1 Algebraic Fractions and Factoring Mar 2019 (No Calculators)

3 pts 1. Simplify: $\dfrac{6x^2+x-2}{9x^2-4}$.

Ans. _____

4 pts 2. Simplify: $\dfrac{\dfrac{5}{x-3}+\dfrac{2}{3-x}}{\dfrac{4}{2x-6}-\dfrac{5}{3x-9}}$

Ans. _____

5 pts 3. Marcia can collate and staple a certain stack of papers in 10 min. Terri can do them in 12 min. Marcia starts by herself and Terri comes to help after a minute. After a short time Marcia is called away for two minutes, but then comes back to help finish. How long did Terri work?

Ans. _____

Solutions – Algebraic Fractions with Factoring

1. $\dfrac{6x^2+x-2}{9x^2-4}=\dfrac{(3x+2)(2x-1))}{(3x+2)(3x-2)}=\dfrac{2x-1}{3x-2}$.

Ans. $\dfrac{2x-1}{3x-2}$

2. $\dfrac{\dfrac{5}{x-3}+\dfrac{2}{3-x}}{\dfrac{4}{2x-6}-\dfrac{5}{3x-9}}=\dfrac{\dfrac{5}{x-3}-\dfrac{2}{x-3}}{\dfrac{4}{2(x-3)}-\dfrac{5}{3(x-3)}}=\dfrac{\dfrac{3}{x-3}}{\dfrac{2}{6(x-3)}}=\dfrac{3}{x-3}\cdot\dfrac{6(x-3)}{2}=9.$

Ans. 9

3. Mary does $\frac{1}{10}$ of the job per hour. Terri does $\frac{1}{12}$ of the job per hour. Since the whole is equal to the sum of its parts and Terri works T hours, then $\frac{1}{10}(1+T-2)+\frac{1}{12}T=1$.

$\frac{T-1}{10}+\frac{T}{12}=1 \rightarrow 6(T-1)+5T=60 \rightarrow 11T=66$, so Terri works for 6 minutes. **Ans. 6 min**

1 Algebraic Fractions and Factoring Mar 2016 – 17 (No Calculators)

3 pts 1. Find the only value of k such that $\frac{-x^2+kx+16}{x-8}$ will reduce to a binomial expression.

Ans. _____

4 pts 2. Simplify $\frac{x^3+2x^2-9x-18}{9-x^2}$, where $x \neq \pm 3$.

Ans. _____

5 pts 3. Solve: $\frac{2x-1}{x^2-5x+6}-\frac{x-2}{x^2-4x+3}=\frac{3x-5}{x^2-3x+2}$.

Ans. _____

Solutions – Algebraic Fractions and Factoring

1. $\frac{-x^2+kx+16}{x-8}=\frac{-(x^2-kx-16)}{x-8}=\frac{-(x-8)(x+2)}{x-8}=\frac{-(x^2-6x-16)}{x-8}$. **Ans. k = 6**

2. $\frac{x^3+2x^2-9x-18}{9-x^2}=\frac{x^2(x+2)-9(x+2)}{(3-x)(3+x)}=\frac{(x-3)(x+3)(x+2)}{-1(x-3)(3+x)}=-(x+2)$ **Ans. –x – 2**

3. $\frac{2x-1}{(x-3)(x-2)}-\frac{x-2}{(x-3)(x-1)}=\frac{3x-5}{(x-2)(x-1)}$ → $(2x-1)(x-1)-(x-2)(x-2)=(3x-5)(x-3)$

$2x^2-3x+1-(x^2-4x+4)=3x^2-14x+15$ → $x^2+x-3=3x^2-14x+15$ → $0=2x^2-15x+18$

$0=(2x-3)(x-6)$, so $x=6$ or $3/2$. **Ans. 6 or 3/2**

1 Algebraic Fractions and Factoring Mar 2016 (No Calculators)

pts 1. Simplify: $\frac{1}{x-1}-\frac{1}{x+1}+\frac{2x}{x^2-1}$

Ans. _____

4 pts 2. Factor completely: $abc - abz + acy - ayz - bcx + bxz - cxy + xyz$

Ans. _____

5 pts 3. Solve the following equation where $x \neq 0$: $\frac{x+1}{x^2}-\frac{1}{x^3}=\frac{x+3}{4x}$

Ans. _____

Solutions – Algebraic Fractions with Factoring

1. $\frac{1}{x-1}-\frac{1}{x+1}+\frac{2x}{x^2-1} = \frac{x+1-x+1+2x}{(x+1)(x-1)} = \frac{2(x+1)}{(x+1)(x-1)} = \frac{2}{x-1}$. **Ans.** $\frac{2}{x-1}$

2. $abc - abz + acy - ayz - bcx + bxz - cxy + xyz = (c - z)(ab + ay - bx - xy) =$
$(c - z)[a(b + y) - x(b + y)] = (c - z)(a - x)(b + y)$. **Ans. $(c - z)(a - x)(b + y)$**

3. $\frac{x+1}{x^2}-\frac{1}{x^3}=\frac{x+3}{4x}$ ➔ $4x(x + 1) - 4 = x^2(x + 3)$ ➔ $4x^2 + 4x - 4 = x^3 + 3x^2$ ➔
$x^3 - x^2 + 4x - 4 = 0$ ➔ $x^2(x - 1) - 4(x - 1) = 0$ ➔ $(x^2 - 4)(x - 1) = 0$. **Ans. 1, 2, -2**

1 Algebraic Fractions and Factoring Mar 2015 (No Calculators)

3 pts 1. Solve for x: $\frac{x+1}{6} = \frac{2}{x}$

Ans. _____

4 pts 2. In a certain city a birth occurs on the average every 24 minutes, and a death every half hour. A resident moves out of the city every 1.5 hours and a new person moves in every 4.5 hours. How long, in hours, does it take on average for the population to increase by one person?

Ans. _____

5 pts 3. An experienced plumber made \$800 for working on a certain job. His apprentice, who makes \$5 less per hour also made \$800. However, the apprentice worked 8 hours more than the plumber. How much does the plumber make per hour?

Ans. _____

Solutions – Algebraic Fractions and Factoring

1. $\frac{x+1}{6} = \frac{2}{x} \rightarrow x^2 + x = 12 \rightarrow x^2 + x - 12 = 0 \rightarrow (x+4)(x-3) = 0.$ **Ans. 3 or -4**

2. Let x = # minutes it takes for 1 person increase in population.
   Birth – Death – Out - In
   $\frac{x}{24} - \frac{x}{30} - \frac{x}{90} + \frac{x}{270} = 1 \rightarrow \frac{x}{24} - \frac{9x}{270} - \frac{3x}{270} + \frac{x}{270} = \frac{x}{24} - \frac{11x}{270} = 1 \rightarrow$
   $270x - 264x = 24(270) \rightarrow 6x = 24(270) \rightarrow x = 4(270).$ $4(270)/60 = 18.$ **Ans. 18 hr**

3. Let R = Plumber's hourly rate, and T = Time plumber worked.
   (1) $RT = 800$, and (2) $(R-5)(T+8) = 800 \rightarrow RT - 5T + 8R - 40 = 800 \rightarrow$
   $8R = 5T + 40 \rightarrow R = \frac{5T+40}{8}$. In (1) $(\frac{5T+40}{8})T = 800 \rightarrow 5T^2 + 40T = 6400 \rightarrow$
   $T^2 + 8T - 1280 = 0 \rightarrow (T-32)(T+40) = 0.$ $T = 32.$ In (1) $R(32) = 800.$ **Ans. \$25**

1 Algebraic Fractions and Factoring Mar 2014 (No Calculators)

3 pts 1. Simplify: $\frac{3}{x+3}+\frac{2}{2x+1}-\frac{6x+8}{2x^2+7x+3}$ **Ans.** _____

4 pts 2. Find $7B-3A$, if $\frac{A}{2x-3}+\frac{B}{2x+3}=\frac{24x+6}{4x^2-9}$

Ans. _____

5 pts 3. Find all x such that $\frac{x-4}{x+3}-\frac{x-10}{x-3}=\frac{x-8}{2x+6}$

Ans. _____

Solutions – Algebraic Fractions

1. $\frac{3(2x+1)}{(x+3)(2x+1)}+\frac{2(x+3)}{(2x+1)(x+3)}-\frac{(6x+8)}{(2x+1)(x+3}=\frac{2x+1}{(x+3)(2x+1)}=\frac{1}{x+3}$ **Ans.** $\frac{1}{x+3}$

2. $A(2x+3)+B(2x-3)=24x+6 \rightarrow 2Ax+3A+2Bx-3B=24x+6$. Therefore $2A+2B=24$, or (1) $A+B=12$, and $3A-3B=6$, or (2) $A-B=2$. Adding (1) + (2): $2A=14$ or $A=7$. In (1), $(7)+B=12$, so $B=5$. $7B-3A=7(5)-3(7)=14$. **Ans. 14**

3. $\frac{x-4}{x+3}-\frac{x-10}{x-3}=\frac{x-8}{2(x+3)} \rightarrow (x-4)(2x-6)-(x-10)(2x+6)=(x-8)(x-3) \rightarrow$
$2(x^2-7x+12)-(2x^2-14x-60)=x^2-11x+24$ or $0=x^2-11x-60 \rightarrow$
$0=(x-15)(x+4)=0$. So $x=15$ or -4. **Ans. 15 or – 4**

1 Algebraic Fractions and Fractional Equations Mar 2013 (No Calcula tors)

3 pts 1. Find, in simplest form, as a single fraction with no parentheses, for x > 0:

$$\frac{x^4+2x^3+x^2}{4} \div \frac{2x^3+3x^2+x}{6}$$

Ans. _____

4 pts 2. Simplify and write as a binomial without parentheses:

$$\frac{8x^4-24x^3-32x^2+96x}{8x-\frac{32}{x}}$$

Ans. _____

5pts 3. Find all values of x such that $\frac{x-2}{2x-3}+\frac{2x-7}{1-x}=\frac{x^2-3x-3}{2x^2-5x+3}$.

Ans. _____

Solutions - Algebraic Fractions and Factoring

(1) $\frac{x^4+2x^3+x^2}{4} \div \frac{2x^3+3x^2+x}{6} = \frac{x^2(x+1)^2}{4} \cdot \frac{6}{x(2x+1)(x+1)} = \frac{3x(x+1)}{2(2x+1)}$. **Ans.** $\frac{3x^2+3x}{4x+2}$

(2) $\frac{8x^4-24x^3-32x^2+96x}{8x-\frac{32}{x}} = \frac{8x(x^3-3x^2-4x+12)}{8\left(\frac{x^2-4}{x}\right)} = \frac{x^2(x^2[x-3]-4[x-3])}{x^2-4} = x^2(x-3) =$

$x^3 - 3x^2$, without parentheses.

Ans. $x^3 - 3x^2$

(3) $\frac{x-2}{2x-3}+\frac{2x-7}{1-x}=\frac{x^2-3x-3}{2x^2-5x+3} \rightarrow \frac{x-2}{2x-3}-\frac{2x-7}{x-1}=\frac{x^2-3x-3}{2x^2-5x+3} \rightarrow$

$(x-2)(x-1)-(2x-7)(2x-3) = x^2 - 3x - 3 \rightarrow x^2 - 3x + 2 - 4x^2 + 20x - 21 = x^2 - 3x - 3$

$= 4x^2 - 20x + 16 \rightarrow 0 = x^2 - 5x + 4 = (x-4)(x-1)$. $x = 4$ or 1, but x can't be 1. **Ans.** 4

1 Algebraic Fractions with Factoring Mar 2012 (No Calculators)

pts 1. Find the lowest common denominator for the following expression:

$$\frac{2x+3y}{6x^3y^7}+\frac{x-y}{4x^7y^3z}-\frac{x+y}{8x^8yz^7}$$

Ans. _____

4 pts 2. Simplify: $\dfrac{2x^2+5x+2}{2x^2-5x-3}+\dfrac{15x^2+19x+6}{3x^2-7x-6}-\dfrac{12x^2+7x-12}{4x^2-15x+9}$

Ans. _____

5 pts 3. Solve for x, if: $\dfrac{x}{3}-\dfrac{x-3}{x}=\dfrac{x-3}{x+3}+\dfrac{x+3}{2x}$

Ans. _____

Solutions – Algebraic Fractions with Factoring

1. Self explanatory. **Ans.** $24x^8y^7z^7$

2. $\dfrac{2x^2+5x+2}{2x^2-5x-3}+\dfrac{15x^2+19x+6}{3x^2-7x-6}-\dfrac{12x^2+7x-12}{4x^2-15x+9}=\dfrac{(2x+1)(x+2)}{(2x+1)(x-3)}+\dfrac{(3x+2)(5x+3)}{(3x+2)(x-3)}-\dfrac{(4x-3)(3x+4)}{(x-3)(4x-3)}$

$\dfrac{x+2+5x+3-3x-4}{x-3}=\dfrac{3x+1}{x-3}$ **Ans.** $\dfrac{3x+1}{x-3}$

3. Multiplying $\dfrac{x}{3}-\dfrac{x-3}{x}=\dfrac{x-3}{x+3}+\dfrac{x+3}{2x}$ by 6x(x+3) produces the following:

$2x^2(x+3)-6(x-3)(x+3)=6x(x-3)+3(x+3)(x+3)$ ➔

$2x^3+6x^2-6x^2+54=6x^2-18x+3x^2+18x+27$ ➔ $2x^3-9x^2+27=0$.

This factors into $(x-3)(x-3)(2x+3)=0$. So x = 3 or - 3/2. **Ans. 3 or -3/2**

1 Algebraic Fractions with Factoring Mar 2011 (No Calculators)

3 pts 1. The reciprocal of half a number increased by half the reciprocal of the number is ½. Find the number.

Ans. _____

4 pts 2. Simplify: $\dfrac{x^{2n}-2x^n y^n+y^{2n}}{x^{2n}+3x^n y^n-4y^{2n}}$

Ans. _____

5 pts 3. Find all real values of x such that $\dfrac{\frac{1}{x^2}-x^2}{\frac{1}{x}+x}=\dfrac{3}{2}$.

Ans. _____

Solutions – Algebraic Fractions and Factoring

1. Let n = the number. ½ n = ½ a number. 2/n = reciprocal of ½ n. Thus $\dfrac{2}{n}+\dfrac{1}{2}\cdot\dfrac{1}{n}=\dfrac{1}{2}$.

Multiplying by 2n: 4 + 1 = n. Thus n = 5. **Ans. 5**

2. $\dfrac{x^{2n}-2x^n y^n+y^{2n}}{x^{2n}+3x^n y^n-4y^{2n}}=\dfrac{(x^n-y^n)(x^n-y^n)}{(x^n+4y^n)(x^n-y^n)}=\dfrac{x^n-y^n}{x^n+4y^n}$. **Ans.** $\dfrac{x^n-y^n}{x^n+4y^n}$

3. $\dfrac{\frac{1}{x^2}-x^2}{\frac{1}{x}+x}=\dfrac{3}{2}\rightarrow\dfrac{\left(\frac{1}{x}-x\right)\left(\frac{1}{x}+x\right)}{\frac{1}{x}+x}=\dfrac{3}{2}\rightarrow\dfrac{1}{x}-x=\dfrac{3}{2}\rightarrow 2-2x^2=3x\rightarrow 2x^2+3x-2=0.$

Factoring: (x + 2)(2x – 1) = 0. Thus x = -2 or ½. **Ans. -2 or ½**

1 Algebraic Fractions with Factoring Mar 2010 (No Calculators)

pts 1. Express the following as a single fraction in simplest form.

$$\cfrac{1}{x+\cfrac{1}{2+\cfrac{1}{x+2}}}$$

Ans. _____

4 pts 2. Simplify: $\dfrac{x+2}{x-3}-\dfrac{3x-5}{2x+1}+\dfrac{3x^2+2x+23}{2x^2-5x-3}$

Ans. _____

5 pts 3. Find all value(s) of x such that $\dfrac{x+1}{x-2}-\dfrac{x-2}{x+1}=\dfrac{5}{6}$.

Ans. _____

Solutions – Algebraic Fractions and Factoring 2009-2010 Meet 4

1. $\cfrac{1}{x+\cfrac{1}{2+\cfrac{1}{x+2}}}=\cfrac{1}{x+\cfrac{1}{\cfrac{2x+5}{x+2}}}=\cfrac{1}{x+\cfrac{x+2}{2x+5}}=\cfrac{1}{\cfrac{2x^2+6x+2}{2x+5}}=\dfrac{2x+5}{2x^2+6x+2}$. **Ans.** $\dfrac{2x+5}{2x^2+6x+2}$

2. $\dfrac{x+2}{x-3}\cdot\dfrac{2x+1}{2x+1}-\dfrac{3x-5}{2x+1}\cdot\dfrac{x-3}{x-3}+\dfrac{3x^2+2x+23}{(2x+1)(x-3)}=\dfrac{2x^2+5x+2-(3x^2-14x+15)+3x^2+2x+23}{(2x+1)(x-3)}=$

$\dfrac{2x^2+21x+10}{(2x+1)(x-3)}=\dfrac{(2x+1)(x+10)}{(2x+1)(x-3)}=\dfrac{x+10}{x-3}$. **Ans.** $\dfrac{x+10}{x-3}$

3. $\dfrac{x+1}{x-2}-\dfrac{x-2}{x+1}=\dfrac{5}{6}$ ➔ Let $\dfrac{x+1}{x-2}=a$, then $a-1/a=5/6$ or $6a^2-6=5a$ ➔ $6a^2-5a-6=0$.

$(3a+2)(2a-3)=0$ ➔ $a=-2/3$ or $a=3/2$. So (1) $\dfrac{x+1}{x-2}=-\dfrac{2}{3}$ or (2) $\dfrac{x+1}{x-2}=\dfrac{3}{2}$.

In (1) $3x+3=-2x+4$ ➔ $x=1/5$. In (2) $2x+2=3x-6$ ➔ $x=8$. **Ans. 1/5 or 8**