2 Logs and Log Equations Feb 2019 (No Calculators)

3 pts 1. Express $\log_3 8 - \log_3 32 + \log_3 16$ in simplest form.

Ans. _____

4 pts 2. Simplify: $(\log_9 625)\,(\log_4 \frac{1}{243})\,(\log_{125} 8)$.

Ans. _____

5 pts 3. If $15x^{2p} = 38x^{p} - 24$, find p in terms of x.

Ans. _____

Logs and Log Equations

(1) $\log_3 8 - \log_3 32 + \log_3 16 = \log_3 \frac{8\cdot 16}{32} = \log_3 4.$ **Ans. $\log_3 4$**

(2) $(\log_9 625)\,(\log_4 \frac{1}{243})(\log_{125} 8) = \left(\frac{\log 625}{\log 9}\right)\left(\frac{\log \frac{1}{243}}{\log 4}\right)\left(\frac{\log 8}{\log 125}\right) = \left(\frac{\log 625}{\log 125}\right)\left(\frac{\log \frac{1}{243}}{\log 9}\right)\left(\frac{\log 8}{\log 4}\right)$

$= (\log_{125} 625)(\log_9 \frac{1}{243})(\log_4 8) = \frac{4}{3}\cdot\frac{-5}{2}\cdot\frac{3}{2} = -5.$ **Ans. -5**

(3) $15x^{2p} = 38x^{p} - 24 \rightarrow 15(x^p)^2 - 38x^p + 24 = 0 \rightarrow (5x^p - 6)(3x^p - 4) = 0.$ Either

(1) $5x^p - 6 = 0$ or (2) $3x^p - 4 = 0$. In (1) $x^p = 6/5$ or $p \log x = \log 6/5$ or $p = \frac{\log 6/5}{\log x} = \log_x 6/5$

Similarly for (2) $p = \log_x 4/3$.

Ans. $\log_x 6/5$ or $\log_x 4/3$

2 Logs and Log Equations Feb 2018 (No Calculators)

3 pts 1. Solve the following for x: $\log 6 + \log (x + 5) = \log 96$.

Ans. _____

4 pts 2. Let k be a positive number not equal to one. If $(\log_k x)(\log_5 k) = 2\frac{1}{2}$, find x.

Ans. _____

5 pts 3. If $\log_8 3 = p$ and $\log_3 5 = q$, then express $\log_{10} 5$ in terms of p and q.

Ans. _____

Logs and Log Equations

1. $\log 6 + \log (x + 5) = \log 96$, thus $6(x + 5) = 96 \rightarrow x + 5 = 16$, so $x = 11$. **Ans. 11**

2. $(\log_k x)(\log_5 k) = 2\frac{1}{2}$ or $\frac{\log x}{\log k}\cdot\frac{\log k}{\log 5} = \frac{5}{2} \rightarrow \frac{\log x}{\log 5} = \frac{5}{2} \rightarrow \log_5 x = 2\frac{1}{2}$. $x = 5^{2\frac{1}{2}} = 25\sqrt{5}$.

Ans. $25\sqrt{5}$

3. $\log_8 3 = p \rightarrow \frac{\log 3}{3\log 2} = p \rightarrow \log 2 = \frac{1}{3p}\log 3$. $\log_3 5 = q \rightarrow \frac{\log 5}{\log 3} = q$, so $\log 5 = q \log 3$.

$$\log_{10} 5 = \frac{\log 5}{\log 10} = \frac{\log 5}{\log 5 + \log 2} = \frac{q\log 3}{q\log 3 + \frac{1}{3p}\log 3} = \frac{q}{q + \frac{1}{3p}} = \frac{3pq}{3pq+1}.$$

Ans. $\frac{3pq}{3pq+1}$

2 Logs and Log Equations Feb 2016-17 (No Calculators)

3 pts 1. Find the product of the solutions of the equation $\log_2(x^2)=6$.

Ans. _____

4 pts 2. Evaluate $\left(\left(5^{\log_5 36}\right)^{\log_6 7}\right)^{\log_{49} 19}$

Ans. _____

5 pts 3. $f(x) = x^2 - rx + s$. When $f(x) = 0$, $x = \log_4 8$ or $x = \log_8 4$. Find $f(1)$.

Ans. _____

Log Equations

1. $\log_2(x^2)=6 \rightarrow 2^6 = x^2$, $x = \pm 8$. The product is - 64. **Ans. -64**

2. $\left(\left(5^{\log_5 36}\right)^{\log_6 7}\right)^{\log_{49} 19} \rightarrow 5^{\frac{2\log 6}{\log 5}\cdot\frac{\log 7}{\log 6}\cdot\frac{\log 19}{2\log 7}} \rightarrow 5^{\frac{\log 19}{\log 5}} \rightarrow 5^{\log_5 19} = 19.$ **Ans. 19**

3. $x = \log_4 8 \rightarrow x = 3/2 \rightarrow 2x - 3 = 0$. $x = \log_8 4 \rightarrow x = 2/3 \rightarrow 3x - 2 = 0$. Thus

$6x^2 - 13x + 6 = 0$, so $f(x) = x^2 - \frac{13}{6}x + 1 = 0$. $f(1) = 1 - 2\frac{1}{6} + 1 = -\frac{1}{6}$. **Ans. -1/6**

2 Logs and Log Equations Feb 2016 (No Calculators)

3 pts 1. Simplify the following: $\log_{27} 9\sqrt[4]{27}$.

Ans. _____

4 pts 2. Solve for x, if $\log_3(5x^2-8x-4)-\log_3(x^2-6x+8)=3$

Ans. _____

5 pts 3. Solve for x, if $\dfrac{\log_2 3\cdot\log_4 5\cdot\log_6 7}{\log_4 3\cdot\log_6 5\cdot\log_8 7}=\log_{10}\left(\dfrac{1}{x}\right)$

Ans. _____

Logs and Log Equations

1. $\log_{27} 9\sqrt[4]{27} = \log_{27} 9+\dfrac{1}{4}\log_{27} 27=\dfrac{2}{3}+\dfrac{1}{4}=\dfrac{11}{12}$. **Ans. 11/12**

2. $\log_3(5x^2-8x-4)-\log_3(x^2-6x+8)=3$ ➔ $\log_3\dfrac{5x^2-8x-4}{x^2-6x+8}=3$ ➔ $5x^2 = 8x-4 = 27x^2 - 162x + 216$

$0 = 22x^2 - 154x + 220 = 22(x^2 - 7x + 10)$ ➔ $(x-5)(x-2)=0$. Only 5 works.

Alt. Sol: If students factor : $\dfrac{5x^2-8x-4}{x^2-6x+8}=\dfrac{(5x+2)(x-2)}{(x-4)(x-2)}$, then $\dfrac{5x+2}{x-4}=27$ ➔ $22x=110$ **Ans. 5**

3. $\dfrac{\log_2 3\cdot\log_4 5\cdot\log_6 7}{\log_4 3\cdot\log_6 5\cdot\log_8 7}=\log_{10}\left(\dfrac{1}{x}\right)$ ➔ $\dfrac{\log 3}{\log 2}\cdot\dfrac{\log 5}{\log 4}\cdot\dfrac{\log 7}{\log 6}\cdot\dfrac{\log 4}{\log 3}\cdot\dfrac{\log 6}{\log 5}\cdot\dfrac{\log 8}{\log 7}=\dfrac{\log 8}{\log 2}=3=\log_{10}\left(\dfrac{1}{x}\right)$.

So $10^3 = 1/x$, thus $x = 1/1000$. **Ans. 1/1000**

2 Logs and Log Equations Feb 2015 (No Calculators)

3 pts 1. If $6 \log_x x = 2n$, solve for n.

Ans. _____

4 pts 2. Solve for x, if $\log_6(x - 2) + \log_6(x + 3) = 2$.

Ans. _____

5 pts 3. $\log A = 2x + 1$, $\log B = 2x - 1$. Find all value(s) of x such that

$$\log_B A - \log_A B = \frac{A}{B}.$$

Ans. _____

Logs and Log Equations

1. $\log_x x = 1$, so $6 \log_x x = 2n \rightarrow 6(1) = 2n$. $n = 3$. **Ans. 3**

2. $\log_6(x - 2) + \log_6(x + 3) = 2 \rightarrow x^2 + x - 6 = 36 \rightarrow x^2 + x - 42 = 0 \rightarrow (x + 7)(x - 6) = 0$.
So x = -7 or 6. But x cannot be -7. **Ans. 6**

3. Since $\log A = 2x + 1$, then $A = 10^{2x+1}$. Likewise $\log B = 2x - 1$, then $B = 10^{2x-1}$.

$$\log_B A - \log_A B = \frac{A}{B} \rightarrow \frac{\log A}{\log B} - \frac{\log B}{\log A} = \frac{10^{2x+1}}{10^{2x-1}} \rightarrow \frac{2x+1}{2x-1} - \frac{2x-1}{2x+1} = 10^2 \rightarrow$$

$$\frac{(2x+1)^2 - (2x-1)^2}{4x^2 - 1} = 100 \rightarrow 8x = 400x^2 - 100 \rightarrow 0 = 400x^2 - 8x - 100 \rightarrow 0 = 100x^2 - 2x - 25.$$

$$x = \frac{2 \pm \sqrt{4 - 4(-2500)}}{200} = \frac{1 \pm \sqrt{2501}}{100}.$$ **Ans.** $\frac{1 \pm \sqrt{2501}}{100}$

2 Logs and Logarithmic Equations Feb 2014 (No Calculators)

3 pts 1. Find the sum: $\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27$.

Ans. _____

4 pts 2. Simplify completely: $8^{\log_2 5}$

Ans. _____

5 pts 3. Solve for x: $3\log_8(9x+5) - 2\log_4(x^2 - 1) = 2$.

Ans. _____

Logs and Log Equations

1. $\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27 = 3 + 3/2 + 1 + 3/4 + 3/5 =$

   $4 + (30 + 15 + 12)/20 = 4 + 57/20 = 6\ 17/20$ or $137/20$. **Ans. 6** $\frac{17}{20}$ **or 137/20**

2. $8^{\log_2 5} = (2^3)^{\log_2 5} = 2^{3\log_2 5} = 2^{\log_2 125} = 125$. **Ans. 125**

3. $3\log_8(9x+5) - 2\log_4(x^2 - 1) = 2$. $3\log_8(9x+5) = \log_8(9x+5)^3 = \log_2(9x+5)$

   $2\log_4(x^2 - 1) = \log_4(x^2 - 1)^2 = \log_2(x^2 - 1)$. Therefore

   $\log_2(9x+5) - \log_2(x^2 - 1) = 2 \rightarrow \log_2 \frac{9x+5}{x^2-1} = 2 \rightarrow \frac{9x+5}{x^2-1} = 4 \rightarrow 9x + 5 = 4x^2 - 4 \rightarrow$

   $4x^2 - 9x - 9 = 0 \rightarrow (4x - 3)(x - 3) = 0$. So $x = 3$ or $-3/4$. $-3/4$ cannot be used. **Ans. 3**

2 Logs and Log Equations Feb 2013 (No calculators)

3 pts 1. Find x, if $\log_x 128 = 7/3$

Ans. _____

4 pts 2. If $\log_4 P = \frac{2}{3}\log_4 27 - \frac{3}{2}\log_4 25 + 4\log_4 \sqrt{15} - 6\log_4 \sqrt{6}$, find P in simplest form.

Ans. _____

5 pts 3. Find all values of x such that

$$\log_5(2x^2 - 5) + \log_5(x - 1) = \log_5(13x^2 - 42x + 35)$$

Ans. _____

Logs and Logarithmic Equations

1. $\log_x 128 = 7/3 \rightarrow x^{\frac{7}{3}} = 128 \rightarrow \left(x^{7/3}\right)^{3/7} = \left(2^7\right)^{3/7} = 2^3 = 8.$ **Ans. 8**

2. $\frac{2}{3}\log_4 27 - \frac{3}{2}\log_4 25 + 4\log_4 \sqrt{15} - 6\log_4 \sqrt{6} \rightarrow$

$\log_4\left(3^3\right)^{2/3} - \log_4\left(5^2\right)^{3/2} + \log_4\left(15^{1/2}\right)^4 - \log_4\left(6^{1/2}\right)^6 = \log_4 \frac{3^2 15^2}{5^3 6^3} = \log_4 \frac{3^2 3^2 5^2}{5^3 3^3 2^3} =$

$\log_4 \frac{3}{40} = \log_4 P$. So $P = \frac{3}{40}$. **Ans.** $\frac{3}{40}$

3. $\log_5(2x^2 - 5) + \log_5(x - 1) = \log_5(13x^2 - 42x + 35) \rightarrow$

$(2x^2 - 5)(x - 1) = 13x^2 - 42x + 35 \rightarrow 2x^3 - 2x^2 - 5x + 5 = 13x^2 - 42x + 35 \rightarrow$

$2x^3 - 15x^2 + 37x - 30 = 0$. Solving synthetically:

3	2	-15	37	-30
		6	-27	30
2	2	-9	10	0
		4	-10	
	2	-5	0	

$\rightarrow 2x - 5 = 0$

Ans. 3, 2 or 5/2

Linear Coordinate G

2 Logs and Log Equations Jan 2012 (No Calculators)

3 pts 1. Find the value of constant k, if $\log_2 8 = k \log_8 2$.

Ans. _____

4 pts 2. Find the value of $(\log_7 8)(\log_6 7)(\log_5 6)(\log_4 5)$.

Ans. _____

5 pts 3. Solve for x if: $8(\log_4 x)^2 + 5 = 7 \log_4 x^2$.

Ans. _____

Logs and Log Equations

1. $\log_2 8 = k \log_8 2 \rightarrow 3 = k(1/3)$, so $k = 9$. **Ans. 9**

2. $(\log_7 8)(\log_6 7)(\log_5 6)(\log_4 5) = \frac{\log 8}{\log 7} \cdot \frac{\log 7}{\log 6} \cdot \frac{\log 6}{\log 5} \cdot \frac{\log 5}{\log 4} = \log_4 8 = 3/2$. **Ans. 3/2 or 1.5**

3. $8(\log_4 x)^2 + 5 = 7 \log_4 x^2 \rightarrow 8(\log_4 x)^2 - 7 \log_4 x^2 + 5 = 0 \rightarrow$
$8(\log_4 x)^2 - 14 \log_4 x + 5 = 0 \rightarrow (2 \log_4 x - 1)(4 \log_4 x - 5) = 0$. Thus either
(1) $2 \log_4 x - 1 = 0$ or (2) $4 \log_4 x - 5 = 0$. In (1) $\log_4 x = 1/2$, thus $4^{1/2} = x$, so $x = 2$.
In (2) $\log_4 x = 5/4$, thus $4^{5/4} = x$, so $x = ((2)^2)^{5/4} = 2^{5/2} = 4\sqrt{2}$. **Ans. 2 or $4\sqrt{2}$**

2 Logs and Log Equations Feb 2011 (No Calculators)

3 pts 1. If $\log a = .21$ and $\log b = .09$, find $\log a^2 b$.

Ans. _____

4 pts 2. $\ln a = 3$, and $\ln b = 2$. Find $\ln\left(\frac{a^2}{\sqrt{be}}\right)$

Ans. _____

5 pts 3. If $\log \sqrt{2x-1} + \log \sqrt{x-9} = 1$, solve for x.

Ans. _____

Logs and Log Equations

1. $\log a^2 b = 2 \log a + \log b = 2(.21) + .09 = .51.$ **Ans. .51**

2. $\ln\left(\frac{a^2}{\sqrt{eb}}\right) = 2 \ln a - \frac{1}{2}(\ln e + \ln b) = 2(3) - \frac{1}{2}(1+2) = 6 - 3/2 = 4\frac{1}{2}.$ **Ans. $4\frac{1}{2}$**

3. $\log \sqrt{2x-1} + \log \sqrt{x-9} = 1 \rightarrow \log \sqrt{(2x-1)(x-9)} = 1 \rightarrow \sqrt{(2x-1)(x-9)} = 10$
   $2x^2 - 19x + 9 = 100 \rightarrow 2x^2 - 19x - 91 = 0 \rightarrow (2x + 7)(x - 13) = 0.$ **Ans. 13**

2 Logs and Log Equations Feb 2010 (No Calculators)

3 pts 1. Find c, if $\log_c 100 = 10$. Express answer in radical form.

Ans. _____

4 pts 2. Solve the following: $\log_2(2x + 3) - \log_2(3x + 2) = 1$

Ans. _____

5 pts 3. Find all values of x such that $2\log_5(12 + x) = \log_5 3 + \log_5(15 - 4x)$

Ans. _____

Logs and Log Equations

1. $\log_c 100 = 10 \rightarrow c^{10} = 100$. $c = 100^{1/10} = (10^2)^{1/10} = 10^{1/5} = \sqrt[5]{10}$ **Ans.** $\sqrt[5]{10}$

2. $\log_2(2x + 3) - \log_2(3x + 2) = 1 \rightarrow \log_2 \frac{2x+3}{3x+2} = 1 \rightarrow \frac{2x+3}{3x+2} = 2 \rightarrow 2x + 3 = 6x + 4$.
Thus $-1 = 4x$ or $x = -1/4$.
Ans. $-1/4$

3. $2\log_5(12 + x) = \log_5 3 + \log_5(15 - 4x) \rightarrow \log_5(12 + x)^2 = \log_5(45 - 12x)$.
Thus $(12 + x)^2 = 45 - 12x \rightarrow 144 + 24x + x^2 = 45 - 12x \rightarrow x^2 + 36x + 99 = 0 \rightarrow$
$(x + 33)(x + 3) = 0$. Thus $x = -3$ or -33. But -33 makes log of negative number in left side of original equation, so cannot be a solution.
Ans. -3