4 Polynomials Nov 2019 (No Calculators)

3 pts 1. Subtract the sum of $3x - 7$ and $2x + 3$ from the product of -3 and $2x - 7$.

Ans. ____________________

4 pts 2. One of the roots of the equation $x^3 + 6x^2 - 25x + c = 0$ is 1. Find the other two roots of the equation.

Ans. ____________________

5 pts 3. A polynomial function $P(x)$ with integral coefficients has roots of 1, $1 + \sqrt{2}$ and $1 - \sqrt{2}$. None of these are double roots. Find $P(-1)$.

Ans. ____________________

Polynomials

1. $-3(2x - 7) - (3x - 7 + 2x + 3) = -6x + 21 - 5x + 4 = -11x + 25.$ **Ans. -11x + 25**

2.
$$\begin{array}{r|rrrr} 1 & 1 & 6 & -25 & c \\ & & 1 & 7 & -18 \\ \hline & 1 & 7 & -18 & c - 18 \end{array}$$

Since $c - 18 = 0$, the resulting polynomial from the division is $x^2 + 7x - 18 = 0 \Rightarrow (x - 2)(x + 9) = 0$. So the other roots are 2 and -9. **Ans. 2 and -9**

3. Roots are 1, $1 + \sqrt{2}$ and $1 - \sqrt{2}$. For $1 + \sqrt{2}$ and $1 - \sqrt{2}$, $x = 1 \pm \sqrt{2}$ or $x - 1 = \pm\sqrt{2}$ or $(x - 1)^2 = 2 \Rightarrow x^2 - 2x + 1 = 2$ or $x^2 - 2x - 1 = 0$. $P(x) = (x^2 - 2x - 1)(x - 1) = x^3 - 3x^2 + x + 1$. $P(-1) = -1 - 3 - 1 + 1 = -4$. **Ans. -4**

4 Polynomials Nov 2018 (No Calculators)

3 pts 1. The product of $(4x^2 - x + 3)(6x^2 + 5x - 2) = ax^4 + bx^3 + cx^2 + dx + e$. Find the value of $a + b + c + d + e$.

Ans. ________________

4 pts 2. Find the sum of the absolute values of the roots of $y = x^4 + 2x^3 - 7x^2 - 8x + 12$.

Ans. ________________

5 pts 3. The cubic polynomial $f(x)$ passes through (0, 4) and has zeroes at x = 1, x = 3, and x = 5. Find $f(4)$.

Ans. ________________

Polynomials

1.

$$\begin{array}{r} 4x^2 - x + 3 \\ 6x^2 + 5x - 2 \\ \hline -8x^2 + 2x - 6 \\ 20x^3 - 5x^2 + 15x \phantom{{}- 6} \\ 24x^4 - 6x^3 + 18x^2 \phantom{{}+ 2x - 6} \\ \hline 24x^4 + 14x^3 + 5x^2 + 17x - 6 \end{array}$$

$24 + 14 + 5 + 17 - 6 = 54$ **Ans. 54**

2.

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -7 & -8 & 12 \\ & & 1 & 3 & -4 & -12 \\ \hline & 1 & 3 & -4 & -12 & 0 \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \end{array} \qquad \begin{array}{r|rrr} -2 & 1 & 5 & 6 \\ & & -2 & -6 \\ \hline & 1 & 3 & 0 \end{array}$$

1, 2, -2, -3

Ans. 8

3. $f(x) = a(x - 1)(x - 3)(x - 5)$. $F(0) = a(-1)(-3)(-5) = 4$. $a = -\frac{4}{15}$. $F(4) = -\frac{4}{15}(3)(1)(-1) = \frac{4}{5}$.

Ans. 4/5

4 Polynomials Nov 2017 (No Calculators)

3 pts 1. Find the product of $4x - 9$ and $3x^2 - 5x + 2$.

Ans. ______________________________

4 pts 2. Find all value(s) of x such that $15x^3 + 71x^2 + 86x + 24 = 0$.

Ans. ______________________________

5 pts 3. If $p(x+3) = 5x^2 + 27x + 38$, find $p(x - 3)$.

Ans. ______________________________

Polynomials

1. $(4x - 9)(3x^2 - 5x + 2) = 12x^3 - 20x^2 + 8x - 27x^2 + 45x - 18$. **Ans. $12x^3 - 47x^2 + 53x - 18$**

2.
$$\begin{array}{r|rrrr} & 15 & 71 & 86 & 24 \\ -3 & & -45 & -78 & -24 \\ \hline & 15 & 26 & 8 & 0 \end{array}$$

So $15x^2 + 26x + 8 = 0$

$(3x + 4)(5x + 2) = 0$. $x = -3, -4/3, -2/5$

Ans. -3, -4/3, -2/5

3. $P(x - 3)$ is 6 units less than $P(x + 3)$ Thus $P(x - 6) = 5(x - 6)^2 + 27(x - 6) + 38$ ➔

$5(x^2 - 12x + 36) + 27x - 162 + 38$ ➔ $5x^2 - 60x + 180 + 27x - 124$. **Ans. $5x^2 - 33x + 56$**

4 Polynomials Nov 2015 (No Calculators)

3 pts 1. From what polynomial must $5x^3 - 2x^2 + 3x$ be subtracted from so as to obtain the polynomial $x^3 - 2$?

Ans. ______________________

4 pts 2. The rational Root Theorem (also called the Rational Zero Theorem) can be used to generate a list of all possible rational roots of a function. What is the product of all the possible rational roots, according to the "theorems", of the function f(x), if

$$f(x) = 2x^4 - 7x^3 + 5x^2 + 14x + 8$$

Ans. ______________________

5 pts 3. If one root of the equation $x^4 - 5x^3 - 22x^2 + 230x - 204 = 0$ is 5 + 3i, find the sum of the remaining roots.

Ans. ______________________

Polynomials

1. $5x^3 - 2x^2 + 3x + (x^3 - 2) = 6x^3 - 2x + 3x - 2$ **Ans. $6x^3 - 2x^2 + 3x - 2$**

2. The factors of 8 are 1, 2, 4, 8 and the factors of 2 are 1, 2. Roots can be positive or negative. So the possible roots are $\pm\left(1, \frac{1}{2}, 2, 4, 8\right)$ The product: $-1\left(\frac{-1}{4}\right)(-4)(-16)(-64) = -1024$. **Ans. -1024**

3. The sum of the roots is 5. The sum of the rest: 5 – (5 + 3i) = -3i. **Ans. -3i**

4 Polynomials Nov 2014 (No Calculators)

3 pts 1. Find all values of x such that $x^2 - 12x + 20 = 0$.

Ans. ______________

4 pts 2. Find the remainder when dividing $x^9 + 2x^8 + 3x^7 + 4x^6 + \ldots + 8x^2 + 9x + 10$ by x + 2.

Ans. ______________

5 pts 3. For any integer values of *a* and *b*, let *S* be the set of all rational roots for all cubic polynomials of the form $6x^3 + ax^2 + bx - 12$. How many elements are in the set?

Ans. ______________

Polynomials

1. $x^2 - 12x + 20 = (x - 10)(x - 2) = 0$. x = 10 or 2. **Ans. 10 or 2**

2. Dividing synthetically using -2:

$$\begin{array}{r|rrrrrrrrrr} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ -2 & & -2 & 0 & -6 & 4 & -18 & 24 & -62 & 108 & -234 \\ \hline & 1 & 0 & 3 & -2 & 9 & -12 & 31 & -54 & 117 & -224. \end{array}$$

Ans. -224

3. By the rational root test, all rational roots of the polynomial with integral coefficients must be of the form m/n, where m is the an integer factor of the constant (-12) and n is an integer factor of the leading coefficient (6).

-12 has 12 factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$; 6 has 8 factors: $\pm 1, \pm 2, \pm 3, \pm 6$

There are 24 rational roots in *S*: $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Ans. 24

4 Polynomials Nov 2013 (No Calculators)

3 pts 1. Let $A = 3x^2y - xy^2 - 4x^3y + 2x^2y^2$ and $B = x^2y^2 - xy^2 - x^2y + x^3y$. Find the polynomial created by $2(A+B) - 3(A-B)$.

Ans. ____________________

4 pts 2. Find the solutions for the equation: $x^3 + 19x = 8x^2 + 12$.

Ans. ____________________

5 pts 3. Determine *a* and *b* such that 2 is a double zero for the polynomial:

$$x^4 + (a-2)x^3 + bx^2 + (a+b)x + 4$$

Ans. ____________________

Polynomials

1. $2(A+B) - 3(A-B) = 2A + 2B - 3A + 3B = 5B - A$. Thus

$$5(x^2y^2 - xy^2 - x^2y + x^3y) - (3x^2y - xy^2 - 4x^3y + 2x^2y^2) =$$

$$5x^2y^2 - 5xy^2 - 5x^2y + 5x^3y - 3x^2y + xy^2 + 4x^3y - 2x^2y^2$$

Ans. $\mathbf{3x^2y^2 - 4xy^2 - 8x^2y + 9x^3y}$

2. $x^3 + 19x = 8x^2 + 12 \Rightarrow x^3 - 8x^2 + 19x - 12 = 0$. By synthetic dividion:

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 19 & -12 \\ & & 4 & -16 & 12 \\ \hline & 1 & -4 & 3 & \end{array}$$

Now factoring $(x-3)(x-1)$. The solutions are 1, 3, 4

Ans. 1, 3, 4

3. Dividing synthetically:

$$\begin{array}{r|rrrrr} 2 & 1 & a-2 & b & a+b & 4 \\ & & 2 & 2a & 4a+2b & 10a+6b \\ \hline 2 & 1 & a & 2a+b & 5a+3b & (10a+6b+4=0) \\ & & 2 & 2a+4 & 8a+2b+8 & \\ \hline & 1 & a+2 & 4a+b+4 & (13a+5b+8=0) & \end{array}$$

Thus $5a + 3b = -2 \Longrightarrow 25a + 15b = -10$

$13a + 5b = -8 \Longrightarrow -39a - 15b = 24$

$-14a = 14$ So $a = -1$. $5(-1) + 3b = -2$, $b = 1$. **Ans. $a = -1$ and $b = 1$**

4 Polynomials Nov 2012 (No Calculators)

3 pts 1. Find the solutions of $2x^2 + 3x - 5 = 0$.

Ans. ____________________

4 pts 2. Let $P(x) = kx^3 + 2k^2x^2 + k^3$. Find the sum of all real numbers k for which $x - 2$ is a factor of P(x).

Ans. ____________________

5 pts 3. If $P(x) = 3x^3 + x^2 - 62x + 40$, find all values of x such that $P(x) = 0$.

Ans. ______________________________

Polynomials

1. $2x^2 + 3x - 5 = 0 \rightarrow (x - 1)(2x + 5) = 0$. $x = 1$ or $-5/2$. **Ans. 1 or -5/2**

2. In order for $(x - 2)$ to be a factor of P(x), $P(2) = 0 = k(8) + k^2(8) + k^3$. Factoring: $k(k^2 + 8k + 8) = 0$. One value of k is 0. The sum of the other two is -8, the opposite of the coefficient of k. Thus the sum is -8. **Ans. -8**

3. Synthetically:

$$\begin{array}{r|rrrr} -5 & 3 & 1 & -62 & 40 \\ & & -15 & 70 & -40 \\ \hline & 3 & -14 & 8 & 0 \end{array}$$

$3x^2 - 14x + 8 = 0 \rightarrow (3x - 2)(x - 4) = 0$

Thus $x = 2/3$ or 4.

Ans. -5, 4 or 2/3

4 Polynomials Nov 2011 (No Calculators)

3 pts 1. Find the product: $(x + 2y)(2x - y)(x + y)$.

Ans. ______________________________

4 pts 2. If $P(x) = 3x^3 + x^2 - 62x + 40$, find all values of x such that $P(x) = 0$.

Ans. ______________________________

5 pts 3. If $21x^5 - 50x^4 + 90x^3 - 43x^2 + kx + p$ is divided by $7x^3 - 5x^2 + 3x + 4$, there is no remainder. Find the values of k and p.

Ans. ______________________________

Polynomials

1. $(x+2y)(2x-y) = 2x^2 + 3xy - 2y^2$. $(2x^2 + 3xy - 2y^2)(x+y) =$
$2x^3 + 3x^2y - 2xy^2 + 2x^2y + 3xy^2 - 2y^3$ **Ans.** $\mathbf{2x^3 + 5x^2y + xy^2 - 2y^3}$

2. $3x^3 + x^2 - 62x + 40 = 0$. Solving by synthetic division:

$$\begin{array}{r|rrrr} -5 & 3 & 1 & -62 & 40 \\ & & -15 & 70 & -40 \\ \hline & 3 & -14 & 8 & 0 \end{array}$$

$3x^2 - 14x + 8 = 0 \Rightarrow (3x - 2)(x - 4) = 0$

Ans. -5, 4, 2/3

3.
$$\begin{array}{r} 3x^2 \quad - \quad 5x \quad + \quad 8 \\ 7x^3 - 5x^2 + 3x + 4 \;\overline{)\; 21x^5 - 50x^4 + 90x^3 - 43x^2 + kx + p} \\ \underline{21x^5 - 15x^4 + 9x^3 + 12x^2} \qquad\qquad \\ -35x^4 + 81x^3 - 55x^2 + kx \quad \\ \underline{-35x^4 + 25x^3 - 15x^2 - 20x} \quad \\ 56x^3 - 40x^2 + (k+20)x + p \\ \underline{56x^3 - 40x^2 + 24x \quad + 32} \\ (k-4)x + p - 32 \end{array}$$

Ans. $\mathbf{k = 4, p = 32}$

4 Polynomials Nov 2010 (No Calculators)

3 pts 1. Find the coefficient of x^3 in the polynomial that results from

$$(x-1)(x-2)(x-3)(x-4).$$

Ans. ____________________

4 pts 2. Find the distance between the two x-intercepts of the graph of

$$y = x^2 - \tfrac{13}{6}x - \tfrac{28}{6}.$$

Ans. ____________________

5 pts 3. Find the product of the greatest and the least roots of

$$x^5 - x^4 - 25x^3 - 11x^2 + 144x + 180$$

Ans. ____________________

Polynomials

1. In any polynomial with real coefficients, the coefficient of the second term, when the powers are ordered from highest to lowest, is the negative of the sum of the roots, when the coefficient of the highest powered term is 1. Thus since the sum of the roots is 10, then the coefficient is -10. **Ans. -10**

2. The x-intercepts are found when y = 0. Thus $x^2 - \frac{13}{6}x - \frac{28}{6} = 0 \Rightarrow 6x^2 - 13x - 28 = 0$. Factoring: $(3x + 4)(2x - 7) = 0$. Thus x = - 4/3 or 7/2. The distance between the intercepts $-1\frac{1}{3}$ and $3\frac{1}{2}$ is $1\frac{1}{3} + 3\frac{1}{2} = 4\frac{5}{6}$. **Ans. $4\frac{5}{6}$**

3. Using synthetic division:

$$\begin{array}{r|rrrrrr}
3 & 1 & -1 & -25 & -11 & 144 & 180 \\
 & & 3 & 6 & -57 & -204 & -180 \\
\hline
5 & 1 & 2 & -19 & -68 & -60 & \\
 & & 5 & 35 & 80 & 60 & \\
\hline
-3 & 1 & 7 & 16 & 12 & & \\
 & & -3 & -12 & -12 & & \\
\hline
-2 & 1 & 4 & 4 & & & \\
 & & -2 & -4 & & & \\
\hline
 & 1 & 2 & & & &
\end{array}$$

The greatest root is 5 and the least root -3. The product of these is -15.

Ans. -15