1 Arithmetic with Literal Equations Feb 2019 (No Calculators)

3 pts 1. A formula for word problems involving rate and time is

$R_1T_1 + R_2T_2 = A$. Solve for R_1. **Ans.** ________________

4 pts 2. Express the following sum in base seven: $231_4 + 314_5 + 152_6$.

Ans. ________________

5 pts 3. The formula for the total surface area of a right hexagonal prism with a regular hexagonal base is $A = 6sh + 3\sqrt{3}s^2$. Solve for s.

Ans. ________________

Solutions – Arithmetic with Literal Equations

(1) $R_1T_1 + R_2T_2 = A \Rightarrow R_1T_1 = A - R_2T_2 \Rightarrow R_1 = \frac{A - R_2T_2}{T_1}$. **Ans.** $\frac{A - R_2T_2}{T_1}$

(2) $231_4 = 2(16) + 3(4) + 1 = 32 + 12 + 1 = 45$, $314_5 = 3(25) + 5 + 4 = 84$, $152_6 = 36 + 5(6) + 2 = 68$. $45 + 84 + 68 = 197$. $197 = 4(49) + 0(7) + 1 = 401_7$. **Ans.** 401_7

(3) $A = 6sh + 3\sqrt{3}s^2 \Rightarrow 3\sqrt{3}s^2 + 6hs - A \Rightarrow s = \frac{-6h \pm \sqrt{36h^2 - 4 \cdot 3\sqrt{3}(-A)}}{6\sqrt{3}} = \frac{-6h \pm 2\sqrt{9h^2 + 3\sqrt{3}A}}{6\sqrt{3}}$

$\frac{-3h \pm \sqrt{9h^2 + 3\sqrt{3}A}}{3\sqrt{3}} = \frac{-3\sqrt{3}h \pm 3\sqrt{\sqrt{3}h^2 + A}}{9} = \frac{-\sqrt{3}h \pm \sqrt{\sqrt{3}h^2 + A}}{3}$. All > 0. **Ans.** ~~$\frac{-\sqrt{3}h + \sqrt{\sqrt{3}h^2 + A}}{3}$~~

$\frac{-h\sqrt{3} + \sqrt{3h^2 + A\sqrt{3}}}{3}$

1 Arithmetic with Literal Equations Feb 2018 (No Calculators)

3 pts 1. Solve the following for m in simplest form: mpr + dh = dmr – pr.

Ans. ____________________

4 pts 2. If each of the variables m, n, and p are not equal to 0, and $4m^2p-5mp^2=n$, solve for p in terms of m and n.

Ans. ____________________

5 pts 3. Ben drove 770 miles to visit his grandparents for Thanksgiving. On the 770 mile return trip home he averaged 15 mph slower and it took him 3 hrs longer. What was his average speed on his return trip?

Ans. ____________________

Solutions – Arithmetic with Literal Equations Feb 2018

1. mpr + dh = dmr – pr ➔ mpr – dmr = -dh – pr ➔ m(pr – dr) = -dh – pr. **Ans.** $\frac{-dh-pr}{pr-dr}$

2. $4m^2p-5mp^2=n$ ➔ $5mp^2 - 4m^2p + n = 0$. Using the quadratic formula:

$$p=\frac{4m^2\pm\sqrt{16m^4-4(5mn)}}{10m}=\frac{4m^2\pm2\sqrt{4m^4-5mn}}{10m}=\frac{2m^2\pm\sqrt{4m^4-5mn}}{5m}$$

Ans. $\frac{2m^2\pm\sqrt{4m^4-5mn}}{5m}$

3. (1) RT = 770 and (2) (r – 15)(T + 3) = 770 or RT - 15T + 3R – 45 = 770. Since RT = 770, then 3R – 15T = 45 and R = 5T + 15. Plugging into (1): (5T + 15)T = 770 or

$5T^2 + 15T - 770 = 0$ ➔ $T^2 + 3T - 154 = 0$ ➔ (T + 14)(T – 11) = 0, so T = 11 and in (1):

R(11) = 770. So R = 70. In (2): R – 15 = 70 – 15 = 55. **Ans. 55 or 55mph**

1 Literal Equations Feb 2016-17 (No Calculators)

3 pts 1. Given that $px + a = tx + e$, where $x \neq 0$, solve for $p - t$.

Ans. ____________________

4 pts 2. The least common multiple of two numbers is 144 and the greatest common divisor is 4. If one of the numbers is 16, what is the other?

Ans. ____________________

5 pts 3. If $9ax + 9by + 105x = 4ay + 66y + 13bx$, find the value of $a - b$, if neither x nor y are zero.

Ans. ____________________

Solutions – Literal Equations – Feb 2016-17

1. $px + a = tx + e \Rightarrow px - tx = e - a \Rightarrow (p - t)x = e - a \Rightarrow p - t = \frac{e-a}{x}$. **Ans.** $\frac{e-a}{x}$

2. $144 = 2^4 \cdot 3^2$, $16 = 2^4$. The other number must be $2^2 \cdot 3^2 = 36$. **Ans. 36**

3. $9ax + 9by + 105x = 4ay + 66y + 13bx \Rightarrow 105x - 66y = (13b - 9a)x - (9b - 4a)y$. Thus (1) $13b - 9a = 105$ and (2) $9b - 4a = 66$. 4(1) – 9(2): $52b - 36a = 420 - (81b - 36a = 594)$ $-29b = -174$, $b = 6$. In (2): $9(6) - 4a = 66 \Rightarrow -4a = 12$, $a = -3$. $a - b = -3 - 6 = -9$. **Ans. -9**

1 Arithmetic with Literal Equations Feb 2016 (No Calculators)

3 pts 1. Solve the following for T_1: $\frac{R_1}{T_1} + \frac{R_2}{T_2} = 1$.

Ans. ______________

4 pts 2. Find the smallest integer k such that $11k$ divided by 19 has a remainder of 10.

Ans. ______________

5 pts 3. In general, S = 1.2F – 24 and H = $\frac{1}{6}$(S + 1), where S = shoe size, F = foot length in centimeters, and H = height in meters. If a person is 1.9 meters tall, how long is his foot in centimeters according to the above formulas? Round answer to nearest tenth.

Ans. ______________

Solutions – Arithmetic with Literal Equations

1. $\frac{R_1}{T_1} + \frac{R_2}{T_2} = 1 \Rightarrow \frac{R_1T_2 + R_2T_1}{T_1T_2} = 1 \Rightarrow R_1T_2 + R_2T_1 = T_1T_2 \Rightarrow R_1T_2 = T_1(T_2 - R_2)$ **Ans.** $\frac{R_1T_2}{T_2 - R_2}$

2. Basically 11k = 19n + 10. So we plug in positive integers for n until we find one that is divisible by 11: 1 ➔ 29, 2 ➔ 48, 3 ➔ 67. Notice the ten's digit increases by 2 and the umit's digit decreases by 1, thus 86, 105, 124, 143. 143/11 = 13. **Ans. 13**

3. $1.9 = \frac{1}{6}(S + 1) \Rightarrow 11.4 = S + 1$, S = 10.4. 10.4 = 1.2F – 24, 1.2F = 34.4, F = 344/12 =

$172/6 = 86/3 = 28\frac{2}{3} = 28.7$ to nearest 10^{th}. **Ans. 28.7**

1 Arithmetic with Literal Equations Feb 2015 (No Calculators)

3 pts 1. If none of the variables in the equation $A - P = \frac{AB}{C}$ are equal to zero, solve for A as a single fraction in simplest form.

Ans. ______________________

4 pts 2. Find the remainder when the product of 342(130)(234) is divided by 7.

Ans. __________

5 pts 3. The surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$. Solve the equation for the radius (r). Express your answer as a single fraction in simplest form.

Ans. ______________________

Solutions – Arithmetic with Literal Equations

1. AC – PC = AB ➔ AC - AB = PC ➔ A(C - B) = PC. $A = \frac{CP}{C-B}$. **Ans.** $\frac{CP}{C-B}$

2. Dividing 342, 130, and 234 by 7yields respective remainders 6,4,3. 6(4)(3) = 72. Dividing by 7 gives remainder of 2. **Ans. 2**

3. $A = 2\pi r^2 + 2\pi rh$ ➔ $0 = 2\pi r^2 + 2\pi rh - A$. Using the quadratic formula:
$r = \frac{-2\pi h \pm \sqrt{4\pi^2h^2 - 4(2\pi)(-A)}}{4\pi} = \frac{-2\pi h \pm 2\sqrt{\pi^2h^2 + 2\pi A}}{4\pi}$. Since the radius cannot be negative,
then $r = \frac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$. **Ans.** $\frac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$

1 Arithmetic with Literal Equations Feb 2014 (No Calculators)

3 pts 1. Solve the following for H: $S = 2LW + 2LH + 2WH$.

Ans. ____________________

4 pts 2. Solve the following for c: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ans. ____________________

5 pts 3. Consider an interesting five-digit number A. If 1 is placed at the end of A to make a six-digit number, it is 3 times the number made by placing 1 in front of A to make a six-digit number. Find A.

Ans. ____________________

Solutions – Arithmetic with Literal Equations

1. $S - 2LW = H(2L + 2W) \Rightarrow H = \dfrac{S - 2LW}{2L + 2W}$ **Ans.** $\dfrac{S-2LW}{2L+2W}$

2. Since this is the solution for x in $ax^2 + bx + c = 0$, then $c = -ax^2 - bx$. **Ans.** $-ax^2 - bx$

3. Let A = the number. Then $10A + 1 = 3(A + 100{,}000) \Rightarrow 10A + 1 = 3A + 300{,}000 \Rightarrow$ $7A = 299{,}999 \Rightarrow A = 42{,}857$. **Ans. 42,857**

1 Arithmetic with Literal Equations Feb 2013 (No Calculators)

3 pts 1. Solve $L = a + (n - 1)d$ for n. Express your answer as a single fraction in simplest form.

Ans. ______________________

4 pts 2. P is increased by 20%. The new value of P is then decreased by 30%. What percent of P is the final result?

Ans. ______________________

5 pts 3. If $\frac{a+2}{m-1} + \frac{b-2}{m+1} = \frac{a-2b+8}{m^2-1}$, where $m \neq 1$ or -1, solve for m. Express your answer as a single fraction in simplest form.

Ans. ______________________

Solutions – Arithmetic with Literal Equations

1. $L = a + (n-1)d \Rightarrow (L-a)/d = n - 1 \Rightarrow n = \frac{L-a}{d} + 1 \Rightarrow n = \frac{L-a+d}{d}$. **Ans.** $\frac{L-a+d}{d}$

2. $P + .2P = 1.2P$. $1.2P - .3(1.2P) = .7(1.2P) = .84P$. $.84 = 84\%$. **Ans. 84%**

3. $\frac{a+2}{m-1} + \frac{b-2}{m+1} = \frac{a-2b+8}{m^2-1} = (a+2)(m+1) + (b-2)(m-1) = a - 2b + 8 =$
$am + 2m + a + 2 + bm - 2m - b + 2 = a - 2b + 8 \Rightarrow m(a+b) + a - b + 4 = a - 2b + 8$
$m(a+b) = -b + 4$. Thus $m = \frac{4-b}{-}$.
Ans. $\frac{4-b}{}$

Arithmetic with Literal Equations Jan 2012 (No Calculators)

3 pts 1. Solve for a, assuming b $\neq$ - 1/2: $2x = 2a + 4ab - 2$.

Ans. ____________________

4 pts 2. Given that $D = rt$ and $r = 2x^2 - 1$, find x if $D = 7$ and $t = 18$. Express your answer as a fraction.

Ans. ____________________

5 pts 3. Solve $a = \frac{1}{a} - ep^D$ for D.

Ans. ____________________

Solutions – Arithmetic with Literal Equations

1. $2x = 2a + 4ab - 2 \Rightarrow 2x + 2 = a(2 + 4b) \Rightarrow a = \frac{2x+2}{2+4b} = \frac{x+1}{2b+1}$. **Ans.** $\frac{x+1}{2b+1}$

2. $D = rt,\ 7 = (2x^2-1)(18) \Rightarrow 36x^2 = 25 \Rightarrow x = \pm 5/6$. **Ans. $\pm 5/6$**

3. $a = \frac{1}{a} - ep^D \Rightarrow a^2 = 1 - aep^D \Rightarrow 1 - a^2 = aep^D$. $\frac{1-a^2}{ae} = p^D$. Taking the log of each side:

$\log \frac{1-a^2}{ae} = D \log p$. So $D = \frac{\log \frac{1-a^2}{ae}}{\log p} = \log_p \frac{1-a^2}{ae}$. **Ans.** $\log_p \frac{1-a^2}{ae}$

1 Arithmetic with Literal Equations Feb 2011 (No Calculators)

3 pts 1. Find the sum of the natural number divisors of 108.

Ans. ____________

4 pts 2. If A and B are negative integers, $\frac{A}{B}(D) = 2C$, $AC = B(D + 4)$ and $D = -8$, find C.

Ans. ____________

5 pts 3. If a = b, find all values for a, b, c, and d that make the following true.

$$\frac{a(c+5)}{d-1} = a - b$$

Ans. ______________________________

Solutions – Arithmetic with Literal Equations

1. 108 = 1(108), 2(54), 3(36), 4(27), 6(18), 9(12). The sum is 280. **Ans. 280**

2. (1): $-8\left(\frac{A}{B}\right) = 2C$ and (2): $AC = B(-8 + 4)$ In (2): $B = \left(\frac{AC}{-4}\right)$. Subbing (2) into (1):

$$-8\left(\frac{A}{\frac{AC}{-4}}\right) = 2C \rightarrow \left(\frac{32}{C}\right) = 2C \rightarrow 16 = C^2.$$ Thus $C = \pm 4$. In (1) since A and B are both negative, C = -4. **Ans. -4**

3. **Ans. If a = b = 0, c = all reals, and d = all reals ≠ 1.**
or If a = b and neither = 0, c = -5 and d = all reals ≠ 1.

 Arithmetic with Literal Equations Feb 2010 (No Calculators)

3 pts 1. If $ax = \frac{a^2bx + abp}{ax + p}$, where $a > 0$ and $ax \neq -p$, find x in simplest form.

Ans. ____________________

4 pts 2. A grocery store had a sale on eggs, selling thirteen eggs for the usual price of a dozen eggs. As a result, the price of the eggs was reduced by 4 cents a dozen. What was the original price for a dozen eggs?

Ans. ____________________

5 pts 3. If $x = \frac{1}{4-y}$, compute $\frac{1}{x} + 4x + y - yx - 1$.

Ans. ____________________

Arithmetic with Literal Equations

1. $ax = \frac{a^2bx + abp}{ax + p} = \frac{ab(ax + p)}{ax + p} = ab$. Thus $x = b$. **Ans. b** or **x = b**

2. Let the price for the dozen eggs be P. Then the price per egg is $\frac{P}{12}$ or $\frac{P}{13}$. Then the price per dozen: $12(\frac{P}{12}) - .04 = 12(\frac{P}{13}) \rightarrow P - .04 = \frac{12}{13}P \rightarrow -.04 = -\frac{1}{13}P$. **Ans. 52 cents**

3. Substituting $x = \frac{1}{4-y}$ into $\frac{1}{x} + 4x + y - yx - 1 \rightarrow 4 - y + \frac{4}{4-y} + y - \frac{y}{4-y} - 1 \rightarrow$
$4 - y + \frac{4-y}{4-y} + y - 1 \rightarrow 4$. **Ans. 4**