3 Circles and Spheres Mar 2019 (No Calculators)

3 pts. 1. A circle of a sphere is 8 units from the center and has a circumference of 12π. What is the volume of the sphere?

Ans. _____

4 pts 2. Two diameters of a circle are perpendicular to each other. A chord of the circle has an endpoint in common with one of the diameter's endpoints and intersects the other diameter at point p. If the length of the chord is 32 and the radius of the circle is 20, how far is p from the center of the circle?

Ans. _____

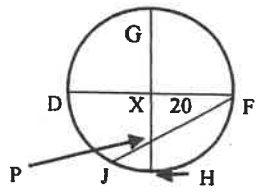
5 pts 3. Each of three circles is externally tangent to the other two. The distances between their centers is 8, 11 and 13. Find the length of the external tangent of the two larger circles.

Ans. _____

Circles and Spheres

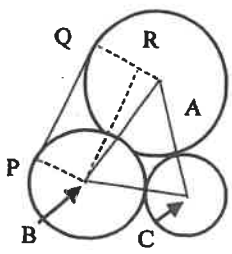
1. The radius of the circle is 6. It is 8 units from the center. So the radius of the sphere is 10, because a 6-8-10 right triangle is formed. Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3$. **Ans.** $\frac{4000\pi}{3}$

2. $\overline{DF}$ and $\overline{GH}$ are diameters, x is the center, $\overline{JF}$ is the chord that intersects $\overline{GH}$ at p. If xp = a, and PF = c, then $(20 + a)(20 - a) = c(32 - c)$ or (1) $400 - a^2 = 32c - c^2$. In right triangle Fxp, (2) $400 + a^2 = c^2$. (1) + (2): $800 = 32c$, so $c = 25$. Using the Pyth. Thm $a = 15$. If you use similar triangles by connecting D to J, you will produce the same answer.



Ans. 15

3. AC = 11, BC = 8, AB = 13. Let the radius of smallest circle be x. The radius of the largest circle from A to C is $11 - x$. The radius of the medium circle from B to C is $8 - x$. The radius of the largest circle from A to B is $3 - (8 - x)$. So $11 - x = 5 + x$. $x = 3$. Radius of medium circle is 5, and the Radius of the largest circle is 8. Connecting from P to B, Q to A, B to R. RA = 3. BR = $\sqrt{169-9} = \sqrt{160} = 4\sqrt{10}$. PR = BR.

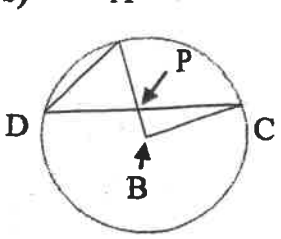


Ans. $4\sqrt{10}$

3 Circles and Spheres Mar 2016 – 17 (No Calculators)

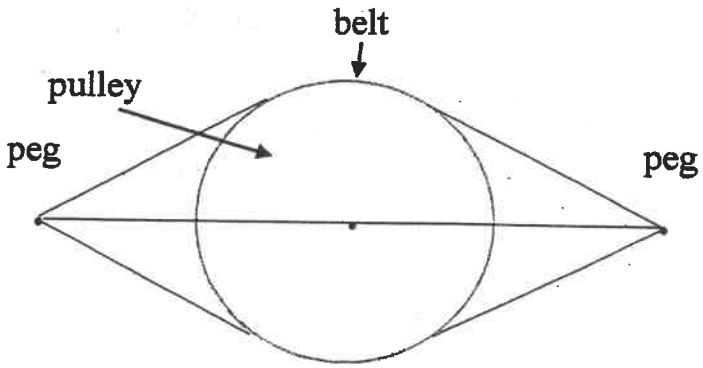
3 pts 1. In the figure, B is the center of the circle, $\overline{AB} \perp \overline{BC}$.

Find the sum: $m\angle APD + m\angle DAP$



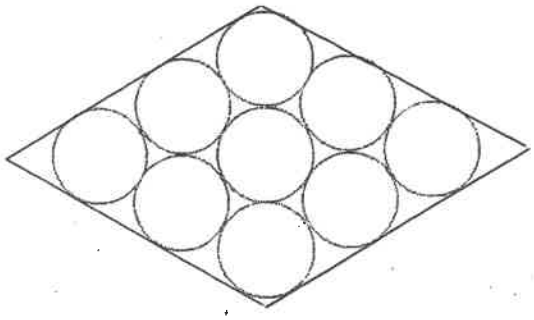
Ans. _____

4 pts 2. A circular belt between two frictionless pegs passes over a rotating circular pulley wheel with radius 1. If the pegs and the center of the pulley are collinear and the pegs are each 1 unit from the closest edge of the pulley, how many units long must the belt be? Assume the belt and pegs have negligible width.



Ans. _____

5 pts 3. In playing "9 Ball", a game in shooting pool, 9 balls are placed in a frame tangent to each other. Assume that the frame is made with two equilateral triangles placed base to base and then discarding the common base. The balls are tangent to the frame. Each ball has a 2 inch diameter. What is the perimeter of the inside of the frame?



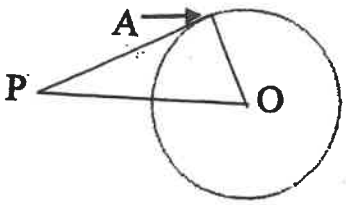
Ans. _____

Circles and Spheres

. $m\angle ADC = 45°$, so the other 2 $\angle$'s of the Δ add to 135°.

Ans. 135°

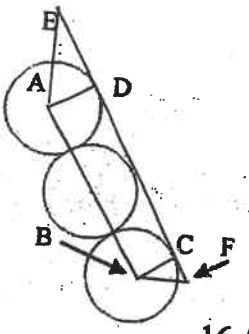
2. Let the center be O, point of tangency be A, and peg point be P. Since $AO = 1$, $PO = 2$, then $AP = \sqrt{3}$. The belt is on the pulley for 120°, which is 1/3 of the circumference or 2/3 π. The belt is $4\sqrt{3} + \frac{2\pi}{3}$ long.



Ans. $4\sqrt{3} + \frac{2\pi}{3}$

3. Working with 3 of these balls tangent to one side of the frame: $AB = 4$, so $CD = 4$. $m\angle EAD = 60°$, $AD = 1$, so $DE = \sqrt{3}$. $m\angle CBF = 30°$, so $CF = \frac{\sqrt{3}}{3}$.

$EF = \sqrt{3} + 4 + \frac{\sqrt{3}}{3} = \frac{3\sqrt{3}+12+\sqrt{3}}{3}$. 4 of these $= \frac{16\sqrt{3}+48}{3}$.



Ans. $\frac{16\sqrt{3}+48}{3}$

3 Circles and Spheres Mar 2016 (No Calculators)

3 pts 1. Find the measure of the arc of a circle which is subtended by a central angle of 150°, if the radius of the circle is 42 cm.

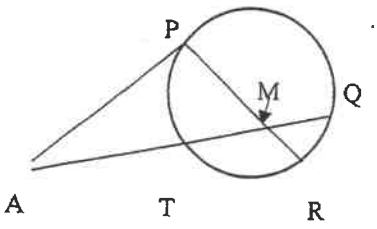
Ans. _____

4 pt 2. Two parallel planes each intersect the same sphere. One passes through the center of the sphere and the other bisects the radius. Find the ratio of the area of the larger circle of intersection to that of the smaller circle of intersection of the planes and the sphere.

Ans. _____

5 pts 3. In the figure, P is the point of tangency of segment AP.

$AT = 3x - 6$, $MR = 5x - 12$, $MQ = 4x - 4$, $PM = 2x + 1$ and $MT = x + 2$. Find AP.



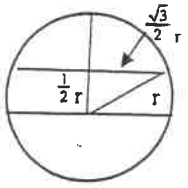
Ans. _____

Circles and Spheres

1. $\frac{150}{360}(84\pi) = \frac{5}{12}(84\pi) = 5(7\pi) = 35\pi$

Ans. 35

2. In the figure at right the radius is r. The distance from the center of the sphere to the other circle of intersection is 1/2 r. Therefore the radius of the other circle is $\frac{\sqrt{3}}{2}$ r. Ratio of areas: $\frac{\pi r^2}{\pi\left(\frac{\sqrt{3}}{2}r\right)^2} = \frac{\pi r^2}{\pi\left(\frac{3}{4}\right)r^2} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$



Ans. 4/3

3. The chords produce: PM MR = TM MQ → $(2x + 1)(5x - 12) = (x + 2)(4x - 4)$ →

$10x^2 - 19x - 12 = 4x^2 + 4x - 8$ → $6x^2 - 23x - 4 = 0$ → $(6x + 1)(x - 4) = 0$, so $x = 4$.

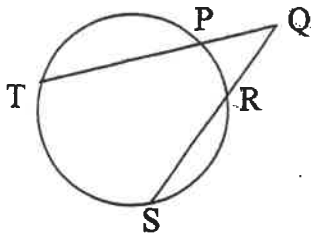
Thus $AT = 3(4) - 6 = 6$, $TQ = TM + MQ = (4) + 2 + 4(4) - 4 = 18$. $(AP)^2 = AT\ AQ = 6\ 24$

$(AP)^2 = 144$, thus $AP = 12$.

Ans. 12

3 Circles and Spheres Mar 2015 (No Calculators)

3 pts 1. In the figure QP = 6, QR = 8, and PT = 22. Find RS.

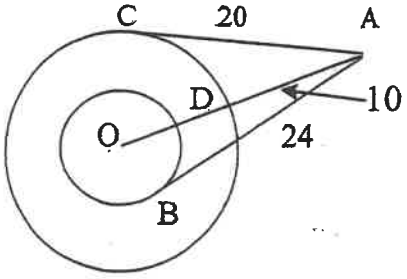


Ans. _____

4 pts 2. A circle is inscribed in a 40° - 60° - 80° triangle. The points of tangency are connected to form a triangle inscribed in the circle. State the measures of the angles of the inscribed triangle in numerical order from least to greatest.

Ans. _____

5 pts 3. $\overline{AB}$ is tangent to the smaller of the two concentric circles at B shown. $\overline{AC}$ is tangent to the larger circle at C. O is the center of the smaller circle. $\overline{OA}$ intersects the larger circle at D. AC is 20, AB = 24, And AD = 10. Find the length of the radius of the smaller circle.



Ans. _____

Circles and Spheres

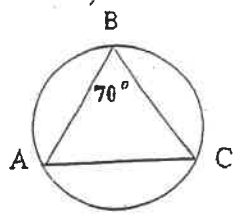
1. $QP \cdot QT = QR \cdot QS = 6 \cdot 28 = 8(8 + RS)$ → $168 = 64 + 8RS$ → $104 = 8RS$. **Ans. 13**

2. The measure of the angle formed by tangents to the circle from the same point is supplementary to the measure of the arc on the circle it intercepts. The measure of an inscribed angle is half the arc it intercepts. So the angles of the circumscribed triangle are $40^\circ, 60^\circ, 80^\circ$. Their corresponding supplements $140^\circ, 120^\circ, 100^\circ$. Their corresponding inscribed angles: $70^\circ, 60^\circ, 50^\circ$. **Ans. $50^\circ, 60^\circ, 70^\circ$**

3. Connect O to C and O to B. OC = OD. Let each = x, then $x^2 + 20^2 = (x + 10)^2$ → $x^2 + 400 = x^2 + 20x + 100$ → $300 = 20x$, So $x = 15$, OA = 25, thus OB = 7. **Ans. 7**

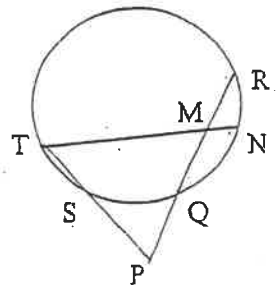
3 Circles and Spheres Mar 2014 (No Calculators)

3 pts 1. Find the measure of arc ABC in the figure.



Ans. _____

4 pts 2. In the figure at right PQ = 8, QR = 7, SP = MT, ST = 2, and MN = 1. Find RM.



Ans. _____

5 pts 3. Point P is 9 cm. from a sphere whose diameter is 16 cm. Find the exact distance from P to the center of the circle made by the locus of points of tangency from P to the sphere.

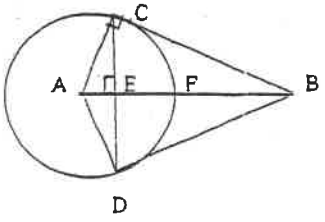
Ans. _____

Circles and Spheres

1. $\angle ABC$ intercepts an arc of $140°$. $360° - 140° = 220°$. **Ans. $220°$**

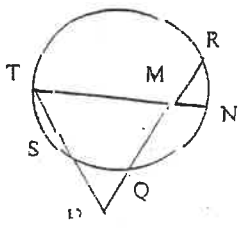
3. At right BF = 9, AF = AC = 8. So BC = 17. Since CD is The diameter of the circle, then CF is the radius. The right Triangles are all similar. So $\frac{AE}{8} = \frac{8}{17} \rightarrow AE = \frac{64}{17} = 3\frac{13}{17}$.

$BE = 17 - 3\frac{13}{17} = 13\frac{4}{17}$. **Ans. $13\frac{4}{17}$**



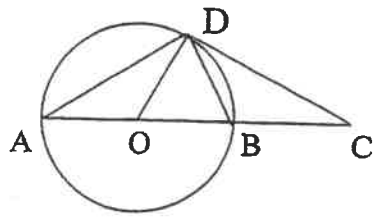
NOTE: SOLUTIONS FOR QUESTION 3 AND 2 ARE SWITCHED IN THIS ROUND.

2. $PQ \cdot PR = PS \cdot PT$. Let PS = x, then $8 \cdot 15 = x(x+2)$. Then $x^2 + 2x = 120 \rightarrow x^2 + 2x - 120 = 0 \rightarrow (x + 12)(x - 10) = 0$. So PS = 10 = MT. $MT \cdot MN = QM \cdot MR$. Let MR = a, then $10(1) = a(7 - a) \rightarrow a^2 - 7a - 10 = 0$ or $(a - 2)(a - 5) = 0$. **Ans. 2 or 5.**



3 Circles and Spheres Mar 2013 (No Calculators)

3 pts 1. Points A, B, and D lie on the circle O. O is the center. Line segment AC passes through points O and B. Line segment CD is tangent to O at point D. Line segment BD is chord of circle O. If angle DCB measures 34°, find the measure of angle OBD.

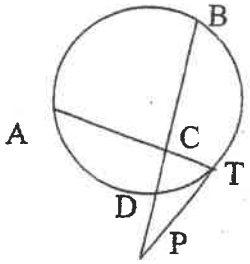


Ans. _____

4 pts 2. Two balls are rolled the length of a 10 foot table. The larger ball makes 10 revolutions and the smaller makes 12 revolutions. How many inches greater is the radius of the larger ball than the radius of the smaller ball?

Ans. _____

5 pts 3. In the drawing, $\overline{PT}$ is a tangent with length 6. $\overline{PB}$ is a secant passing through point D on the circle. $\overline{AT}$ is a chord intersecting with $\overline{PB}$ at C. If PD = 3, DC = 4, and CT = $\sqrt{13}$, find the distance from $\overline{BD}$ to the center of the circle.



Ans. _____

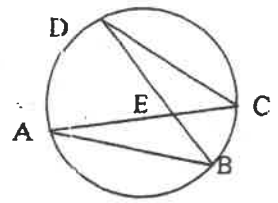
Circles and Spheres

(1) There are several ways to find the measure of angle OBD. Since $\overline{CD}$ is tangent at D. Then angle ODC is a right angle. Thus $m\angle COD = 56$. From here, (1) since OD = OB, then $m\angle OBD = \frac{1}{2}(180 - 56) = 62$, (2) since $m\angle AOD = 124$, then arc AD = 124 and it ibed angle ABD = 62, (3) since $\angle ADB$ is a right angle because it is inscribed in a semicircle and AO = OD, then $m\angle DAO = 28$, and its compliment ABD = 62. **Ans. 62°**

(2) 10 feet = 120 inches. Since $C = 2\pi r$, then Large ball: $C = \frac{120in}{10} = 12$ in. for the Small ball: $C = \frac{120in}{12} = 10$ in. For large ball: $r = \frac{12}{2\pi} = \frac{6}{\pi}$. For small: $r = \frac{10}{2\pi} = \frac{5}{\pi}$. The difference is $1/\pi$ inches. **Ans. $1/\pi$**

3 Circles and Spheres Mar 2012 (No Calculators)

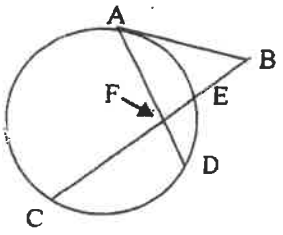
3 pts 1. In the figure, $m\angle A = 20^\circ$, m arc $AD = 100^\circ$. Find the sum of $m\angle D$ and $m\angle AED$.



Ans. _____

4 pts 2. In the figure AB = 2x, BE= x, FE = 2, CF = x + 6, DF = 3 and AF = y.

Find y.



Ans. _____

5 pts 3. Three spheres are placed on the floor. Each tangent to the other two and all have a radius of 8. A fourth sphere, with the same radius is placed in the center atop the other three. How high is it from the top of the highest sphere to the floor.

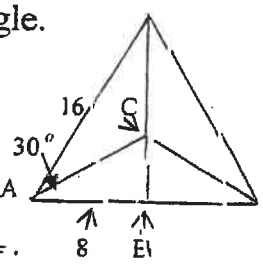
Ans. _____

Circles and Spheres

1. Inscribed angles A and D intercept the same arc, so $m\angle D$ is 20°. Inscribed angle C has a measure of 50. Exterior angle AED has measure 70. **Ans. 50°**

2. $AB^2 = BE(BC) \rightarrow (2x)^2 = x(2x + 8) \rightarrow 4x^2 = 2x^2 + 8x \rightarrow 2x^2 - 8x = 0 \rightarrow$ $2x(x - 4) = 0$, so $x = 4$. $AF(FD) = FE(CF) \rightarrow y(3) = 2(10)$, so $y = 6\frac{2}{3}$. **Ans. $6\frac{2}{3}$**

3. The centers of the base of the lower 3 spheres form an equilateral triangle. Connecting these three vertices with center of the upper sphere forms a regular tetrahedron with edge of 16 cm. From the base of the tetrahedron to the floor is 8 cm, and from the vertex of the tetrahedron to the top of the upper sphere is 8 cm. We need to find the height of the tetrahedron for the final measurement. In the base of the tetrahedron at right, $AC = \frac{16}{\sqrt{3}}$.

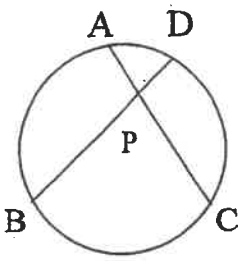


By the Pyth. Thm. the height (h) of the tetrahedron is $h^2 = 16^2 - \left(\frac{16}{\sqrt{3}}\right)^2 = 16^2(2/3)$.

So $h = 16\sqrt{\frac{2}{3}} = \frac{16\sqrt{6}}{3}$. This added to 16 makes $16 + \frac{16\sqrt{6}}{3}$. **Ans. $16 + \frac{16\sqrt{6}}{3}$**

3 Circles and Spheres Mar 2011 (No Calculators)

3 pts 1. In the circle at right, measure of arc AD = 40° and measure of arc BC = 120°. Find the measure of angle DPC.

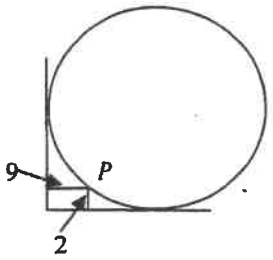


Ans. _____

4 pts 2. Points A and B lie on a sphere of radius 12. The length of the chord joining A and B is $12\sqrt{3}$. What is the length of the shortest path from A to B, if every point on the path lies on the sphere?

Ans. _____

5 pts 3. A circular table in the diagram at right is pushed against two perpendicular walls. The point *P* on the circumference of the table is a distance of 2 feet from one wall and 9 feet from the other wall. What is the radius of the table?

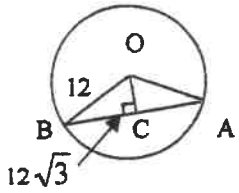


Ans. _____

Circles and Spheres

1. $m\angle BPC = \frac{1}{2}(120 + 40) = 80°$. $m\angle DPC = 180 - 80 = 100$. **Ans. 100° or 100**

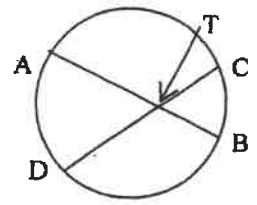
2. Central angle AOB = 120°, since OA = 12 and ½ AB = $6\sqrt{3}$, makes right ΔAOC a $30-60-90\Delta$ with 30° at angle A. Need length of arc from A to B: $1/3\,(2\pi(12)) = 8\pi$. **Ans. 8π**



3. The two radii and the portion on the wall from the corner to the table form a square of side-length equal to the radius. Let $y = r - 9$ and $x = r - 2$. Drawing a radius to a point P forms a right triangle with sides x, y and hypotenuse r, so $x^2 + y^2 = r^2$. Substituting:
$(r-2)^2 + (r-9)^2 = r^2 \rightarrow r^2 - 4r + 4) + r^2 - 18r + 81 = r^2 \rightarrow r^2 - 22r + 85 = 0$.
$(r-17)(r-5) = 0$. $r = 17$ or 5, but $r > 9$. So $r = 17$. **Ans. 17**

3 Circles and Spheres Mar 2010 (No Calculators)

3 pts 1. If the measure of arc BC = 35° and $m\angle BTD = 152^\circ$, Find the measure of arc AD.



Ans. _____

4 pts 2. Point P is 10 cm from a circle in a plane. If one of the tangent segments to the circle is 20 cm long, what is the length of the radius of the circle?

Ans. _____

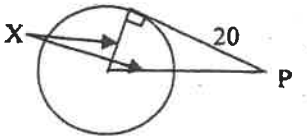
5 pts 3. A point P is 17 cm from the center of a sphere. A tangent segment from P to the sphere is 15 cm long. How long is the radius of the circle of intersection made by all the points of tangency from P to the sphere?

Ans. _____

Circles and Spheres

1. $m\angle ATD = 28$. Let measure arc AD = x, then $28 = \frac{1}{2}(35 + x) \rightarrow 56 = 35 + x$. **Ans. 21**

2. In the figure, let x = the radius. Then $x^2 + 20^2 = (x + 10)^2 \rightarrow$
$x^2 + 400 = x^2 + 20x + 100$
$20x = 300$, thus $x = 15$. **Ans. 15 or 15 cm**



3. In the figure, the radius $\overline{AN}$ of the circle of tangents is perpendicular to $\overline{BP}$ and $\angle BAP$ is a right angle at point of tangency A, so all the triangles are similar. Since BP = 17 and AP = 15, then AB = 8 through the Pyth. Thm. Thus $\frac{AN}{8} = \frac{15}{17} \rightarrow AN = \frac{8 \cdot 15}{17} = \frac{120}{17} = 7\frac{1}{17}$. **Ans.** $7\frac{1}{17}$ **or** $\frac{120}{17}$.