3 Counting Principles and Binomial Theorem Nov 2019 (No Calculators)

3 pts 1. How many 4-digit numbers can be made by rearranging all four of the digits of 2019?

Ans. ____________________

4 pts 2. What is the coefficient of the term containing x^6 in the expansion of

$$\left(4x^2 - \frac{y}{2}\right)^9 ?$$

Ans. ____________________

5 pts 3. How many different permutations are there of the letters of the word eructation (the act of burping), if the letters u and c must be adjacent to each other?

Ans. ____________________

Counting Principles and Binomial Theorem

1. First digit $\neq 0$, second digit can be 0, so $3 \cdot 3 \cdot 2 \cdot 1 = 18$. **Ans. 18**

2. $\binom{9}{3}\left(4x^2\right)^3\left(\frac{-y}{2}\right)^6 = \left(\frac{9 \cdot 8 \cdot 7 \cdot 6!}{3 \cdot 2 \cdot 6!}\right)64x^6\left(\frac{y^6}{64}\right)$. Coefficient $= 3(4)(7) = 84$. **Ans. 84**

3. We lock the u and c together so there are now 9 things to permutate. The u and c can be permutated 2 times. There are 2 t's. So $\frac{2!9!}{2!} = 9! = 362{,}880$. **Ans. 362,880**

3 Counting Principles and Binomial Theorem Nov 2018 (No Calculators)

3 pts 1. How many unique 4-digit whole numbers contain two 8's, one 7 and one 4?

Ans. ____________

4 pts 2. If $(1{,}000{,}000{,}000 + 10)^{10}$ is expanded using the binomial theorem, the middle term can be expressed in the form $2^a \cdot 3^b \cdot 5^c \cdot 7^d$, where a, b, c, d, are positive integers. Find the sum $a + b + c + d$.

Ans. ____________

5 pts 3. A club with N members needs to select a potentate, a scribe, and three centurions from its members. If there are 420 different ways the selections can be made, find N.

Ans. ____________

Counting Principles and Binomial Theorem

1. Wanting to know how many arrangements of AABC $= \frac{4!}{2!} = 12$. **Ans. 12**

2. $1{,}000{,}000{,}000 = 10^9$. The middle term is $\binom{10}{5}\left(10^9\right)^5 (10)^5 = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{5\cdot 4\cdot 3\cdot 2} 10^{45}\, 10^5 =$

$3\cdot 2\cdot 7\cdot 6\cdot 2^{50}\cdot 5^{50} = 3^2\cdot 7^1\cdot 2^{52}\cdot 5^{50}$. $2 + 1 + 52 + 50 = 105$. **Ans. 105**

3. If N = 5 there are 5 ways to select a potentate, 4 ways to select a scribe, then the other three are centurions, for a total of 20 ways. If N = 6, there are $6\cdot 5\cdot\binom{4}{3} = 120$. If N = 7, then

$7\cdot 6\cdot\binom{5}{3} = 7(6)(10 = 420$. **Ans. 7**

3 **Counting Principles and Binomial Theorem** **Nov 2017** **(No Calculators)**

3 pts 1. A set consists of 9 distinct elements. How many different subsets of 3 distinct elements can be formed from this set?

Ans. ________________

4 pts 2. Find the 4th term in the expansion of $(3x-5y)^7$.

Ans. ________________

5 pts 3. How many even 4-digit numbers can be formed using 0, 2, 3, 5, 6, 7, 8 or 9 for each digit in the numbers, and in each number the digits are distinct?

Ans. ________________

Counting Principles and Binomial Theorem

1. ${}_9C_3 = 84.$

Ans. 84

2. $\binom{7}{3}(3x)^4(-5y)^3 = 35(81)x^4(-125)y^3 = -354{,}375x^4y^3$

Ans. $-354{,}375x^4y^3$

3. Since the numbers have to be even, then the unit's digit has to be 0, 2, 6, or 8. If it is 0, then there are 7 possible numbers for the first digit, then 6, and then 5. Multiplying = 210. If 2,6, 8 are the unit's digit, then the first digit cannot be 0, so that leaves 6 for the first digit. Then 0 can go in the next digit's place making 6 and the last digit 5 possible numbers left. Multiplying $6(6)(5)(3) = 30(18) = 540$. $540 + 210 = 750$.

Ans. 750

3 Counting Principles and Binomial Theorem Nov 2015 (No Calculators)

3 pts 1. A team consisting of 3 girls and 2 boys is selected from 10 girls and 5 boys. How many different teams can be made?

Ans. ____________________

4 pts 2. What is the ratio of the coefficients of the x^2 term to the x^3 term in the expansion of $(2x - 3y)^7$?

Ans. ____________________

5 pts 3. Find the number of positive integers less than 10,000 with all distinct digits.

Ans. ____________________

Counting Principles and Binomial Theorem

1. $\binom{10}{3}\binom{5}{2} = 120(10) = 1200.$ **Ans. 1200**

2. $$\frac{\binom{7}{5}(2)^2(-3)^5}{\binom{7}{4}(2)^3(-3)^4} = \frac{21(4)(-243)}{35(8)(81)} = \frac{\cancel{7}(3)(\cancel{4})(-3)(\cancel{81})}{\cancel{7}(5)(2)(\cancel{4})(\cancel{81})} = \frac{-9}{10}.$$ **Ans. -9/10**

3. Distinct one-digit numbers: 1 – 9 = , 2–digit: (9)(9) = 81; 3-digit: (9)(9)(8) = 648; 4-digit: (9)(9)(8)(7) = 4536. The sum is 5274. **Ans. 5274**

3 Counting Principles and Binomial Theorem Nov 2014 (No Calculators)

3 pts 1. President, vice-president, secretary and treasurer are to be chosen from a club of 10 members. How many different choices are possible?

Ans. ________________

4 pts 2. A classroom contains 7 girls, 8 boys, and 1 teacher. If the order of departure from the classroom during a fire drill is recorded in terms of G's, B's, and the T, how many orders are possible?

Ans. ________________

5 pts 3. In the expansion of $(2x + 3y)^9$, what is the sum of the coefficients of the 5th and the 6th terms?

Ans. ________________

Counting Principles and Binomial Theorem

1. $_{10}P_4 = \frac{10!}{6!} = 10(9)(8)(7) = 720(7) = 5040.$ **Ans. 5040**

2. $\frac{16!}{7!\cdot 8!\cdot 1!} = \frac{16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2} = 52(22)90 = 1144(90) = 102{,}960.$ **Ans. 102,960**

3. The 5th term is $\binom{9}{4}(2x)^5(3y)^4$. The 6th term is $\binom{9}{5}(2x)^4(3y)^5$. $\binom{9}{5} = 126.$

$126(2^5 3^4 + 2^4 3^5) = 126(2^4 3^4)(2+3) = 126(80)(81) = 126(6480) = 816{,}480.$ **Ans. 816,480**

3 Counting Principles and Binomial Theorem Nov 2013 (No Calculators)

3 pts 1. How many distinguishable ways can ten A's and three B's be arranged in a row?

Ans. ______________

4 pts 2. Find the third term in the expansion of $\left((2x)^3 - \frac{1}{x^2}\right)^5$.

Ans. ______________

5 pts 3. A committee of three people is randomly selected from a club consisting of 11 females and 10 males. If A is the number of committees comprised of a dominance of females, and B is the number of committees comprised of dominance of males. How many more committees are in A than there are in B?

Ans. ______________

Counting Principles and Binomial Theorem

Ans. 286

1. $\left(\frac{13!}{3!10!}\right) = \frac{13\cdot 12\cdot 11}{3\cdot 2} = \frac{13\cdot 2\cdot 11}{1} = 143(2) = 286.$

Ans. 5120x⁵

2. $\binom{5}{2}(8x^3)\left(-\frac{1}{x^2}\right) = (10)(512x^9)\left(\frac{1}{x^4}\right) = 5120x^5$

3. A is all females or 2 females and one male: ${}_{11}C_3 + {}_{11}C_2 \cdot {}_{10}C_1 = \left(\frac{11!}{8!3!}\right) + \left(\frac{11!}{9!2!}\right)\cdot 10 =$ $\left(\frac{11\cdot 10\cdot 9\cdot 8!}{3\cdot 2\cdot 8!}\right) + \left(\frac{11\cdot 10}{2}\right)\cdot 10 = 11\cdot 5\cdot 3 + 11\cdot 5\cdot 10 = 11\cdot 5\cdot 13 = 715.$ B is all males or 2 males and 1 female: ${}_{10}C_3 + {}_{10}C_2 \cdot {}_{11}C_1 = \left(\frac{10!}{7!3!}\right) + \left(\frac{10!}{2!8!}\right)\cdot 11 = \left(\frac{10\cdot 9\cdot 8\cdot 7!}{3!7!}\right) + \left(\frac{10\cdot 9\cdot 8!}{2!8!}\right)\cdot 11 \rightarrow$ $10\cdot 3\cdot 4 + 5\cdot 9\cdot 11 = 120 + 495 = 615.\quad 715 - 615 = 100.$

Ans. 100

3 Counting Principles and Binomial Theorem Nov 2012 (No Calculators)

3 pts 1. How many distinguishable automobile license plates of 6 digits can be made, if the first digit (on the left) cannot be 0 (zero)?

Ans. ____________________

4 pts 2. Find the coefficient of the term in the expansion of $\left(4x^2 - \frac{1}{2x^3}\right)^8$ that contains x to a power of 1.

Ans. ____________________

5 pts 3. Three fair, standard six-faced dice of different colors are rolled. In how many ways can the dice show a sum of 10 on the top faces?

Ans. ____________________

Counting Principles and Binomial Theorem

1. The first digit has 9 possibilities. Thus $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$. **Ans. 900,000**

2. $\binom{8}{3}(4x^2)^5\left(-\frac{1}{2x^3}\right)^3 = -56(4^5)x^{10}\left(\frac{1}{8x^9}\right) = -56(2^7)x = -7168x.$ **Ans. -7,168**

3. To get a sum of 10, you could get 1 + 3 + 6 (six ways) or 1 + 4 + 5 (six ways) or 2 + 3 + 5 (six ways) or 2 + 2 + 6 (three ways) or 3 + 3 + 4 (three ways) or 2 + 4 + 4 (3 ways). Total is 27 ways. **Ans. 27 ways**

3 Counting Principles and Binomial Theorem Nov 2011 (No Calculators)

3 pts 1. How many 4-letter distinguishable "words" can be made from the letters M, A, M, L. Words such as MMAL do not have to make a real word.

Ans. ______________

4 pts 2. An antique car dealer featured 10 cars from the 1940's. 3 of the cars will be chosen at random to go to auction to be sold. How many different groups of cars can go to the auction?

Ans. ______________

5 pts 3. In the expansion of $(3x^2 - 5y^3)^5$, each term has an $x^p y^q$. Find the sum of the coefficients of the two terms where $p + q$ has the largest values.

Ans. ______________

Counting Principles and Binomial Theorem

1. Each time the M's are side-by-side (which are 3 places), the A and L can be switched, thus making 6. Like wise when the M's are not side-by side (3 places) there are 6 more. Otherwise if students have prepared for the category: $\frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2}{2} = 12$. **Ans. 12**

2. ${}_{10}C_3 = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$. **Ans. 120**

3. Setting up exponents for sum: $(x^2)^5(y^3)^0 \Rightarrow 10$; $(x^2)^4(y^3)^1 \Rightarrow 11$; it appears that the sum is increasing and the highest sums should be 14 and 15 resulting from the last two terms: $\binom{5}{4}(3x^2)^1(-5y^3)^4$ and $\binom{5}{5}(3x^2)^0(-5y^3)^5$. The 5th term's coefficient is $5(3)(-5)^4 = 15(625) = 9375$. The 6th term's coefficient is $(-5)^5 = -3125$. **Ans. 6250**

3 Counting Principles and Binomial Theorem Nov 2010 (No Calculators)

3 pts 1. Find the coefficient of x in the expansion of $\left(x+\frac{1}{x}\right)^7$.

Ans. ________________

4 pts 2. A high school Interact Club has 12 members, 7 of which are girls. The club has been asked to be represented by 7 members at a convention. How many different groups of members can be sent, if 4 of the group are girls and 3 are boys?

Ans. ________________

5 pts 3. Mr. Trendiless has one of those cars that has 3 separate seats in the front and 3 separate seats in the back. If 5 persons, only two of which can drive, get in to take a joy-ride, how many possible ways can they be seated?

Ans. ________________

Counting Principles and Binomial Theorem

1. The term is $\binom{7}{3}(x)^4\left(\frac{1}{x}\right)^3 = \frac{7\cdot6\cdot5}{3\cdot2}x = 35x$. Coefficient is 35. **Ans. 35**

2. ${}_5C_3\cdot{}_7C_4 = \frac{5\cdot4}{2}\cdot\frac{7\cdot6\cdot5}{3\cdot2} = 10\cdot35 = 350$. **Ans. 350**

3. The driver's seat has 2 possibilities. That leaves 4 persons to be seated in 5 seats. The first person has 5 seats to choose from. The next has 4 to choose from. The next 3. And the last 2. Thus $5\cdot4\cdot3\cdot2$. Or ${}_5P_4 = 120$. $2(120) = 240$. **Ans. 240**