4 Functions Feb 2019 (No Calculators)

3 pts 1. f(x) = 7x – 8, g(x) = 5x – 11 and h(x) = 3x + 7. Find the value of $\frac{f(8)+g(7)}{h(3)}$.

Ans. ____________________

4 pts 2. In the food chain, barracuda feed on bass and bass feed on shrimp. Suppose that the size of the barracuda population is estimated by the function r(b) = 1000 + $\sqrt{2b}$, where b is the size of the bass population. Also suppose that the size of the bass population is estimated by the function a(s) = 2050 + $\sqrt{s}$, where s is the size of the shrimp population. About how many barracuda are there when the size of the shrimp population is 4,000,000?

Ans. ____________________

5 pts 3. If f(2x+1) = $\frac{6x+3}{4x^2-2x-1}$, find f(x - 1).

Ans. ____________________

Functions

1. f(x) = 7x – 8, f(8) = 7(8) -8 = 48; g(x) = 5x – 11, g(7) = 5(7) – 11 = 24; h(x) = 3x+ 7, h(3) = 3(3) +7 = 16. $\frac{48+24}{16} = \frac{72}{16} = 4\frac{1}{2}$. **Ans. 4 1/2**

2. a(s) = 2050 + $\sqrt{4{,}000{,}000}$ = 4050. r(b) = 1000 + $\sqrt{2(4050)}$ = 1000 + $\sqrt{8100}$ **Ans. 1090**

3. f(2x+1) = $\frac{6x-3}{4x^2-2x-1}$. In the numerator f(x) = 3x. The denominator takes on the form of a quadratic $ax^2 + bx + c$. To get $4x^2$, it must be x^2, so $(2x + 1)^2 = 4x^2 + 4x + 1$. To get -2x in the f(2x + 1) function, then b must be -3, because -3(2x + 1) = -6x – 3 and this added to 4x would give the -2x in the f(2x + 1). Now we have -3 and +1 left over which = -2. To get the – 1 in the f(2x + 1) function we would have to add 1. Thus making c = 1. So f(x) = $\frac{3x}{x^2-3x+1}$.

f(x – 1) = $\frac{3(x-1)}{(x-1)^2-3(x-1)+1} = \frac{3x-3}{x^2-2x+1-3x+3+1} = \frac{3x-3}{x^2-5x+5}$. **Ans.** $\frac{3x-3}{x^2-5x+5}$

4 Functions Feb 2018 (No Calculators)

3 pts 1. Let the function f be defined as $f(x, y) = 9x - 11y + 32$. Find the value of $f(-6, -7)$.

Ans. ____________________

4 pts 2. For $f(x) = x^3 - 10x^2 + Ax + B$, A and B are real numbers. Given that f has one root of $x = 2$ and the sum of A and B is 9, find the value of $f(3)$.

Ans. ____________________

5 pts 3. If $f(x) = x^3 + 3x^2 - 13x - 15$, find the largest root of $f(\frac{x}{2})$.

Ans. ____________________

Functions

1. $f(-6,-7) = 9(-6) - 11(-7) + 32 = -54 + 77 + 32 = 109 - 54 = 55$. **Ans. 55**

2. For root 2, $0 = 8 - 10(4) + 2A + B$ or (1) $2A + B = 32$. Since (2) $A + B = 9$, then (1) – (2) = $A = 23$ and $B = -14$. Thus $f(x) = x^3 - 10x^2 + 23x - 14$ and $f(3) = 27 - 90 + 69 - 14$. **Ans. -8**

3. $f(x) = x^3 + 3x^2 - 13x - 15$, $f(\frac{x}{2}) = (\frac{x}{2})^3 + 3(\frac{x}{2})^2 - 13(\frac{x}{2}) - 15 = \frac{x^3}{8} + \frac{3x^2}{4} - \frac{13x}{2} - 15$. Finding the roots: $\frac{x^3}{8} + \frac{3x^2}{4} - \frac{13x}{2} - 15 = 0 \Rightarrow x^3 + 6x^2 - 52x - 120 = 0 \Rightarrow (x + 10)(x - 6)(x + 2) = 0$. **Ans. 6**

4 Functions Feb 2016-17 (No Calculators)

3 pts 1. Let f(x) = 4 – 3x, g(x) = 5x – 1, and h(x) = f(g(x)). If Jack plugs in a number for x and gets h(x) = 2, what was the value for x that Jack plugged in?

Ans. ____________________

4 pts 2. Find the range of the function f(x), if $f(x) = \frac{x}{1-x}$, where x is not equal to 1.

Ans. ____________________

5 pts 3. The quadratic function f(x) passes through the points (1, 2), (2, 5), and (3, 7).

Find the value of f(10).

Ans. ____________________

Functions

1. h(x) = f(g(x)) = 4 – 3(5x – 1) = -15x + 7 = 2 ➔ - 15x = -5, so x = 1/3. **Ans. 1/3**

2. $f(x) = \frac{x}{1-x}$. Let f(x) = y, interchange x and y ➔ $x = \frac{y}{1-y}$. Solving for y: x – xy = y ➔ xy + y = x ➔ $y = \frac{x}{x+1} = f^{-1}(x)$. Range is all Reals except -1. **Ans. All Reals except -1**

3. f(x) = ax² + bx + c. For (1, 2): (1) a + b + c = 2. For (2, 5): (2) 4a + 2b + c = 5. For (3, 7): (3) 9a + 3b + c = 7. (2) – (1): (4) 3a + b = 3; (3) – (1): (5) 8a + 2b = 5. (5) - 2(4): 2a = -1, so a = -1/2. In (4): 3(-1/2) + b = 3, so b = 9/2. In (1): (-1/2) + (9/2) + c = 2, so c = -2. Thus f(x) = -1/2 x² + 9/2 x – 2. F(10) = -1/2 (100) + 9/2 (10) – 2 = -50 + 45 – 2 = -7. **Ans. – 7**

4 Functions Feb 2016 (No Calculators)

3 pts 1. If f(x) = 5x – 3, g(x) = 6x – 4 and h(x) = 7x – 5, find the value of

$$\frac{f(7)-g(5)+h(6)-7}{f(5)-g(4)+h(3)-6}.$$

Ans. ________________

4 pts 2. f(x) = 1 – x^2, g(x) = $\frac{1}{x+1}$ and k(x) = $\frac{x}{x-1}$. Find the domain of

g(x) divided by $\frac{k(x)}{f(x)+3}$.

Ans. ________________________________

5 pts 3. If f(n) = 2f(n-1) – 3 and f(2) = 1, find the value of f(11) – f(12).

Ans. ______________________

Functions

1. $\frac{f(7)-g(5)+h(6)-7}{f(5)-g(4)+h(3)-6} = \frac{32-26+37-7}{22-20+16-6} = \frac{69-33}{38-26} = \frac{36}{12} = 3.$ **Ans. 3**

2. f(x) + 3 = 4 – x^2, thus x ≠ 2 or -2. k(x) = $\frac{x}{x-1}$, x ≠ 1 or 0. G(x) = $\frac{1}{x+1}$, thus x ≠ -1.

Ans. All reals except -2, -1, 0, 1, 2

3. f(3) = 2f(2) -3 = 2(1) – 3 = -1. f(4) = 2f(3) – 3 = 2(-1) – 3 = -5. f(5) = 2f(4) – 3 = 2(-5) – 3 = -13, f(5) = 2(-13) – 3 = -29. f(2) – f(3) = 1 – (-1) = 2. f(3) – f(4) = -1 – (-5) = 4. f(4) – f(5) = -5 – (-13) = 8. Notice the pattern:

f(2) – f(3) = 2^1, f(3) – f(4) = 2^2, f(4) – f(5) = 2^3, so f(11) – f(12) = 2^{10} = 1024. **Ans. 1024**

4 Functions Feb 2015 (No Calculators)

3 pts 1. If f(x) = 5x + 3 and g(x) = 7 – 2x. Find $\frac{f(x)-g(x)}{f(g(x))}$. Express answer as a single fraction in simplest form.

Ans. ________________

4 pts 2. f(x) = 5x + 2, g(x) = x^2 - 1, find all value(s) of x such that

f o g (x) + g o f (x) = 10

Ans. ________________

5 pts 3. Suppose that f(x) is a linear function such that 3 f(x) + 2 f(1 – x) = 2x + 9 for every real number x. What is the value of f(2)?

Ans. ________________

$\frac{4-7x}{10x-38}$

Functions

1. $\frac{f(x)-g(x)}{f(g(x))} = \frac{5x+3-(7-2x)}{5(7-2x)+3} = \frac{7x-4}{35-10x+3} = \frac{7x-4}{38-10x}$.

Ans. $\frac{7x-4}{38-10x}$

2. f o g (x) + g o f (x) = 10 → $5(x^2 - 1) + 2 + (5x + 2)^2 - 1 = 10$ → $5x^2 - 3 + 25x^2 + 20x + 3 = 10$ → $30x^2 + 20x - 10 = 0$ → $3x^2 + 2x - 1 = 0$ → (3x – 1)(x + 1) = 0, so x = -1 or 1/3.

Ans. -1 or 1/3

3. f(x) must be a linear function, thus f(x) = ax + b. 3(ax + b) + 2(a(1 – x) + b) = 2x + 9. 3ax + 3b + 2a – 2ax + 2b = 2x + 9. 3a – 2a = 2, so a = 2. 3b + 2a + 2b = 9 → 5b = 5, so b = 1. Thus f(x) = 2x + 1, and f(2) = 2(2) + 1 = 5.

Ans. 5

4 Functions Feb 2014 (No Calculators)

3 pts 1. If $f(x) = x^2 - 5x - 8$, find all values of x such that $f(x) = 6$.

Ans. ______________________

4 pts 2. If $f(x) = \frac{x-3}{x+2}$, find $f^{-1}(2)$.

Ans. ___________________

5 pts 3. $f(x) = \frac{2x-3}{3x+2}$. $g(x) = \frac{5x+2}{4x-3}$. Find the domain of fog(x).

Ans. __________________________

Functions

1. $6 = x^2 - 5x - 8 \Rightarrow 0 = x^2 - 5x - 14 \Rightarrow 0 = (x-7)(x+2)$. So x = 7 or -2. **Ans. 7 or -2**

2. $f(x) = \frac{x-3}{x+2}$, to find $f^{-1}(2)$, simply set $\frac{x-3}{x+2} = 2$ and solve for x: $x - 3 = 2x + 4 \Rightarrow$ $-7 = x$. The alternative is to find $f^{-1}(x)$ and plug in 2 for x. To find $f^{-1}(x)$ we switch the x for y and y for x and solve for y: $x = \frac{y-3}{y+2} \Rightarrow x(y+2) = y - 3 \Rightarrow xy + 2x = y - 3 \Rightarrow$ $xy - y = -2x - 3 \Rightarrow y(x-1) = -2x - 3 \Rightarrow y = \frac{-2x-3}{x-1} \Rightarrow f^{-1}(x) = \frac{-2x-3}{x-1}$, $f^{-1}(2) = \frac{-7}{1}$

Ans. -7

3. $f(x) = \frac{2x-3}{3x+2}$, $g(x) = \frac{5x+2}{4x-3}$. $fog(x) = \frac{2\left(\frac{5x+2}{4x-3}\right) - 3}{3\left(\frac{5x+2}{4x-3}\right) + 2} = \frac{2(5x+2) - 3(4x-3)}{3(5x+2) + 2(4x-3)} =$

$\frac{10x + 4 - 12x + 9}{15x + 6 + 8x - 6} = \frac{-2x+13}{23x}$. The domain is: **Ans. All reals ≠ -2/3, 3/4, or 0**

4 Functions Feb 2013 (No Calculators)

3 pts 1. $f(x) = x^2 - 3x + 7$. If $x > 0$ and $f(x) = 11$, find x.

Ans. ____________

4 pts 2. When Alice gets a number, she doubles it and tells the result to Bill. Bill adds 7 and tells the result to Carol. Carol subtracts what she gets from 100 and tells the result to Don. Don adds 65 and says the result. Call these functions A(x), B(x), C(x), D(x), respectively and let $E(x) = D(C(B(A(x))))$. Find $E^{-1}(52)$.

Ans. ____________

5 pts 3. Let f be a real-valued function such that $f(x) + 2f(2002/x) = 3x$. Find f(2) in simplest form.

Ans. ____________

Functions

1. $11 = x^2 - 3x + 7 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0$. $x = 4$ for $x > 0$. **Ans. 4**

2. Working backward: 52 => 13 => 113 => 106 => 53. **Ans. 53**

3. $f(x) + 2f(2002/x) = 3x$. (1) $f(2) + 2f(1001) = 6$. (2) $f(1001) + 2f(2) = 3003$.
In (2): $f(1001) = 3003 - 2f(2)$. Subbing this into (1): $f(2) + 2[3003 - 2f(2)] = 6$.
Thus $-3f(2) = 6 - 6006 \Rightarrow f(2) = 2000$. **Ans. 2000**

Trigonometric Mechanics

4 Functions Jan 2012 (No Calculators)

3 pts 1. If $f(x) = x^2 - 6x + 2$, find $f(x + 2)$.

Ans. ______________________

4 pts 2. If $f(2x - 3) = 6x - 14$, find $f(3x + 2)$.

Ans. ______________________

5 pts 3. Function f(t) is defined recursively as follows:

$4^{f(t)} = f(t - 1)$ and $f(0) = 4^{(2^{512})}$.

If the range of f(t) is limited to real numbers, how many whole numbers are in the domain of f(t)?

Ans. ______________

Functions

1. $f(x) = x^2 - 6x + 2$. $f(x+2) = (x+2)^2 - 6(x+2) + 2 = x^2 + 4x + 4 - 6x - 12 + 2$

Ans. $\mathbf{x^2 - 2x - 6}$

2. Let $f(x) = ax + b$, then $f(2x - 3) = a(2x-3) + b = 2ax - 3a + b$. Since this is suppose to equal $6x - 14$, then (1) $2a = 6$ and (2) $-3a + b = -14$. In (1) $a = 3$. In (2) $-3(3) + b = -14$ or $b = -5$. Thus $f(x) = 3x - 5$ and $f(3x + 2) = 3(3x + 2) - 5 = 9x + 1$ **Ans. 9x + 1**

3. $4^{f(t)} = f(t - 1)$ ➔ Taking log of both sides base 4: $\log_4 4^{f(t)} = \log_4 f(t - 1)$.
So $f(t) = \log_4 f(t - 1)$. Since $f(0) = 4^{2^{512}}$, then $f(1) = \log_4 f(0) = 2^{512}$, $f(2) = 256$, $f(3) = 4$, $f(4) = 1$, $f(5) = 0$. There are 6, since log 0 does not exist. **Ans. 6**

4 Functions Feb 2011 (No Calculators)

3 pts 1. Suppose $f(x) = \frac{x}{2}$ and $g(x) = x - 3$. Find f(g(-5)).

Ans. ________________

4 pts 2. A function f(n) defined for all positive integers has the property that $f(m) + f(n) = f(mn)$ for any two positive integers m and n. If f(2) = 7 and f(3) = 10, then calculate f(12).

Ans. ________________

5 pts 3. Let f(x) be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = x$ for all x not equal to 0 or 1. Find the exact value of f(2).

Ans. ________________

Functions

1. $g(-5) = -5 - 3 = -8$. $f(-8) = \frac{-8}{2} = -4$. **Ans. -4**

2. If f(m) + f(n) = f(mn) then f(2) + f(3) = f(6) ➔ Since f(2) = 7 and f(3) = 10, then f(6) = 17. f(2) + f(6) = f(12) ➔ 7 + 17 = f(12). **Ans. 24**

3. $f(x) + f\left(\frac{1}{1-x}\right) = x$ ➔ $f(x) = x - f\left(\frac{1}{1-x}\right)$. $f(2) = 2 - f\left(\frac{1}{1-2}\right)$ ➔ (1): $f(2) = 2 - f(-1)$
$f(-1) = -1 - f\left(\frac{1}{1-(-1)}\right)$ ➔ (2): $f(-1) = -1 - f(\frac{1}{2})$. $f(\frac{1}{2}) = \frac{1}{2} - f\left(\frac{1}{1-(\frac{1}{2})}\right)$ ➔ (3): $f(\frac{1}{2}) = \frac{1}{2} - f(2)$.
Now plugging (3) into (2) into (1): $f(2) = 2 - (-1 - (\frac{1}{2} - f(2)))$ ➔ $f(2) = 2 - (-1\frac{1}{2} + f(2))$
$f(2) = 2 + 1\frac{1}{2} - f(2)$ ➔ $2f(2) = 3\frac{1}{2}$ ➔ $f(2) = 1\frac{3}{4}$ or $\frac{7}{4}$ **Ans. $1\frac{3}{4}$ or $\frac{7}{4}$**

4 Functions Feb 2010 (No Calculators)

3 pts 1. If f(x + 1) = 2x – 3, where f is a linear function, find f(x + 3).

Ans. ______________________

4 pts 2. $f(x) = \frac{x-2}{x+3} - \frac{x+3}{x-2}$, find the domain of *f*.

Ans. ______________________

5 pts 3. The number r is said to be a *fixed point* of the function *f*, if f(r) = r. Find all ordered pairs (a, b) for which the function $f(x) = x^2 + ax + b$ has exactly one *fixed point*.

Ans. ______________________

Functions

1. If f(x + 1) = 2x - 3 and f is a linear function, then f(x) = ax + b = a(x + 1) + b ax +a + b = 2x - 3. Thus a = 2 and a + b = -3 or b = -5. f(x + 3) = 2(x + 3) – 5 = 2x + 1. Alternate solution: since x + 3 is 2 unit more than x + 1, then f(x + 2) = 2(x + 2) – 3 = 2x + 1.

Ans. 2x + 1

2. $f(x) = \neq \frac{x-2}{x+3} - \frac{x+3}{x-2}$, denominators cannot be -3 or 2. **Ans. All reals ≠ -3 or 2**

3. By definition $f(x) = x^2 + ax + b = x$, thus $f(x) = x^2 + (a-1)x + b$. Using the quadratic formula and knowing that the discriminant must be zero if there is to be only one real solution: $(a-1)^2 - 4b = 0$. $(a-1)^2 = 4b$, thus $a = 1 \pm 2\sqrt{b}$ or $b = \left(\frac{a-1}{2}\right)^2$.

Ans. $(1 \pm 2\sqrt{b}, b)$ or $(a, (\frac{a-1}{2})^2)$