2 Series and Sequences Nov 2019 (No Calculators)

3 pts 1. Carl Gauss, one of the most prodigious mathematicians ever to live, was asked in kindergarten to add all the integers from 1 to 100, inclusive. He did so in less than a minute. Can you? Find the sum.

Ans. _____

4 pts 2. Consider all 4-element arithmetic sequences of increasing positive integers with a last term of 12. Add the 4 elements of each sequence. Find the total sum of all these sequence sums.

Ans. _____

5 pts 3. In a geometric series of positive terms, the 5^{th} term minus the 4^{th} term is 576. The 2^{nd} term minus the 1^{st} term is 9. What is the sum of the first two terms of the sequence?

Ans. _____

Series and Sequences

1. $(100 + 0) + (99 + 1) + (98 + 2) + \ldots + (51 + 49) + 50 = 100(50) + 50 = 5050.$ **Ans. 5050**

2. $9 + 10 + 11 + 12 = 42$; $6 + 8 + 10 + 12 = 36$; $3 + 6 + 9 + 12 = 30$. $42 + 36 + 30 =$ **Ans. 108**

3. Five terms are a, ar, ar^2, ar^3, ar^4. So $ar^4 - ar^3 = 576$ or $r^3(ar - a) = 576$. Since $ar - a = 9$, then $r^3(9) = 576$ and $r^3 = 64$, so $r = 4$. $a(4) - a = 9$, $3a = 9$, so $a = 3$. $3 + 3(4) = 15$. **Ans. 15**

2 Series and Sequences Nov 2018 (No Calculators)

3 pts 1. Find the 10^{th} term in the sequence -46, -35, -24, -13, … .

Ans. _____

4 pts 2. Find the sum of the first eight terms of the sequence whose first three terms are 243, 162, and 108.

Ans. _____

5 pts 3. 13 segments are drawn and labeled. Each segment, in order is t units shorter than the next. The segments are numbered in order from 1 to 13. If the sum of the lengths of the segments from 1 through 8 is equal to 148, and the sum of the lengths of the segments from 3 through 13 is equal to 319, find the length of segment 10.

Ans. _____

Series and Sequences

1. $-46 + 9(11) = -46 + 99 = 53$. **Ans. 53**

2. The sequence has a common ratio of 2/3. The 4^{th} term is 72, the 5^{th} is 48, the 6^{th} is 32. The sum of these is 665. The 7^{th} is $64/3 = 21\frac{1}{3}$, and the 8^{th} is $128/9 = 14\frac{2}{9}$. This sum is $35\frac{5}{9}$. The sum of the eight terms is $700\frac{5}{9}$. **Ans.** $700\frac{5}{9}$

3. 1^{st} segment = n, 2^{nd} = n + t, 3^{rd} = n + 2t, 4^{th} = n = 3t, 5^{th} = n + 4t, 6^{th} = n + 5t, 7^{th} = n + 6t, 8t = n = 7t, 9^{th} = n = 8t, 10^{th} = n + 9t, 11^{th} = n + 10t, 12^{th} = n + 11t, 13^{th} = 12t. Sum of 1 through 8 terms ➔ (1) 8n + 28t = 148. The sum of 3 through 13 ➔ (2) 11n + 77t = 319.

(1) ÷4 ➔ 2n + 7t = 37, (2) ÷11 ➔ n + 7t = 29. Subtracting n = 8, t = 3. 10^{th} segment = n + 9t = 8 + 9(3) = 35. **Ans. 35**

2 Series and Sequences Nov 2017 (No Calculators)

3 pts 1. In the sequence 7, 15, 23, …, the *n*th term is 143. Find *n*.

Ans. _____

4 pts 2. The 10^{th} term of an arithmetic sequence is 142. The 15^{th} term is 187. Find the 25^{th} term.

Ans. _____

5 pts 3. The 5^{th} term of a geometric sequence is 648. The 8^{th} term is 2187. Find the sum of the first three terms.

Ans. _____

Series and Sequences

1. $143 = 7 + (n-1)8 \rightarrow 136 = (n-1)8 \rightarrow 17 = n-1 \rightarrow n = 18.$ **Ans. 18**

2. (1) $142 = a + 14d$, (2) $187 = a + 24d$. (2) – (1): $45 = 5d$, $d = 9$. $187 = a + 14(9) = a + 126$, $a = 61$. 25^{th} term: $61 + 24(9) = 61 + 216 = 277$. Alternate solution: Since the 10^{th} term is 143 and the 15^{th} term is 187, then the difference is 45 for 5 terms. For 10 terms the difference is 90. $187 + 90 = 277$.

Ans. 277

3. $648n^3 = 2187$, $n^3 = \frac{2187}{648} = \frac{243}{72} = \frac{27}{8}$. So $n = 3/2$. $648 = a\left(\frac{3}{2}\right)^4 = a\left(\frac{81}{16}\right)$. $a = 648\left(\frac{16}{81}\right) = 8(16) = 128$. $128(3/2) = 64(3) = 192$. $192(3/2) = 96(3) = 288$. $288 + 192 + 128$. **Ans. 608**

2 Series and Sequences Nov 2015 (No Calculators)

3 pts 1. The 24th term of this sequence marks the years that one of the MAML Board members was born. What year was it?

2015, 2012, 2009, … **Ans.** _____

4 pts 2. An infinite geometric series has its fourth term equal to 3 and its common ratio equal to two-thirds. Find the sum of the series.

Ans. _____

5 pts 3. The first three terms of an arithmetic sequence are 76, 72, and 68. There are two values of n, such that the sum of the first n terms is 448. Find the sum of the two values of n.

Ans. _____

Series and Sequences

1. Age $= 2015 - 3(23) = 1946$. **Ans. 1946**

2. $3 = a(\frac{2}{3})^3 \rightarrow 3 = \frac{8}{27}a$, so $a = \frac{81}{8}$. Sum $= \frac{81/8}{1-2/3} = \frac{81}{8}\cdot\frac{3}{1} = \frac{243}{8}$. **Ans.** $\frac{243}{8}$

3. $448 = \frac{n}{2}(2(76) + (n-1)(-4)) \rightarrow \frac{n}{2}(152 - 4n + 4) \rightarrow 896 = 156n - 4n^2 \rightarrow$

$n^2 - 39n + 224 = 0 \rightarrow (n-32)(n-7) = 0$. $n = 32$ or 7, the sum being 39. **Ans. 39**

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2 Series and Sequences Nov 2014 (No Calculators)

3 pts 1. Find the 47^{th} term of the sequence -47, -36, -25, …

Ans. _____

4 pts 2. An infinite geometric series has a sum of 100 and a first term of 40. What is the fifth term in the series?

Ans. _____

5 pts 3. The 4^{th} term of a geometric sequence is 567. The 8^{th} term is 45,927. What is the sum of the first three terms?

Ans. _____

Series and Sequences

1. $-47 + 46(11) = -47 + 506 = 459$. **Ans. 459**

2. $\frac{40}{1-r} = 100$. $40 = 100 - 100r$. $r = 3/5$. $40(3/5)^4 = \frac{40 \cdot 81}{5 \cdot 125} = \frac{8 \cdot 81}{125} = \frac{648}{125}$. **Ans.** $\frac{648}{125}$

3. Multiplying the 4^{th} term by r^4 will give the 8^{th} term: $567r^4 = 45{,}927$.
$r^4 = 45{,}927/567 = 81$. So $r = \pm 3$. Finding the 1^{st} term a: $567 = a(3)^3$, $a = 567/27 = 21$.
So the 1^{st} three terms are 21, ± 63, 189. The sums are 273 or 147. **Ans. 273 or 147**

2 Series and Sequences Nov 2013 (No Calculators)

3 pts 1. An arithmetic sequence is defined as $A_{N+1} = A_N + 3/4$. If $A_1 = 1/2$, find the value of A_9.

Ans. _____

4 pts 2. One-fourth of the air in a tank is removed with each stroke of a vacuum pump. What fractional part of the air remains in the tank after 4 strokes?

Ans. _____

5 pts 3. The 9th term of an arithmetic sequence is 16. The 42nd term is 214. What is the sum of the first 50 terms of the sequence?

Ans. _____

Series and Sequences

1. $a_9 = 1/2 + 8(3/4) = 1/2 + 6 = 6\ 1/2$. **Ans. 6 1/2**

2. $a_4 = \frac{3}{4}\left(\frac{3}{4}\right)^3 = \frac{3^4}{4^4} = \frac{81}{256}$. **Ans. 81/256**

2. 9th term: (1) $16 = a_1 + 8d$; 42nd term: (2) $214 = a_1 + 41d$. (2) – (1): $198 = 33d$, $d = 6$. In (1): $16 = a_1 + 8(6)$, $a = -32$. The 50th term is $-32 + 49(6) = 262$. Sum $= \frac{50}{2}(-32 + 262) = 25(230) = 5750$

Ans. 5750

2 Series and Sequences Nov 2012 (No Calculators)

3 pts 1. Find the sum of the series (-300) + (-297) + (-294) + … + (306) + (309).

Ans. _____

4 pts 2. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

Ans. _____

5 pts 3. w, x, y, and z are consecutive terms of an arithmetic sequence with common difference of 2012. Find the following: $\frac{Z^2 - w^2}{y^2 - x^2}$.

Ans. _____

Series and Sequences

1. The last three terms are not cancelled: 303 + 306 + 309 = 918. **Ans. 918**

2. 24 = 2(8 + x), x being the 8th term. x = 4. y = 2(24 + 4) , y being the 10th term. y = 56. z = 2(24 + 56), z being the 11th term. z = 160. **Ans. 160**

3. The common difference is going to be eliminated. By factoring the numerator, you get (z + w)(z – w) and since z = w + 3d, this yields (2w + 3d)(3d). By factoring the denominator, you get (y + x)(y – x) and since y = w + 2d and x = w + d, this yields factors

3 pts 1. Find the 11^{th} term of the sequence which begins 13, 20, 27, … .

Ans. _____

4 pts 2. Alexa's father gave her a nickel after the first day she was good all day. The next day he gave her 2 nickels for being good all day. The third day he gave here 3 nickels. She put them all in her piggy bank. What would be the total amount of money in dollars and cents (such as $5.20), that Alexa would have in her bank after 40 days of being good?

Ans. _____

5 pts 3. Find the sum of the multiples of 6 from 200 to 300 which are not divisible by 9.

Ans. _____

Series and Sequences

1. $13 + 10(7) = 83$. **Ans. 83**

2. $S = \frac{40(5+5+39\cdot 5)}{2} = 20(5\cdot 41) = 4100.$ **Ans. $41.00**

3. Multiples of 6 → 204 is the first, 300 is the last. → $300 = 204 + (n-1)6$ → $96 = (n-1)6$

$n - 1 = 16$, so 17 terms. Sum $= \frac{17(204+300)}{2} = \frac{17(504)}{2} = 17(252) = 4284$.

Those which are multiples of 9 are multiples of 18, the LCM. The first is 216 and the last is 288. $288 = 216 + (n-1)\,18$ → $72 = (n-1)18$ → $4 = n - 1$, so $n = 5$.

Sum $= \frac{5(216+288)}{2} = \frac{5(504)}{2} = 5(252) = 1260$. $4284 - 1260 = 3024$. **Ans. 3024**