $x = -1$

6. Consider

$$\sum_{n=1}^{\infty} \frac{1}{2^n\sqrt{n}}(x-5)^n$$

a. Does the series converge or diverge at $x = 3$?

$$\sum_{n=1}^{\infty} \frac{(3-5)^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-2)^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$\boxed{\text{converges}}$ by alt series @ $x = 3$

* converges conditionally

b. What does your answer from part (a) imply about the radius of convergence of the series?

centered @ $x=5$ converges @ $x = 3$

$$\boxed{R \geq 2}$$

c. Find the interval of convergence of the power series.

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{2^{n+1}\sqrt{n+1}} \cdot \frac{2^n\sqrt{n}}{(x-5)^n}\right| = \lim_{n\to\infty}\left|\frac{(x-5)\sqrt{n}}{2\sqrt{n+1}}\right| = \left|\tfrac{1}{2}(x-5)\right|$$

$\left|\frac{1}{2}(x-5)\right| < 1$

$-1 < \frac{1}{2}(x-5) < 1$

$-2 < x-5 < 2$

$3 < x < 7$

$\boxed{3 \leq x < 7}$

check endpoints

when $x = 3$

$\sum_{n=1}^{\infty} \frac{(-2)^n}{2^n\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges

when $x = 7$

$\sum_{n=1}^{\infty} \frac{2^n}{2^n\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series

7. You are given that the power series $\sum_{n=0}^{\infty} C_n(x+3)^n$ converges when $x = -6$ and diverges when $x = 1$. Write an inequality that represents the value R could be. (R represents the radius of convergence).

centered @ $x = -3$

✓ -6 -3 ✗ 1

3   4

$$\boxed{3 \leq R \leq 4}$$

8. Determine the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n(x-5)^{4n}}{n^5(16)^n}$

$$\lim_{n\to\infty} \left| \frac{(x-5)^{4(n+1)}}{(n+1)^5 \; 16^{n+1}} \cdot \frac{n^5 \; 16^n}{(x-5)^{4n}} \right| = \lim_{n\to\infty} \left| \frac{(x-5)^4 \; n^5}{16 \; (n+1)^5} \right| = \left| \frac{1}{16}(x-5)^4 \right|$$

$-1 < \frac{1}{16}(x-5)^4 < 1$

$\underbrace{-16 <}_{\text{no solutions}} (x-5)^4 < 16$

$(x-5)^4 < 16$

$|x-5| < \sqrt[4]{16}$

$-2 < x-5 < 2$

$3 < x < 7$

$\boxed{3 \le x \le 7}$

check endpoints

when $x = 3$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-2)^{4n}}{n^5 \; 16^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \; 16^n}{n^5 \; 16^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$$

converges by alt series

when $x = 7$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \; 2^{4n}}{n^5 \; 16^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \; 16^n}{n^5 \; 16^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$$

converges by alt series

9. The power series $\sum_{n=0}^{\infty} \frac{(n+2)x^n}{n^4+1}$ has radius of convergence 1. Determine the interval of convergence of this power series.

centered @ $x = 0$ $\quad R = 1$

$-1 < x < 1$

$\boxed{-1 \le x \le 1}$

when $x = -1$

$$\sum_{n=0}^{\infty} \frac{(n+2)(-1)^n}{n^4+1}$$

converges by alt series

when $x = 1$

$$\sum_{n=0}^{\infty} \frac{(n+2)\,1^n}{n^4+1}$$

$$\sum_{n=0}^{\infty} \frac{n+2}{n^4+1}$$

compare to

$$\sum_{n=0}^{\infty} \frac{n}{n^4} = \sum_{n=0}^{\infty} \frac{1}{n^3}$$

converges by p-series

$$\lim_{n\to\infty} \frac{1/n^3}{n+2/n^4+1} = \lim_{n\to\infty} \frac{n^4+1}{n^4+2n^3} = 1$$

converges by limit comparison