4 Conics Mar 2019 (No Calculators)

3 pts 1. What is the closest distance that the circle $x^2 + y^2 - 24x + 10y + 133 = 0$ comes to the origin?

Ans. ____________________

4 pts 2. What are the coordinates of the endpoints of the major axis of the ellipse whose equation is $25x^2 + 16y^2 - 300x - 224y + 1284 = 0$?

Ans. ____________________

5 pts 3. The equations of the asymptotes of a hyperbola are $4x - 7y = -67$ and $7y + 4x = 3$. If one of its vertices is (-8, 1), what is the equation of the hyperbola?

Ans. ______________________________

Conics

1. $x^2 + y^2 - 24x + 10y + 133 = 0 \Rightarrow (x^2 - 24x + 144) + (y^2 + 10y + 25) = -133 + 144 + 25 \Rightarrow$ $(x - 12)^2 + (y + 5)^2 = 36$. Center (12, -5) is 13 units from the origin. Since radius is 6, **Ans. 7**

2. $25x^2 + 16y^2 - 300x - 224y + 1284 = 0 \Rightarrow 25(x - 12x + 36) + 16(y - 14x + 49) = -1284 +$ $900 + 784 = 400$. So $\frac{(x-6)^2}{16} + \frac{(y-7)^2}{25} = 1$. Major axis endpoints, $(6, 7 \pm 5)$ **Ans. (6, 12), (6, 2)**

3. The equations $4x - 7y = -67$ and $7y + 4x = 3$ intersect at the center. Adding we get $8x = -64$, so $x = -8$. $4(-8) - 7y = -67$, $-7y = -35$, so $y = 5$, and the center is (-8, 5). Since one of the vertices is (-8, 1), then the hyperbola opens up and down and has form: $\frac{(y-5)^2}{16} - \frac{(x+8)^2}{b^2} = 1$.

The slopes of the asymptotes are $\pm\frac{4}{7}$, then $b^2 = 49$. **Ans.** $\frac{(y-5)^2}{16} - \frac{(x+8)^2}{49} = 1$

4 Conics Mar 2016 – 17 (No Calculators)

3 pts 1. If the center of the circle $x^2 + y^2 - 4x - 6y = 12$ is moved 1 unit to the right, while its radius stays the same, what will be the sum of the coordinates of its new center?

Ans. ______________

4 pts 2. Find the two points on the conic $16x^2 + 36y^2 - 160x + 216y + 148 = 0$ which are the furthest apart.

Ans. ______________

5 pts 3. A hyperbola has asymptotes $3x - 2y = 16$ and $3x + 2y = 8$. In the same plane, a rectangle is constructed with sides parallel to the x- and y-axes such that all four vertices are on the asymptotes and the rectangle is tangent to the hyperbola at both of the hyperbola's vertices. If an ellipse is inscribed inside the rectangle, find all possible ratios of the eccentricity of the hyperbola to the eccentricity of the ellipse.

Ans. ______________

Conics

1. $x^2 + y^2 - 4x - 6y = 12 \rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = 12 + 13 = 25$. Circle's center is at (2, 3) with radius of 5. One unit to the right is (3, 3).

Ans. c = 6

2. $16x^2 + 36y^2 - 160x + 216y + 148 = 0 \rightarrow 16(x^2 - 10x + 25) + 36(y^2 + 6y + 9) = -148 + 724 = 576 \rightarrow \frac{(x-5)^2}{36} + \frac{(y+3)^2}{16} = 1$. Center is (5, -3). 6 each way on x: (11, -3), (-1, -3)

Ans. (11, -3) and (-1, -3)

3. For the ellipse, the major and minor axes are in the ratio of 3:2, so a = 3n and b = 2n for some n. Thus $c^2 = 9n^2 - 4n^2 = 5n^2$, so $c = \sqrt{5}n$. The ecc. $= \frac{c}{a} = \frac{\sqrt{5}}{3}$.

There are two possible hyperbolas. $c^2 = a^2 + b^2$, where a = 3 and b = 2 or a = 2 and b = 3. In either case $c = \sqrt{13}$. So the ecc. is either $\frac{\sqrt{13}}{3}$ or $\frac{\sqrt{13}}{2}$. So the possible ratios are $\frac{\frac{\sqrt{13}}{3}}{\frac{\sqrt{5}}{3}} = \frac{\sqrt{65}}{5}$ or $\frac{\frac{\sqrt{13}}{2}}{\frac{\sqrt{5}}{3}} = \frac{3\sqrt{65}}{10}$.

Ans. $\frac{3\sqrt{65}}{10}$ **or** $\frac{\sqrt{65}}{5}$

4 Conics Mar 2016 (No Calculators)

3 pts 1. Find the center in the form (x, y) and the radius of the circle whose equation is

$x^2 + y^2 + 6x - 6y + 14 = 0$. **Ans.** ______________________

4 pts 2. Find the equation of the parabola whose directrix is y = - 4 and focus is (-2, 2).

Write the equation in the form $y = ax^2 + bx + c$ or $x = ay^2 + by + c$.

Ans. ______________________

5 pts 3. The endpoints of the major axis of an ellipse are (-7, 5) and (1, 5). The eccentricity is $\frac{\sqrt{3}}{2}$. What is the equation of the ellipse in the form $\frac{(x-h)}{a^2} + \frac{(y-k)}{b^2} = 1$?

Ans. ______________________

Conics

1. $x^2 + y^2 + 6x - 6y + 14 = 0$ ➔ $(x^2 + 6x + 9) + (y^2 - 6y + 9) = -14 + 18 = 4$. **Ans. C(-3, 3), r = 2**

2. The vertex of the parabola is halfway from the focus (-2, 2) to the directrix y = -4. The vertex is (-2, -1) and the parabola opens up, so it has the form $(x - h)^2 = 4p(y - k)$.

$(x + 2)^2 = 4(3)(y + 1)$ ➔ $x^2 + 4x + 4 = 12y + 12$ ➔ $12y = x^2 + 4x - 8$. **Ans.** $y = \frac{1}{12}x^2 + \frac{1}{3}x - \frac{2}{3}$

3. The midpoint of (-7, 5) and (1, 5) is (-3, 5) which is the center of the hyperbola. $\frac{f}{v} = \frac{\sqrt{3}}{2} = \frac{f}{4}$

$f = 2\sqrt{3}$. For $a^2 + b^2 = c^2$ ➔ $16 + b^2 = 12$, $b^2 = 4$. **Ans.** $\frac{(x+3)^2}{16} + \frac{(y-5)^2}{4} = 1$

 Conics Mar 2015 (No Calculators)

3 pts 1. Find the center and the radius of the circle whose equations is

$$2x^2 + 2y^2 + 2x - 6y - 3 = 0$$

Ans. ____________________

4 pts 2. Find the equation of the conic whose center is (-5, 3), focus is (-5, -12) and whose vertex is (-5, 15).

Ans. ____________________

5 pts 3. Find the equation of the ellipse with foci (-4, 0) and (4, 0) that passes through the point $(3, \sqrt{15})$. Give answer in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

Ans. ____________________

Conics

1. $2x^2 + 2y^2 + 2x - 6y - 3 = 0 \Rightarrow (x^2 + x + \frac{1}{4}) + (y^2 - 3y + \frac{9}{4}) = \frac{3}{2} + \frac{1}{4} + \frac{9}{4} = 4.$

   $(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = 4.$ **Ans. Center $(-\frac{1}{2}, \frac{3}{2})$, radius 2**

2. The vertex is 12 units away from the center. The focus is 15 units from the center. This must be a hyperbola. The endpoint of the conjugate axis is 9 units from the center, since it involves a 9 – 12 – 15 right triangle. The hyperbola opens up and down thus the equation: $\frac{(y+5)^2}{144} - \frac{(x-3)^2}{81} = 1$ **Ans.** $\frac{(y+5)^2}{144} - \frac{(x-3)^2}{81} = 1$

3. Center is at (0, 0), thus the equation takes on the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $a^2 - b^2 = 16 \Rightarrow$ $a^2 = b^2 + 16$. Plugging in the point $(3, \sqrt{15})$: $\frac{9}{a^2} + \frac{15}{b^2} = 1, \Rightarrow \frac{9}{b^2 + 16} + \frac{15}{b^2} = 1 \Rightarrow$ $9b^2 + 15b^2 + 240 = b^4 + 16b^2 \Rightarrow b^4 - 8b^2 - 240 = 0 \Rightarrow (b^2 - 20)(b^2 + 12) = 0.$

   So $b^2 = 20$ and $a^2 = 36$. **Ans.** $\frac{x^2}{36} + \frac{y^2}{20} = 1$

4 Conics Mar 2014 (No Calculators)

pts **1.** Each of three different circles, centered at the origin, contains one of the following points (4, 7), (-3, 8) and (5, -6). What is the equation of the smallest circle?

Ans. ____________________

4 pts 2. Find the eccentricity of the ellipse whose equation is

$$x^2 + 16y^2 + 14x - 288y + 1329 = 0.$$

Ans. __________

5 pts 3. Find the coordinates of the foci of the hyperbola

$$16y^2 - 9x^2 + 72x + 160y + 832 = 0.$$

Ans. ____________________

Conics

1. The radius from (0, 0) to (-5, 6) is $\sqrt{(-5)^2 + (6)^2} = \sqrt{25+36} = \sqrt{61}$. **Ans.** $\mathbf{x^2 + y^2 = 61}$

2. Completing the square for x and y for $x^2 + 16y^2 + 14x - 288y + 1329 = 0 \rightarrow$
$(x^2 + 14x + 49) + 16(y^2 - 18y + 81) = -1329 + 49 + 1296 = 16$. Thus $\frac{(x+7)^2}{16} + \frac{(x - -9)^2}{1} = 1$
The eccentricity is $\sqrt{15}/4$. **Ans.** $\mathbf{\sqrt{15}/4}$

3. Completing the square for $16y^2 - 9x^2 + 72x + 160y + 832 = 0$:
$16(y^2 + 10y + 25) - 9(x^2 - 8x + 16) = -832 + 400 - 144 = -576$. So $\frac{(x-4)^2}{64} - \frac{(y+5)^2}{36} = 1$.
The distance to the foci from the center is $\sqrt{64+36} = 10$. Center is (4, -5). Since the foci are horizontal, then they are $(4 \pm 10, -5)$. **Ans. (14, -5), (-6, -5)**

4 Conics Mar 2013 (No Calculators)

3 pts 1. Find the distance from the origin, (0, 0), to the center of the circle whose equation is

$$x^2 + 8x + y^2 - 6y + 21 = 0.$$

Ans. ____________________

4 pts 2. An ellipse with major axis parallel to the x-axis is inscribed in a rectangle with vertices (0, 15), (0, 17), (8, 17) and (8, 15). Find *d* if the equation of the ellipse is written in the form $x^2 + ay^2 + bx + cy = d$.

Ans. ____________________

5 pts 3. The vertices of a hyperbola are (-7, 5) and (1, 5). The slopes of the asymptotes are $\pm\frac{1}{2}$. Find the equation of the hyperbola.

Ans. ____________________

√13 13 **Ans.** $\frac{}{13}$

Conics

(1) The center of the circle is (-4, 3). It is 5 units from the origin.

Ans. 5

(2) The center of the ellipse is (4, 16). The semi-major axis is 4 and the semi-minor is 1.
Equation: $\frac{(x-4)^2}{16} + \frac{(y-16)^2}{1} = 1 \rightarrow x^2 - 8x + 16 + 16y^2 - 512y + 4096 = 16 \rightarrow$
$x^2 + y^2 - 8x - 512y = -4096.$

Ans. -4096

(3) Since vertices are (-7, 5) and (1, 5), then center is at (-3, 5) and semi transverse axis is 4. Slopes of asymptotes are ½, so $\frac{1}{2} = \frac{b}{4}$, thus b = 2(the semi-conjugate axis).

Ans. $\frac{(x+3)^2}{16} - \frac{(y-5)^2}{4} = 1$

Arithmetic with Statisti

4 Conics Mar 2012 (No Calculators)

3 pts 1. Find the endpoints of the latus rectum of the parabola $y^2 + 12x = 0$.

Ans. ____________________

4 pts 2. The eccentricity of an ellipse is .6 and the minor axis is 2 units shorter than the major axis. The minor axis is horizontal. The center is at the intersection of the lines $5x - y = 2$ and $x + 2y = 7$. Find the equation of the ellipse.

Ans. ____________________

5 pts 3. One axis of symmetry of a hyperbola is $y + 8 = 0$, which the hyperbola does not intersect. One of the asymptotes of the hyperbola has the equation $2x + y = 0$. The distance between the foci is $6\sqrt{5}$. Find the equation of the hyperbola.

Ans. ____________________

Conics

1. The parabola opens to the left, with the focus at (-3, 0). The latus rectum is 12 units long, which is 6 units up and down from the focus. **Ans. (-3, 6), (-3, -6)**

2. Solving (1) $5x - y = 2$ and (2) $x + 2y = 7$ for center: Mult. (1) by 2: $10x - 2y = 4$. Add this to (2) yields $11x = 11$. So $x = 1$ and $y = 3$: center (1, 3). Ecc. = .6 = 3/5, so the ratio of the semi-minor to the semi-major is 4 to 5, a difference of 1 unit. This means that the minor axis is 2 units shorter than the major axis at 8 and 10. Since the minor axis is horizontal, then the equation is $\frac{(x-1)^2}{16} + \frac{(y-3)^2}{25} = 1$. **Ans.** $\frac{(x-1)^2}{16} + \frac{(y-3)^2}{25} = 1$

3. The asymptotes intersect the center of the hyperbola on the axis of symmetry, so $y = -8$ is the y-coordinate of the center and plugging this into $2x + y = 0$, makes $x = 4$ the x value for the center. The hyperbola opens up and down thus has the form $\frac{(y+8)^2}{b^2} - \frac{(x-4)^2}{a^2} = 1$, because it does not intersect $y + 8 = 0$. The slope of the asymptotes from $2x + y = 0$: $y = -2x$ is $\frac{2}{1}$ or $\frac{2x}{1x}$, where a = 1x and b = 2x. By th Pyth. Thm. $(2x)^2 + x^2 = (3\sqrt{5})^2$ ➔ $4x^2 + x^2 = 45$ ➔ $5x^2 = 45$, so $x = 3$. Thus: **Ans.** $\frac{(y+8)^2}{36} - \frac{(x-4)^2}{9} = 1$

4 Conics Mar 2011 (No Calculators)

3 pts 1. Find an equation for the ellipse with foci at (8, 0) and (-8, 0) and y-intercepts at 6 and -6.

Ans. ____________________

4 pts 2. Find an equation of the parabola with vertex (1/2, -1/4) and directrix on the line having equation y = - ½.

Ans. ____________________

5 pts 3. Find an equation for the hyperbola with foci at (0, 4) and (0, -4) and passing through the point ($\sqrt{15}$, 3).

Ans. ____________________

Conics

1. c = 8, b = 6, therefore c = 10. Major axis parallel to x – axis. **Ans.** $\frac{x^2}{100}+\frac{y^2}{36}=1$

2. The equation is of the form $(x-h)^2 = 4p(y-k)$. The distance from the vertex to the directrix is ¼ and the parabola opens up, since the vertex is above the directrix. So $(x-½)^2 = y + ¼ \rightarrow x^2 - x + ¼ = y + ¼$. Thus $y = x^2 - x$. **Ans.** $y = x^2 - x$

3. Center is at the origin, c = 4, therefore (1) $a^2 + b^2 = 16$, and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

The point ($\sqrt{15}$, 3) will produce (2) $\frac{9}{a^2}-\frac{15}{b^2}=1$. In (1): $a^2 = 16 - b^2$. Subbing (1) into (2): $\frac{9}{16-b^2}-\frac{15}{b^2}=1 \rightarrow 9b^2 - 15(16-b^2) = b^2(16-b^2) \rightarrow 9b^2 - 240 + 15b^2 = 16b^2 - b^4$.

Thus $b^4 + 8b^2 - 240 = 0$. $(b^2 + 20)(b^2 - 12) = 0$. Thus $b^2 = 12$ and from (1): $a^2 = 4$.

Ans: $\frac{y^2}{4}-\frac{x^2}{12}=1$

4 Conics Mar 2010 (No Calculators)

3 pts 1. Find the coordinates of the endpoints of the major axis of the ellipse

$$16x^2 + 25y^2 = 10{,}000$$

Ans. ____________

4 pts 2. The vertices of a hyperbola are (-3, 9) and (-3, 1). Its eccentricity is $\frac{\sqrt{5}}{2}$. Find its equation.

Ans. ____________

5 pts 3. A circle is tangent to the lines 5x + 12y = -21 and 5x + 12y = 83. Its center is on the line 2x + 3y = 7. Find the equation of the circle.

Ans. ____________

Conics

1. $16x^2 + 25y^2 = 10{,}000 \Rightarrow \frac{x^2}{625} + \frac{y^2}{400} = 1 \Rightarrow \frac{x^2}{(25)^2} + \frac{y^2}{(20)^2} = 1.$ **Ans. (±25, 0)**

2. If the vertices are (-3, 9) and (-3, 1), the center is at (-3, 5) and opens up and down. The form of the equation is $\frac{(y-5)^2}{16} - \frac{(x+3)^2}{b^2} = 1$. Ecc = $\frac{\sqrt{5}}{2} = \frac{f}{v} = \frac{f}{4}$. Thus $f = 2\sqrt{5}$.

$f^2 = a^2 + b^2 \Rightarrow 20 = 16 + b^2$, so $b^2 = 4$. **Ans.** $\frac{(y-5)^2}{16} - \frac{(x+3)^2}{4} = 1$

3. The distance between the parallel lines 5x + 12y = -21 and 5x + 12y = 83 = $\frac{|83-(-21)|}{\sqrt{5^2+12^2}}$ $= \frac{104}{13} = 8$. Thus the radius of the circle is 4. The line of centers between the two parallel lines is (1) 5x + 12y = 31. Since the center is also on the line (2) 2x + 3y = 7, we need to find the intersection. 2(1) -5(2) → 10x + 24y = 62 and -10x – 15y = -35, thus 9y = 27, so = 3. Subbing into (1): 5x + 12(3) = 31, so x = -1. **Ans.** $(x+1)^2 + (y-3)^2 = 16$