Sequences and Summation Notation

Writing Terms of a Sequence from the General Form

Explicit Formula

1. Write the first four terms of the sequence whose nth term, or general term, is given:

a. $a_n = 3^n$ $\quad n = 1, 2, 3, 4$

$3^1, 3^2, 3^3, 3^4$

$\boxed{3, 9, 27, 81}$

b. $a_n = \frac{3n}{n+5}$

$\frac{3(1)}{1+5}, \frac{3(2)}{2+5}, \frac{3(3)}{3+5}, \frac{3(4)}{4+5}$

$\boxed{\frac{1}{2}, \frac{6}{7}, \frac{9}{8}, \frac{4}{3}}$

Using a Recursion Formula

2. The sequences below are defined using recursion formulas. Write the first four terms of each sequence.

a. $a_1 = 12$ and $a_n = a_{n-1} + 4$ for $n \geq 2$

$a_1 = 12$

$a_2 = a_{2-1} + 4 = a_1 + 4 = 12 + 4 = 16$

$a_3 = a_{3-1} + 4 = a_2 + 4 = 16 + 4 = 20$

$a_4 = a_{4-1} + 4 = a_3 + 4 = 20 + 4 = 24$

$\boxed{12, 16, 20, 24}$

b. $a_1 = 5$ and $a_n = 3a_{n-1} - 1$ for $n \geq 2$

$a_1 = 5$

$a_2 = 3(5) - 1 = 14$

$a_3 = 3(14) - 1 = 41$

$a_4 = 3(41) - 1 = 122$

$\boxed{5, 14, 41, 122}$

Factorial Notation

If n is a positive integer, the notation $n!$ (read "n factorial") is the product of all positive integers from n down through 1. By definition, $0! = 1$.

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

3. The general term of a sequence is given and involves a factorial. Write the first four terms of each sequence.

a. $a_n = \frac{(n+1)!}{n^2}$

$\frac{(1+1)!}{1^2}, \frac{(2+1)!}{2^2}, \frac{(3+1)!}{3^2}, \frac{(4+1)!}{4^2}$

$2, \ 6/4, \ 24/9, \ 120/16$

$\boxed{2, \ 3/2, \ 8/3, \ 15/2}$

4. Evaluate each factorial expression.

a. $\frac{17!}{15!}$

$= \frac{17 \cdot 16 \cdot \cancel{15 \cdot 14 \cdot 13 \cdot 12 \cdots 2 \cdot 1}}{\cancel{15 \cdot 14 \cdot 13 \cdot 12 \cdots 2 \cdot 1}}$

$= 17 \cdot 16$

$\boxed{= 272}$

b. $\frac{20!}{2!\,18!}$

$= \frac{20 \cdot 19 \cdot \cancel{18 \cdot 17 \cdot 16 \cdots 2 \cdot 1}}{(2 \cdot 1)(\cancel{18 \cdot 17 \cdot 16 \cdots 2 \cdot 1})}$

$= \frac{20 \cdot 19}{2}$

$\boxed{= 190}$

Summation Notation

The sum of the first n terms of a sequence is represented by the **summation notation**

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

where i is the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation**. Any letter can be used for the index of summation. The letters i, j, and k are used commonly. The lower limit of summation can be an integer other than 1.

5. Find each indicated sum.

a. $\sum_{i=1}^{5} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

$= 1 + 8 + 27 + 64 + 125$

$\boxed{= 225}$

b. $\sum_{k=1}^{4} (k-3)(k+2)$

$= (1-3)(1+2) + (2-3)(2+2) + (3-3)(3+2) + (4-3)(4+2)$

$= (-2)(3) + (-1)(4) + (1)(6)$

$= -6 - 4 + 6$

$\boxed{= -4}$

Writing Sums in Summation Notation

6. Express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

a. $1^4 + 2^4 + 3^4 + \ldots + 12^4$

$\sum_{i=1}^{12} i^4$

b. $\frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \ldots + \frac{n}{9^n}$

$\sum_{i=1}^{n} \frac{i}{9^i}$

Arithmetic Sequence

Definition of an Arithmetic Sequence
An **arithmetic sequence** is a sequence in which each term after the first differs from the preceding term by a constant amount. The difference between consecutive terms is called the **common difference** of the sequence.

7. Write the first six terms of each arithmetic sequence.

a. $a_1 = 300,\ d = 50$

300, 350, 400, 450, 500, 550

b. $a_n = a_{n-1} + 4,\ a_1 = -7$ $d = 4$ $a_1 = -7$

-7, -3, 1, 5, 9, 13

Explicit Form

General Form of an Arithmetic Sequence
The nth term (the general term) of an arithmetic sequence with the first term a_1 and common difference d is

$$a_n = a_1 + (n-1)d.$$

8. Find the indicated term of the arithmetic sequence with the first term, a_1, and the common difference, d.

a. Find a_{150} when $a_1 = -60,\ d = 5$

$a_n = -60 + (n-1)(5)$

$a_{150} = -60 + (150-1)(5)$

$= \boxed{685}$

b. Find a_{60} when $a_1 = 35,\ d = -3$

$a_n = 35 + (n-1)(-3)$

$a_{60} = 35 + (60-1)(-3)$

$= \boxed{-142}$

9. Write a formula for the general term (the nth term) of each arithmetic sequence. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

a. 2, 7, 12, 17, ... $d = 5$ $a_1 = 2$

$\boxed{a_n = 2 + (n-1)(5)}$

$a_{20} = 2 + (20-1)(5)$

$\boxed{= 97}$

b. 6, 1, -4, -9, ... $d = -5$ $a_1 = 6$

$\boxed{a_n = 6 + (n-1)(-5)}$

$a_{20} = 6 + (20-1)(-5)$

$\boxed{= -89}$

c. $a_1 = 6,\ d = 3$

$\boxed{a_n = 6 + (n-1)(3)}$

$a_{20} = 6 + (20-1)(3)$

$\boxed{= 63}$

d. $a_n = a_{n-1} + 5,\ a_1 = 6$ $d = 5$

$\boxed{a_n = 6 + (n-1)(5)}$

$a_{20} = 6 + (20-1)(5)$

$\boxed{= 101}$

The Sum of the First *n* Terms of an Arithmetic Sequence
The sum, S_n, of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

in which a_1 is the first term and a_n is the nth term.

10. Find the sum of:
    a. the first 25 terms of the arithmetic sequence: 7, 19, 31, 43, ...

$d = 12$
$a_1 = 7$

$a_n = 7 + (n-1)(12)$

$a_{25} = 295$

$S_{25} = \frac{25}{2}(7 + 295)$

$\boxed{= 3775}$

    b. the first 80 positive even integers.

$a_1 = 2 \quad d = 2$

$a_n = 2 + (n-1)(2)$
$a_{80} = 2 + (80-1)(2)$
$= 160$

$a_{80} = 160$

$S_{80} = \frac{80}{2}(2 + 160)$

$\boxed{= 6480}$

11. Write out the first three terms and the last term. Then use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

    a. $\sum_{i=1}^{20}(6i - 4) = (6-4) + (6(2)-4) + (6(3)-4) + \ldots + (6(20)-4)$

$= 2 + 8 + 14 + \ldots + 116$

$S_{20} = \frac{20}{2}(2 + 116)$

$\boxed{= 1180}$

Geometric Sequences

Definition of a Geometric Sequence
A **geometric sequence** is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant. The amount by which we multiply each time is called the **common ratio** of the sequence.

13. Write the first 5 terms of each geometric sequence.

a. $a_1 = 4,\ r = 3$

4, 12, 36, 108, 324

b. $a_n = -6\,a_{n-1};\ a_1 = -2$ $r = -6$

-2, 12, -72, 432, -2592

General Term of a Geometric Sequence
The *n*th term (the general term) of a geometric sequence with the first term a_1 and the common ratio r is

$$a_n = a_1\, r^{n-1}.$$

14. Find a_{20} when $a_1 = 8000$ and $r = -\frac{1}{2}$.

$a_n = 8000(-1/2)^{n-1}$

$a_{20} = 8000(-1/2)^{20-1}$

$= -0.01525...$

$= -125/8192$

15. Write the general term of the geometric sequence. Then find the 12th term in the sequence.

3, -15, 75, -375, ... $r = -5$

$\boxed{a_n = 3(-5)^{n-1}}$

$a_{12} = 3(-5)^{12-1}$

$\boxed{= -146{,}484{,}375}$

Sum of the first n terms of a geometric sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}$$