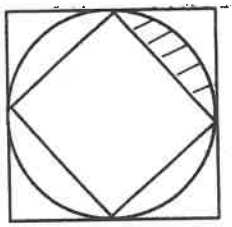
3 pts 1. Four different triangles share a common edge, the segment from (3,0) to (5, 0) in the coordinate plane. The third vertices of the triangles are at the points (-1, 3), (3, -3), (4, 4) and (13, 2). Let T be the triangle with the greatest area. Find the area of T in square units.

Ans. _____

4 pts 2. A circle is inscribed in a big square and a small square is inscribed inside the circle as shown. If the big square has an area of 64, find the area of the shaded region.



Ans. _____

5 pts 3. If the volume of the region bounded by the circumscribed sphere and the inscribed sphere of a cube, whose volume 512, is used to make a third sphere, what would the radius of that third sphere be?

Ans. _____

Areas and Volumes

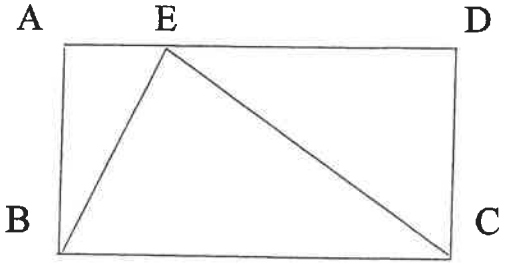
1. Treat (3, 0) and (5, 0) as the base, which is on the x-axis. The highest point is above the axis is (4, 4). Since the base is 2 units long and the height is 4, the area is ½(2)(4) = 4. **Ans. 4**

2. The radius of the circle is 4, so area is 16π. The diagonal of the inner square is 8, so the area of this square is ½(8)(8) = 32. Four segments area is 16π - 32, one is 4π - 8. **Ans. 4π - 8**

3. The inscribed sphere has a radius of 4. The circumscribed sphere has a radius of $4\sqrt{3}$. So the volume of the region bounded by the spheres is $\frac{4}{3}\pi(4\sqrt{3})^3 - \frac{4}{3}\pi(4)^3 = \frac{4}{3}\pi(4)^3\left(3\sqrt{3}-1\right)$. Thus the resulting sphere must have a radius of $4\sqrt[3]{3\sqrt{3}-1}$. **Ans. $4\sqrt[3]{3\sqrt{3}-1}$**

5 Areas and Volumes Nov 2018 (You may use calculators)

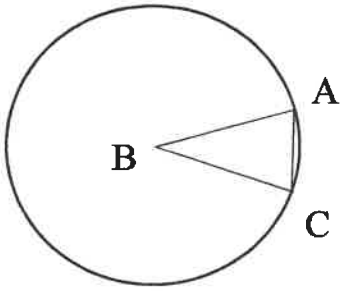
3 pts 1. Given the rectangle ABCD and point E on $\overline{AD}$. AB = 8 and AD = 12. Find the sum of the areas of ΔABE and ΔCDE.



Ans. _____

4 pts 2. In the circle the measure of angle ABC = 30^o and radius AB = 10.

Find the area of the segment of the circle bounded by segment AC and minor arc AC.



Ans. _____

5 pts 3. Find the volume of a regular tetrahedron whose edges are 18 inches long. A tetrahedron is a triangular pyramid, the solid made with the least number of faces.

Ans. _____

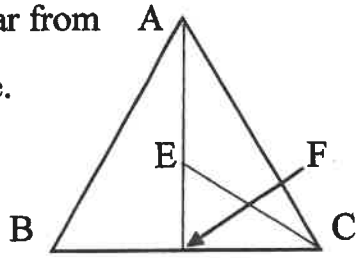
Areas and Volumes

1. The area of triangle BEC = ½ (8)(12) = 48. The area of the rectangle is 96. Subtracting we get 48. **Ans. 48**

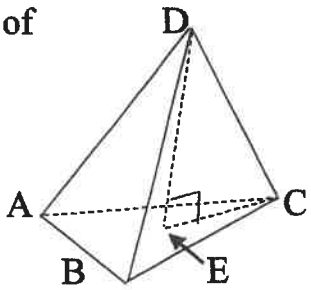
2. The area of the sector = $\frac{30}{360}\pi(100) = \frac{100}{12}\pi = \frac{25}{3}\pi$. In triangle ABC if the perpendicular is drawn from C to side AB, its length would be 5. Thus the area of the triangle is ½ (5)(10) = 25.

The area of the segment of the circle is $\frac{25}{3}\pi$ - 25. **Ans.** $\frac{25}{3}\pi$ **- 25**

3. Let E be the center of triangle ABC, the base. Draw a perpendicular from E to BC at F and a segment from E to C. ΔCEF is a 30-60-90 triangle. FC = 9, so FE = $3\sqrt{3}$ and CE = $6\sqrt{3}$. The area of ΔABC = $81\sqrt{3}$.



In the tetrahedron ABCD, EC = $6\sqrt{3}$, DC = 18, so by the Pyth. Thm. DE = $\sqrt{18^2 - (6\sqrt{3})^2} = \sqrt{324-108} = \sqrt{216} = 6\sqrt{6}$. The volume of the tetrahedron is $\frac{1}{3}(81\sqrt{3})(6\sqrt{6}) = 486\sqrt{2}$.



Ans. $\mathbf{486\sqrt{2}}$

5 Areas and Volumes Nov 2017 (You may use Calculators)

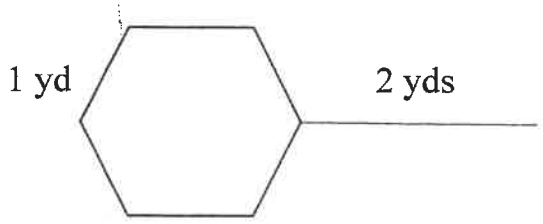
3 pts 1. A right circular cone is 12 inches high and has a base diameter of 10 inches. Find its volume.

Ans. _____

4 pts 2. A rhombus is formed by two radii and two chords of a circle whose diameter is 32. What is the area of the rhombus?

Ans. _____

5 pts 3. Ruby's doghouse is in the shape of a regular hexagon. Ruby's rope, attached to the front of the doghouse as shown, is 2 yards long. Each side of the doghouse is 1 yard long. What is the area that the rope can reach outside of the doghouse?



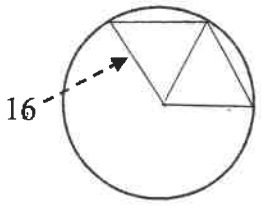
Ans. _____

Areas and Volumes

1. $V = \frac{1}{3}(5)^2 \pi(12) = 100\pi$.

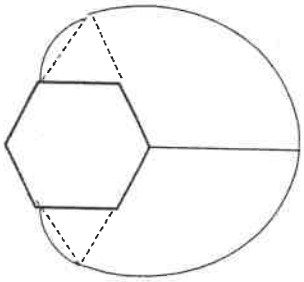
Ans. 100π

2. The figure at right shows two equilateral triangles with 16 ft.



Area $= 2\left(\frac{16^2\sqrt{3}}{4}\right) = 128\sqrt{3}$. **Ans. $128\sqrt{3}$**

3. The area will be 2/3 of a 2 yard radius circle, then 1/6 of a 1 yard radius circle two times gives $\frac{2}{3}\pi(4) + 2\left(\frac{1}{6}\pi(1)\right) =$



$\frac{8}{3}\pi + \frac{1}{3}\pi = 3\pi$. **Ans. 3π**

5 Areas and Volumes Nov 2015 (You may use calculators)

3 pts 1. A 6 inch by 8 inch by 12 inch box is setting on its smallest face. What is the total surface area of the box, if it is open on the top?

Ans. _____

4 pts 2. Spherical planet A has a surface area of 270,000,000 square miles. Spherical planet B has a volume 8/27 of planet A's volume. Find the surface area of planet B in square miles.

Ans. _____

5 pts 3. A fancy salt shaker in the shape of a pyramid has an equilateral triangle measuring 2 cm on a side for a base and a height of 8 cm. Find the capacity of the shaker in grains of salt if there are 27 grains of salt per mm^3. Assume all grains of salt are whole grains. Round answer to nearest hundred grains.

Ans. _____

Areas and Volumes

1. $6(8) + 2(8)(12) + 2(6)(12) = 48 + 192 + 144 = 384.$ **Ans. 384**

2. Since the volumes are in a ratio of 8 to 27, then the radii are in a ratio of 2 to 3. This makes the surface areas ratio 4 to 9. Thus $\frac{4}{9}(270,000,000) = 120,000,000.$ **Ans. 120,000,000**

3. If the density of salt is 27 grains per mm^3, then this would be 27,000 grains per cm^3.

The volume of the salt shaker is $\frac{1}{3}\left(\frac{\sqrt{3}}{4}\cdot 2^2\right)(8) = \frac{8}{3}\sqrt{3}$ cm^3. Number of grains:

$\frac{8}{3}\sqrt{3}(27,000) = 124,707.658$ → Rounded: 124,700. **Ans. 124,700**

5 Areas and Volumes Nov 2014 (You may use calculators)

3 pts 1. Cube A has base edges of 6. Cube B has edges half as long as cube A. What is their combined volumes?

Ans. _____

4 pts 2. A rectangular pyramid has base edges of 6 and 8 and a height of 4. All the lateral edges are congruent. Find the total surface area of the pyramid.

Ans. _____

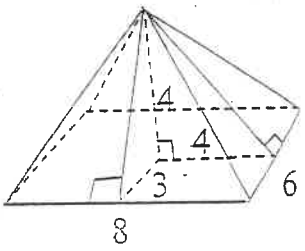
5 pts 3. . A spherical balloon has a volume of 288π cubic cm. If it begins losing volume at the rate of 1.8π cubic cm. per hour, how many hours will it take until the number for its surface area equals the number for its volume?

Ans. _____

Areas and Volumes Nov 2014

1. $6^3 + 3^3 = 216 + 27 = 243$. **Ans. 243**

2. In the figure at right, the right triangle whose hypotenuse is the altitude of the triangle whose base is 6, is $4\sqrt{2}$. The right triangle whose hypotenuse is the triangle whose base is 8 is 5. Both results of Pythagorean Theorems. The triangle Whose base is 8 has area 20. The triangle whose base is 6 is $12\sqrt{2}$. TSA = $48 + 40 + 24\sqrt{2}$.

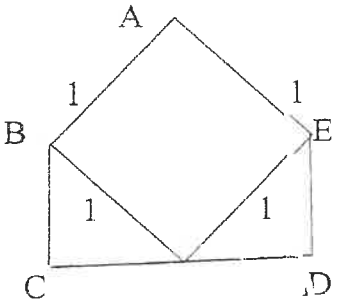


Ans. $88 + 24\sqrt{2}$

3. $V = SA \rightarrow 4/3\ \pi r^3 = 4\pi r^2 \rightarrow 1/3r = 1$, so $r = 3$. $V = 4/3\ \pi 3^3 = 36\pi$. Volume change: $288\pi - 36\pi = 252\pi$. $252\pi/18\pi = 14$ hrs. **Ans. 14 hrs**

5 Areas and Volumes Nov 2013 (You may use calculators)

3 pts 1. Home plate used in baseball is in the shape of the pentagonal region ABCDE at right. The square with 1 foot sides has two right isosceles triangles attached as shown in the figure. What is the area of home plate?



Ans. _____

4 pts 2. A hemisphere and an inscribed right cone have the same base. What is the ratio of the volume of the cone to the volume of the hemisphere?

Ans. _____

5 pts 3. Two circles are concentric. A chord ***C*** units long of the larger circle is tangent to the smaller circle. Express the area between the two circles, in terms of ***C***.

Ans. _____

Areas and Volumes

1. The area of the square is 1. The two triangles can be put together to make a square whose diagonals are 1, the area being half the product of the diagonals. **Ans. 1.5**

2. The cone and hemisphere have same base and same height. The volume of the cone is $1/3\ \pi r^3$, the hemisphere is $2/3\ \pi r^3$. Ratio is 1 to 2. **Ans. 1 to 2**

3. Let R be the radius of the large circle and r be the radius of the smaller circle. Then the area of the annulus is $\pi R^2 - \pi r^2$ or $\pi(R^2 - r^2)$. $(1/2C)^2 = R^2 - r^2$. **Ans.** $\frac{\pi}{4}C^2$

5 Areas and Volumes Nov 2012 (You may use calculators)

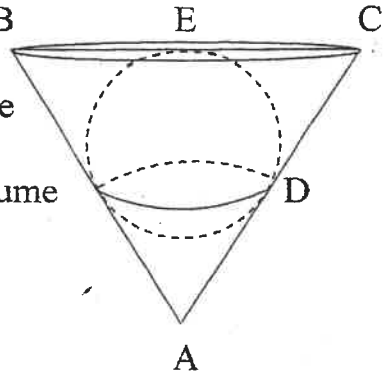
3 pts 1. A slanted pentagonal pyramid has a base area of 32 cm^2. If it stands 3 cm high, what is its volume?

Ans. _____

4 pts 2. The area of a cross-section of a pyramid is 36 cm^2. The cross-section is 4 cm from the base and the height of the pyramid is 16 cm. What is the volume of the pyramid?

Ans. _____

5 pts 3. $\overline{BC}$ is the diameter of the base of the right circular cone. The measure of angle BAC is 60^o. A metallic ball is placed in the cone tangent to the lateral surface such that AD = 20. The ball is also tangent to the base of the cone at point E. Find the exact volume of the cone or round the volume to the nearest 100^{th}.



Ans. _____

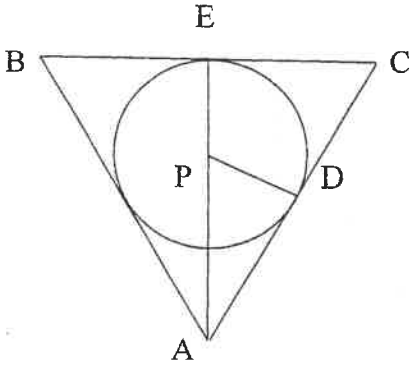
Areas and Volumes

1. Volume = 1/3 area of base times the height: 1/3 (32) 3 = 32 **Ans. 32cm^3**

2. $\frac{k^2}{h^2}A_B = A_{XC} \rightarrow \frac{3^2}{4^2}(A_B) = 36$ $A_B = 36\left(\frac{16}{9}\right) = 64$. $V = \frac{1}{3}(64)(16) = 1024/3$

Ans. = 1024/3 or 341 ⅓ or 341.33

3. Draw a perpendicular from D to the center of the sphere meeting at P. ΔAPD is a 30-60-90, so $PD = 20/\sqrt{3}$ and $AP = 40/\sqrt{3}$. This makes $AE = 60/\sqrt{3}$. Since ΔAEC is also a 30-60-90, then $\frac{AD}{AP} = \frac{AE}{EC}$ or $\frac{20}{20/\sqrt{3}} = \frac{60/\sqrt{3}}{EC}$. Thus $20EC = \frac{60}{\sqrt{3}} \cdot \frac{20}{\sqrt{3}} = 20(20)$ or $EC = 20$. So the volume of the cone is $\frac{1}{3}\pi(400)\frac{60}{\sqrt{3}} =$ **Ans.** $8000\sqrt{3}\,\pi/3$



5 Areas and Volumes Nov 2011 (You may use Calculators)

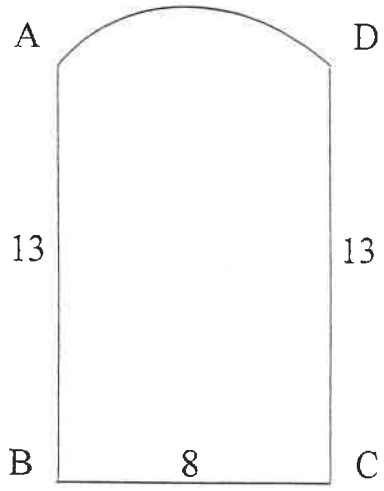
3 pts 1. How many little cubical blocks with side lengths of 2 units does it take to make a large solid cube with a side length of 8 units?

Ans. _____

4 pts 2. Assume that all bears have the same body structure. A bear weighing 648 lbs. stands 42 inches high from the bottom of his paws to the tip of his shoulders when on all fours. How tall does a bear weighing 375 lbs. stand?

Ans. _____

5 pts 3. Given: AB = CD = 13, BC = 8, and arc AD is an arc of the circle centered at the midpoint of side BC. Find the area of the figure in the drawing. Round your answer to 4 decimal places.



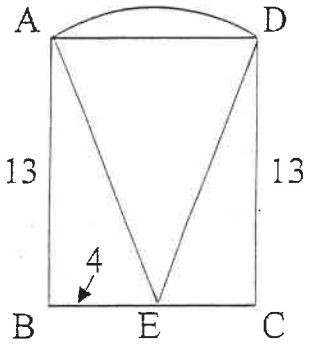
Ans. _____

Areas and Volumes

1. It would take 4 cubes to fill the bottom of one side, 16 cubes to fill one row 4 cubes high, and it will take 64 to fill all four rows of the base. **Ans. 64**

2. $\frac{375}{648} = \frac{125}{216} = \frac{5^3}{6^3}$. The weight of the bears is in three dimensions. The height is in one dimension. So $\frac{5}{6} = \frac{x}{42}$ or $x = 35$. **Ans. 35 inches**

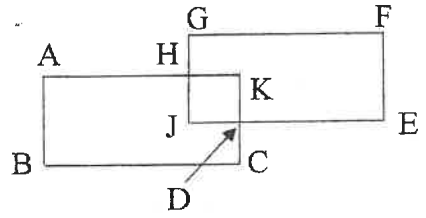
3. Let E be the midpoint of $\overline{BC}$. Draw $\overline{AE}$ and $\overline{DE}$.
Then $m\angle AEB = m\angle DEC = \tan^{-1}(13/4)$ and
$m\angle AED = 180 - 2\tan^{-1}(13/4)$. Sector AED then has radius $\sqrt{4^2+13^2} = \sqrt{185}$ and area $\frac{180 - 2\tan^{-1}(13/4)}{360}(185\pi)$ and the area of the figure is $(2)(1/2)(4)(13) + \frac{180 - 2\tan^{-1}(13/4)}{360}(185\pi)$.
Using calculator: 107.2223



Ans. 107.2223

5 Areas and Volumes Nov 2010 (You may use calculators)

3 pts 1. In the figure: AB = GJ = AH = 2 units. ABCK and GFEJ are rectangles. $\overline{GJ} \perp \overline{AK}$. HJ = JD. GF = BC = 3 units. Find the area of ABCDEFGH.

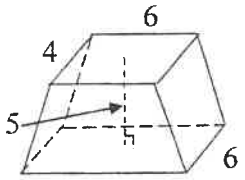


Ans. _____

4 pts 2. A circle intersects a sphere 7 inches from its center. If the area of the circle is 576π sq. in., what is the surface area of the sphere? Give answer to nearest square inch.

Ans. _____

5 pts 3. The opposite lateral faces of this frustum are congruent. Find the volume of the frustum. The height of the frustum is 5.



Ans. _____

Areas and Volumes

1. The areas of the two large rectangles 12. The overlap is 1. 12 – 1 = 11. **Ans. 11**

2. The radius of the circle is $\sqrt{576} = 24$. The radius of the sphere is 25, since it forms a 7-24-25 Δ. The surface area of the sphere is $4\pi(25)^2 = 2500\pi = 7853.98$. **Ans. 7854**

3. To find the height of the prism that the frustum was made from, let x be the height of the pyramid on top of the frustum that completed the rest of the pyramid. By ratios: $\frac{x}{x+5} = \frac{4}{6}$ ➔ 6x = 4x + 20, thus x = 10. Other base edge is 9. The volume of the frustum is the difference of the volumes of the two pyramids: $\frac{1}{3}(54)(15) - \frac{1}{3}(24)(10) = 190$. **Ans. 190**