2 Trigonometric Equations and Identities Mar 2019 (No Calculators)

3 pts 1. Find all values of θ, such that $0^o \leq \theta \leq 360^o$ and $\cos\theta = -\frac{1}{2}\sqrt{2}$.

Ans. ______________________

4 pts 2. Express the following as a single trig function of t in simplest form: $\dfrac{\cos(\frac{\pi}{2}-t)}{\tan t}$.

Ans. ______________________

5 pts 3. Find the value of $\sin^2\left[\frac{1}{2}\tan^{-1}\left(\frac{5}{12}\right)\right]$.

Ans. ______________________

Trigonometric Equations and Identities

1. $\cos\theta = -\frac{1}{2}\sqrt{2}$, so the reference angle for θ is 45^o. cos is negative in the second and third quadrants, so that means 135^o and 225^o.

Ans. 135^o and 225^o

2. $\dfrac{\cos(\frac{\pi}{2}-t)}{\tan t} = \dfrac{\sin t}{\frac{\sin t}{\cos t}} = \sin t\left(\dfrac{\cos t}{\sin t}\right) = \cos t$. Alt. sol. For numerator: $\cos\frac{\pi}{2}\cos t + \sin\frac{\pi}{2}\sin t$ simplifies to $\sin t$.

Ans. cos t

3. $\sin^2\left[\frac{1}{2}\tan^{-1}\left(\frac{5}{12}\right)\right]$. $\sin\frac{1}{2}\theta = \sqrt{\dfrac{1-\cos\theta}{2}}$, so $(\sin\frac{1}{2}\theta)^2 = \dfrac{1-\cos\theta}{2}$. Thus

$$\sin^2\left[\frac{1}{2}\tan^{-1}\left(\frac{5}{12}\right)\right] = \frac{1-\cos(\tan^{-1}(5/12)}{2} = \frac{1-(12/13)}{2} = \frac{1/13}{2} = 1/26.$$

Ans. 1/26

 Trigonometric Equations and Identities Mar 2016 – 17 (No Calculators)

3 pts 1. Simplify: $\sin 20° \tan 40° \sec 70°$.

Ans. ______________

4 pts 2. $\sin^2\theta + 1 + \tan^2\theta$ can be written as $\frac{A}{\cos^2\theta}$. Find A where A involves only the trig function cosine along with any powers or constants, if they occur.

Ans. ______________

5 pts 3. If $\sin\theta = 1/3$, calculate the exact value of $\sin 4\theta$.

Ans. ______________

Trigonometric Equations and Identities

1. $\sin 20° \tan 40° \sec 70° = \sin 20° \tan 40° \csc 20° = \tan 40°$ Ans. $\tan 40°$

2. $\sin^2\theta + 1 + \tan^2\theta = \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{(1-\cos^2\theta)(\cos^2\theta)+1}{\cos^2\theta} =$

$\frac{\cos^2\theta - \cos^4\theta + 1}{\cos^2\theta}$. Ans. $\cos^2\theta - \cos^4\theta + 1$

3. $\sin\theta = 1/3$, so $\cos\theta = \frac{2\sqrt{2}}{3}$. $\sin 2\theta = 2\sin\theta\cos\theta$, $\sin 4\theta = 2\sin 2\theta\cos 2\theta =$

$2(2\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta) = 2(2)\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{8}{9}-\frac{1}{9}\right) = \frac{56\sqrt{2}}{81}$. Ans. $\frac{56\sqrt{2}}{81}$

3 pts 1. Find all values of x, where $0^o \le x < 360^o$ and tan x = 1.

Ans. ______________________

4 pts 2. Express $\frac{\sin\theta - 1}{\cos\theta} - \frac{\cos\theta}{\sin\theta + 1}$ in simplest form.

Ans. ______________________

5 pts 3. Find all values of θ, where $0^o \le \theta < 360^o$, if $\sqrt{3}\sec^2\theta + 2\tan\theta = 2\sqrt{3}$.

Ans. ______________________

Trigonometric Equations and Identities

1. tan x = 1 at 45° and 225°. **Ans. 45° or 225°**

2. $\frac{\sin\theta - 1}{\cos\theta} - \frac{\cos\theta}{\sin\theta + 1}$ ➔ $\frac{(\sin\theta - 1)(\sin\theta + 1) - \cos^2\theta}{\cos(\sin\theta + 1)}$ ➔ $\frac{\sin^2\theta - 1 - (1 - \sin^2\theta)}{\cos\theta(\sin\theta + 1)}$ ➔ $\frac{2\sin^2\theta - 2}{\cos\theta(\sin\theta + 1)}$ ➔

$\frac{2(\sin^2\theta - 1)}{\cos\theta(\sin\theta + 1)}$ ➔ $\frac{2(\sin\theta - 1)(\sin\theta + 1)}{\cos\theta(\sin\theta + 1)}$ ➔ $\frac{2(\sin\theta - 1)}{\cos\theta}$ ➔ $\frac{2\sin\theta - 2}{\cos\theta}$. **Ans.** $\frac{2\sin\theta - 2}{\cos\theta}$

Working with cos instead of sin produces $\frac{-2\cos\theta}{\sin\theta + 1}$, which is equivalent to above answer.

3. $\sqrt{3}\sec^2\theta + 2\tan\theta = 2\sqrt{3}$ ➔ $\sqrt{3}(\tan^2\theta + 1) + 2\tan\theta = 2\sqrt{3}$ ➔ $\sqrt{3}\tan^2\theta + \sqrt{3} + 2\tan\theta = 2\sqrt{3}$

$\sqrt{3}\tan^2\theta + 2\tan\theta - \sqrt{3} = 0$ ➔ $(\sqrt{3}\tan\theta - 1)(\tan\theta + \sqrt{3}) = 0$, thus (1) $\tan\theta = \frac{1}{\sqrt{3}}$ or (2) $\tan\theta = -\sqrt{3}$.

In (1): θ = 30° or 210°. In (2): θ = 120° or 300°. **Ans. 30°, 120°, 210°, 300°**

2 Trigonometric Equations and Identities Mar 2015 (No Calculators)

3 pts 1. Find all values of x from $0°$ to $360°$ for which $\tan x = -\frac{1}{3}\sqrt{3}$.

Ans. ____________________

4 pts 2. Compute the value of the following:

$$\cos(\text{Arccos}\,\frac{3}{5} + \text{Arc}\sin\,\frac{3}{5})$$

Ans. ____________________

5 pts 3. What is the value of the sum:

$$\cos\frac{\pi}{6} + \cos\frac{\pi}{3} + \cos\frac{\pi}{2} + \ldots + \cos\frac{2017\pi}{6}?$$

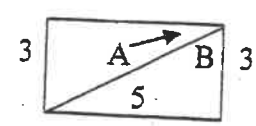
Ans. ____________________

Trigonometric Equations and Identities

1. $\tan x = -\frac{1}{3}\sqrt{3} = -\frac{1}{\sqrt{3}}$, so related angle is $30°$.

Ans. $150°$ or $330°$

2. Mechanically putting the two angles at the same vertex, shows angle sum to be $90°$ (at right). Alternate solution: Let A = Arccos 3/5 and let B = Arc sin 3/5, then cos (A + B) = cos A cos B − sin A sin B: $\frac{3}{5}\cdot\frac{4}{5} - \frac{4}{5}\cdot\frac{3}{5} = 0.$



Ans. 0

3. The sum of the cosines of all these angles to $2\pi = 0$. $\frac{2017\pi}{6} = 336\frac{1}{6}\pi$. Many 2π's Plus $\frac{1}{6}\pi$, $\cos\frac{1}{6}\pi = \frac{\sqrt{3}}{2}$.

Ans. $\frac{\sqrt{3}}{2}$

2 Trigonometric Equations and Identities Mar 2014 (No Calculators)

pts 1. In right triangle ABC with $m\angle C = 90°$, $\tan \angle B = 15/8$. Find $\sin \angle A$.

Ans. ______________________

4 pts 2. Find the value of $\tan 157°30'$.

Ans. ______________________

5 pts 3. Find all θ, where $0° \le \theta < 360°$, for which

$$\sqrt{3}\sec^2\theta - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1$$ [redacted]

Ans. ______________________

Trigonometric Equations and Identities

$\frac{\text{opposite}}{\text{adjacent}}$, so 15 is opp. $\angle B$ and 8 is opp $\angle A$. Hyp = 17. $\sin A = \frac{8}{17}$. **Ans.** $\frac{8}{17}$

$= \tan\frac{315°}{2} = -\sqrt{\frac{1-\cos 315°}{1+\cos 315°}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = -\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}} = -\frac{2-\sqrt{2}}{\sqrt{2}} = 1-\sqrt{2}$

$\sqrt{2}+2$ $= -\sqrt{3-2\sqrt{2}}$.

Ans. 1 - $\sqrt{2}$ or $-\sqrt{3-2\sqrt{2}}$

$\tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1 = 0$ ➔ $\sqrt{3}(\tan^2\theta + 1) - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1 = 0$ ➔

$- \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1$ ➔ $\sqrt{3}\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + 1 = 0$ ➔

$1) - (\tan\theta - 1) = 0$ ➔ $(\sqrt{3}\tan\theta - 1)(\tan\theta - 1) = 0$. So either $\tan\theta = \frac{1}{\sqrt{3}}$, in which

$210°$; or $\tan\theta = 1$, in which case $\theta = 45°$ or $225°$.

Ans. 30°, 45°, 210°, 225°

2 **Trigonometric Equations and Identities Mar 2013 (No Calculators)**

3 pts 1. If $0° \le \theta < 360°$, find all values of θ satisfying:

$$\frac{\sin\theta}{5} - \frac{\sin\theta}{3} = \frac{\sqrt{3}}{15}$$

Ans. ______________________

4 pts 2. Let $A = \sec x$. Find both values of A for which $\cot x = 2$.

Ans. ______________________

5 pts 3. On the domain $0 \le x < \pi$, find all values of x satisfying

$$4\sin^2 x + 6\cos x = 8\sin^2 x\cos x + 3$$

Ans. ______________________

$0 = 4x^2 - 20x + 16 \Rightarrow 0 = x^2 - 5x + 4 = (x-4)(x-1)$. $x = 4$ or 1, but x can't be 1. **Ans. 4**

Trigonometric Equations and Identities

(1) $\frac{\sin\theta}{5} - \frac{\sin\theta}{3} = \frac{\sqrt{3}}{15} \Rightarrow 3\sin\theta - 5\sin\theta = \sqrt{3} \Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$. **Ans. 240° or 300°**

(2) $\cot x = 2 = \frac{2}{1}$, where 2 is the adjacent side and 1 is the opposite side of a right triangle.

Therefore the hypotenuse is $\sqrt{5}$. $\sec x = \frac{\sqrt{5}}{2}$. Since there are no restrictions on x, then x could be a first or a third quadrant angle, thus $\sec x = \pm\frac{\sqrt{5}}{2}$. **Ans.** $\pm\frac{\sqrt{5}}{2}$

(3) $4\sin^2 x + 6\cos x = 8\sin^2 x\cos x + 3 \Rightarrow 8\sin^2 x\cos x - 4\sin^2 x - 6\cos x + 3 = 0$
$4\sin^2 x(2\cos x - 1) - 3(2\cos x - 1) = 0 \Rightarrow (4\sin^2 x - 3)(2\cos x - 1) = 0$. Therefore $\sin x = \pm\frac{\sqrt{3}}{2}$ or $\cos x = 1/2$. $x = \pi/3$ or $2\pi/3$. **Ans.** $\pi/3$ **or** $2\pi/3$

2 **Trigonometric Equations and Identities Mar 2012 (No Calculators)**

5 pts 1. Solve for x, such that $0 \le x \le \frac{\pi}{2}$: $3 \sin x = 1.5$

Ans. ____________________

4 pts 2. If $\tan x = 3/4$ and $\tan y = 7/24$, and x and y are first quadrant angles, find $\sin(x - y)$.

Ans. ____________________

5 pts 3. Find all values of θ, where $0° \le \theta < 360°$, such that

$$\sin 2\theta = \sin \theta$$

Ans. ____________________

Trigonometric Equations and Identities

1. $3 \sin x = 1.5 \rightarrow \sin x = 1/2$. Thus $x = 30° = \pi/6$. **Ans.** $\pi/6$

2. $\sin(x - y) = \sin x \cos y - \cos x \sin y$. Since $\tan x = 3/4$ and $\tan y = 7/24$, then $\sin x = 3/5$, $\cos x = 4/5$, $\sin y = 7/25$ and $\cos y = 24/25$. Thus $\sin(x - y) = \frac{3}{5}\cdot\frac{24}{25} - \frac{4}{5}\cdot\frac{7}{25} = \frac{72-28}{125} = \frac{44}{125}$. **Ans. 44/125**

3. $\sin 2\theta = \sin\theta \rightarrow 2 \sin\theta \cos\theta = \sin\theta$ or $2 \sin\theta \cos\theta - \sin\theta = 0$. Factoring: $\sin\theta\,(2\cos\theta - 1) = 0$. Thus (1) $\sin\theta = 0$, or (2) $\cos\theta = 1/2$. In (1): $\sin\theta = 0$ at 0° or 180°. In (2): $\cos\theta = 1/2$ at 60° or 300°. **Ans. 0°, 60°, 180°, 300°**

2 Trigonometric Equations and Identities Mar 2011 (No Calculators)

3 pts 1. Find the value in simplest form of: $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

Ans. ______________

4 pts 2. If $\tan\alpha = -\frac{8}{15}$, and $\tan\beta = \frac{3}{4}$, find the value of $\tan(\alpha+\beta)$.

Ans. ______________

5 pts 3. Simplify the following to an expression in terms of a single trig function of x in simplest form.

$$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

Ans. ______________

Trigonometric Equations and Identities

1. $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}=\cos\left(\frac{\pi}{4}-\frac{\pi}{12}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ **Ans.** $\frac{\sqrt{3}}{2}$

2. $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-\frac{8}{15}+\frac{3}{4}}{1-\frac{-8}{15}\cdot\frac{3}{4}}=\frac{\frac{-32+45}{60}}{\frac{60+24}{60}}=\frac{13}{84}$. **Ans: 13/84**

3. $\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}}=\frac{\frac{2\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{1}{\cos^2\frac{x}{2}}}=2\sin\frac{x}{2}\cos\frac{x}{2}=\sin x$. **Ans. sin x**

2 Trigonometric Equations and Factoring Mar 2010 (No Calculators)

3 pts 1. Express $\sin\theta + \tan\theta - \tan\theta\cos\theta$ as a single trig function in simplest form.

Ans. ______________

4 pts 2. Solve the following for x, where $0° \le x < 360°$:

$$2\csc^2 x - 3\csc x = 2$$

Ans. ______________

5 pts 3. Find all values of x where $0° \le x \le 180°$:

$$\tan^3 2x = \tan 2x$$

Ans. ______________

Trigonometric Equations and Identities

1. $\sin\theta + \tan\theta - \tan\theta\cos\theta \Rightarrow \sin\theta + \tan\theta - \frac{\sin\theta}{\cos\theta}\cdot\cos\theta = \tan\theta$. **Ans. $\tan\theta$**

2. $2\csc^2 x - 3\csc x = 2 \Rightarrow 2\csc^2 x - 3\csc x - 2 = 0 \Rightarrow (2\csc x + 1)(\csc x - 2) = 0$. Thus $\csc x = -1/2$ which it cannot, or $\csc x = 2$. Here x = 30 or 150. **Ans. 30° or 150°**

3. $\tan^3 2x = \tan 2x \Rightarrow \tan^3 2x - \tan 2x = 0 \Rightarrow \tan 2x(\tan^2 2x - 1) = 0$. Thus (1) $\tan 2x = 0$ or (2) $\tan^2 2x = 1$. In (1) 2x = 0 or 180, thus $x = 0°$ or $90°$ and since it repeats every $90°$, $180°$ is also a solution. In (2) $\tan 2x = \pm 1$, so $2x = 45°, 135°, 225°, 315°$ so $x = 22\frac{1}{2}°, 67\frac{1}{2}°, 112\frac{1}{2}°, 157\frac{1}{2}°$. **Ans.** $0°, 90°, 180°, 22\frac{1}{2}°, 67\frac{1}{2}°, 112\frac{1}{2}°, 157\frac{1}{2}°$