Midterm Review

* No Calculator *

1) The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

2) The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

3) If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$

4) If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

5) Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

6) The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

7) For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

8) If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

9) If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

10) If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

11) What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

12) Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

13) If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

14) If $\ln(2x + y) = x + 1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) $y - \frac{y}{x}$

15) The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\frac{3}{2}$. For $6 < c < 9$, there is no value of c such that $f'(c) = -\frac{3}{2}$. Of the following, which must be true?

(A) $\frac{1}{3}\int_6^9 f(x)\,dx = -\frac{3}{2}$

(B) $\int_6^9 f(x)\,dx$ does not exist.

(C) $\frac{f'(6) + f'(9)}{2} = -\frac{3}{2}$

(D) $f'(x) < 0$ for all x in the open interval $(6, 9)$.

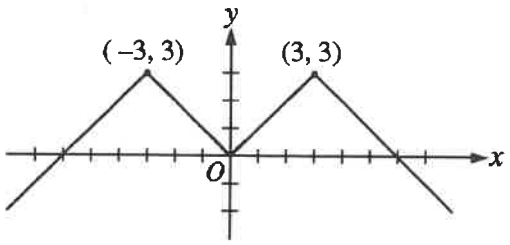
(E) f is not differentiable on the open interval $(6, 9)$.

16) If $\lim_{h \to 0} \frac{\arcsin(a + h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2

17) Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

(A) -5 (B) $-\frac{15}{4}$ (C) -1 (D) 6 (E) 17



18) The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x \to 0} (f(x) - f(0)) = 0$

(B) $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \to 0} \frac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ does not exist.

19) Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

20) $\lim\limits_{x \to 0} \dfrac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

* calculator *

21) The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

22) The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

23) Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

24) Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

25) The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only

(B) II only

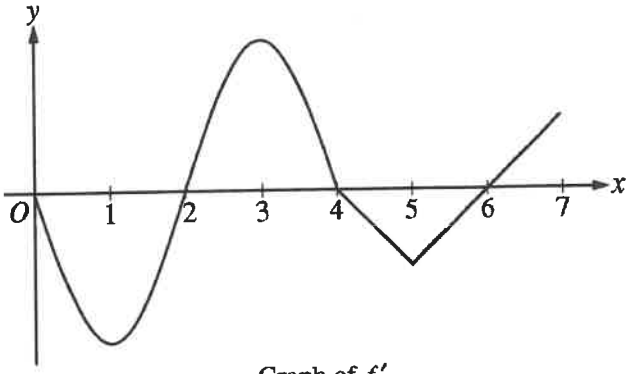
(C) III only

(D) I and III only

(E) I, II, and III

26) A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin(\sqrt{t+1})$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668



Graph of f'

27) The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) $[2, 4]$ only

(B) $[3, 5]$ only

(C) $[0, 1]$ and $[3, 5]$

(D) $[2, 4]$ and $[6, 7]$

(E) $[0, 2]$ and $[4, 6]$

28) If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

29) The derivative of the function f is given by $f'(x) = x^3 - 4\sin\left(x^2\right) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

(A) -1.970 and 0

(B) -1.467 and 1.075

(C) -0.475, 0.542, and 1.396

(D) -0.475 and 1.396 only

(E) 0.542 only

30) The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

31) A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t + 2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

32) The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

Mid Term Review Extra Problems

Name

1. At $x = 3$, the function given by

$$f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases} \text{ is}$$

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

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2. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$

(B) $-\frac{3}{4e}$

(C) $-\frac{1}{4e}$

(D) $\frac{1}{4e}$

(E) $\frac{1}{e}$

---

3. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

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(A) 0.246

(B) 0.430

(C) 0.447

(D) 0.790

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4. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is decreasing only when $b<h$.

(D) A is decreasing only when $b>h$.

(E) A remains constant.

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5. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of 4/9 meter per second, at what rate, in meters per second, is the person walking?

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(A) 4/27

(B) 4/9

(C) 3/4

(D) 4/3

(E) 16/9

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6. Sand is deposited into a pile with a circular base. The volume V of the pile is given by $V = \frac{r^3}{3}$, where r is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of 5π feet per hour. When the circumference of the base is 8π feet, what is the rate of change of the volume of the pile, in cubic feet per hour?

(A) $\frac{8}{\pi}$

(B) 16

(C) 40

(D) 40π

(E) 80π

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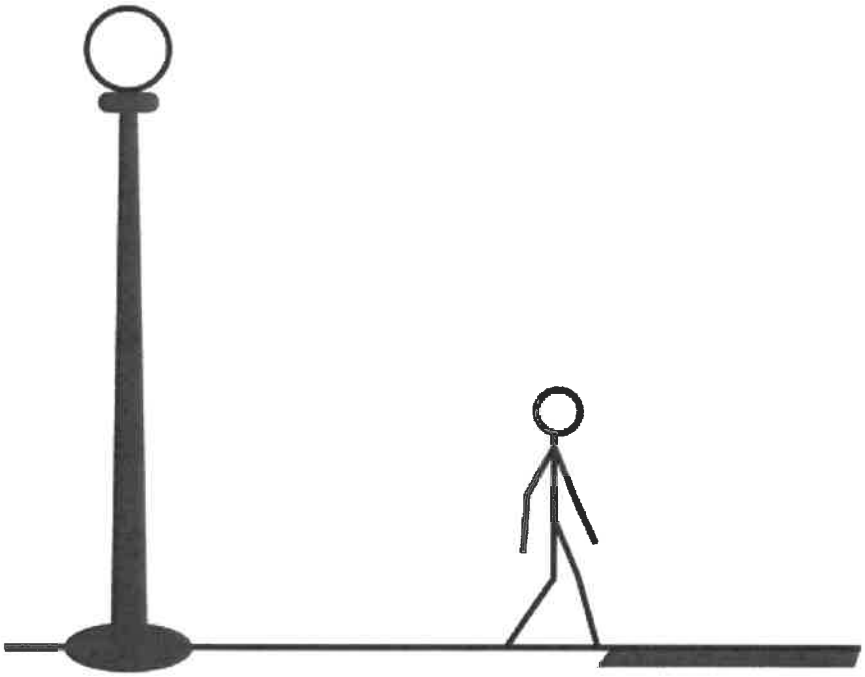
Mid Term Review Extra Problems

7.



A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec

(B) 2.667 ft/sec

(C) 3.75 ft/sec

(D) 6 ft/sec

(E) 10 ft/sec

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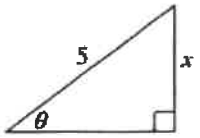
8.



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3

(B) $\frac{15}{4}$

(C) 4

(D) 9

(E) 12

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9. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4

(B) 0.6

(C) 0.7

(D) 1.3

(E) 1.4

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10. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6

(B) 3.4

(C) 4.2

(D) 4.6

(E) 4.64

---

11. Let $y = f(x)$ be a differentiable function such that $\frac{dy}{dx} = \frac{x}{y}$ and $f(8) = 2$. What is the approximation of $f(8.1)$ using the line tangent to the graph of f at $x = 8$?

(A) 0.4

(B) 2.025

(C) 2.4

(D) 6

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12. Let f be a twice-differentiable function such that $f''(x) < 0$ for all x. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of f at $x = 4$. Which of the following is true?

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(A) $f\,(4.2) < S(4.2) < T(4.2)$

(B) $f\,(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f\,(4.2) < T(4.2)$

(D) $T(4.2) < f\,(4.2) < S(4.2)$

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13. Let f be a function that is continuous on the closed interval [1,3] with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

(A) $10 \leq f(2) \leq 18$

(B) f is increasing on the interval [1, 3].

(C) $f(x) = 17$ has at least one solution in the interval [1, 3].

(D) $f'(x) = 8$ has at least one solution in the interval (1, 3).

(E) $\int_1^3 f(x)dx > 20$

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14. Let g be a continuous function on the closed interval [0,1]. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

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(A) There exists a number h in [0,1] such that $g(h) \geq g(x)$ for all x in [0,1].

(B) For all a and b in [0,1], if $a=b$, then $g(a)=g(b)$

(C) There exists a number h in [0,1] such that $g(h) = \frac{1}{2}$

(D) There exists a number h in [0,1] such that $g(h) = \frac{3}{2}$

(E) For all h in the open interval (0,1) , $\lim_{x \to h} g(x) = g(h)$

15.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval [0,2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0,2] if $k=$

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 3

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16.

x	$f(x)$
–1	–30
0	–2
3	10
5	18

The table above gives selected values for a twice-differentiable function f. Which of the following must be true?

(A) f has no critical points in the interval -1< x < 5.

(B) f' (x) = 8 for some value of x in the interval -1<x < 5.

(C) $f'(x) > 0$ for all values of x in the interval -1<x<5

(D) $f''(x) < 0$ for for all values of x in the interval -1 < x < 5

(E) The graph of f has no points of inflection in the interval -1< x < 5

17.

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

The table above shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times $f(x) = 4$?

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(A) one

(B) two

(C) three

(D) four

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18. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at x=2.

II. f is continuous at x=2.

III. f is differentiable at x=2.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

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19. $f(x) = \begin{cases} 3x+5 & \text{when } x < -1 \\ -x^2+3 & \text{when } x \geq -1 \end{cases}$

If f is the function defined above, then $f'(-1)$ is

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(A) −3

(B) −2

(C) 2

(D) 3

(E) nonexistent

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20. $f(x) = \begin{cases} 2x+5 & \text{for } x < -1 \\ -x^2+6 & \text{for } x \geq -1 \end{cases}$

If f is the function defined above, then $f'(-1)$ is

(A) -2

(B) 2

(C) 3

(D) 5

(E) nonexistent

21.

$$f(x) = \begin{cases} x+b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

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(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and and b is any real number

(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b.

---

22. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a is a vertical asymptote of the graph of f.

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23. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

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Mid Term Review Extra Problems

(A) -1

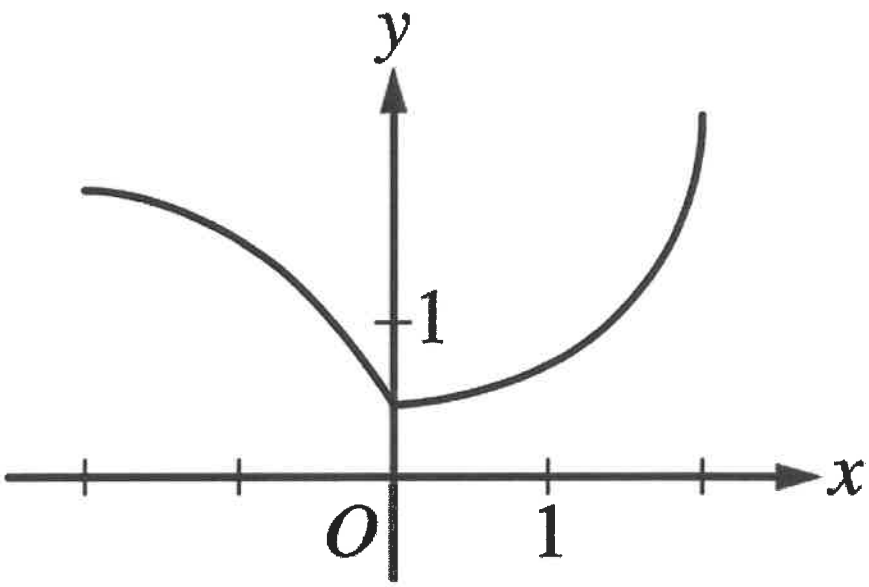
(B) 0

(C) 1

(D) 2

(E) Nonexistent

24.



Graph of f

The function f, whose graph is shown above, is defined on the interval $-2 \leq x \leq 2$. Which of the following statements about f is false?

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(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

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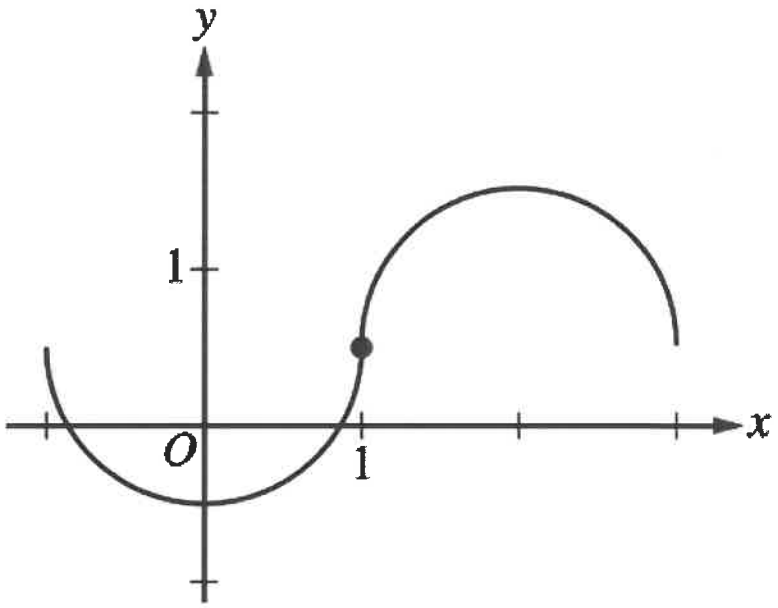
Mid Term Review Extra Problems

25.



Graph of h'

The function h is defined on the closed interval $[-1, 3]$. The graph of h', the derivative of h, is shown above. The graph consists of two semicircles with a common endpoint at $x = 1$. Which of the following statements about h must be true?

1.
$h(-1) = h(3)$

2.
h is continuous at $x = 1$.

3.
The graph of h has a vertical asymptote at $x = 1$.

(A) None

(B) II only

(C) I and II only

(D) I and III only

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26. If *f* is a differentiable function such thatf(3)=8 and $f'(3) = 5$, which of the following statements could be false?

(A) $\lim_{x \to 3} f(x) = 8$

(B) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

(C) $\lim_{x \to 3} \frac{f(x)-8}{x-3} = 5$

(D) $\lim_{h \to 0} \frac{f(3+h)-8}{h} = 5$

(E) $\lim_{h \to 3} f'(x) = 5$

---

27. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

---

28. Let f be the function defined by $f(x) = \frac{x^3-2x^2-3x}{x^3-3x^2+4}$. Which of the following statements about f at $x = 2$ and $x = -1$ is true?

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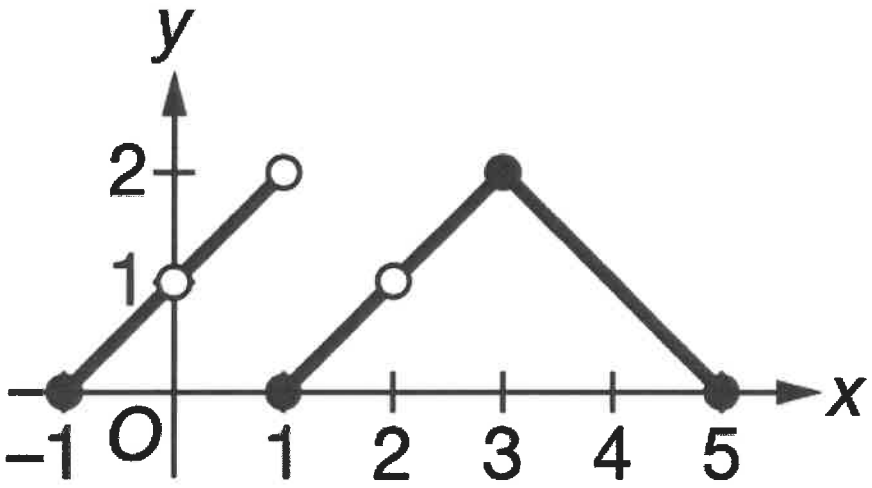
(A) f has a jump discontinuity at $x = 2$, and f is continuous at $x = -1$.

(B) f has a jump discontinuity at $x = 2$, and f has a removable discontinuity at $x = -1$.

(C) f has a discontinuity due to a vertical asymptote at $x = 2$, and f is continuous at $x = -1$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$, and f has a removable discontinuity at $x = -1$.

29.



Graph of f

The graph of the function f is shown above. What are all values of x for which f has a removable discontinuity?

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(A) 0 only

(B) 1 only

(C) 0 and 2 only

(D) 0, 1, and 2

---

30. Let f be the function defined by $f(x) = \frac{x^3-9x}{x^3+x^2-8x-12}$. Which of the following statements about f at $x = -2$ and $x = 3$ is true?

(A) f has a jump discontinuity at $x = -2$, and f is continuous at $x = 3$.

(B) f has a jump discontinuity at $x = -2$, and f has a removable discontinuity at $x = 3$.

(C) f has a discontinuity due to a vertical asymptote at $x = -2$, and f is continuous at $x = 3$.

(D) f has a discontinuity due to a vertical asymptote at $x = -2$, and f has a removable discontinuity at $x = 3$.

---

31. The values $f(x)$ of a function f can be made arbitrarily large by taking x sufficiently close to 2 but not equal to 2. Which of the following statements must be true?

(A) $f(2)$ does not exist.

(B) f is continuous at $x = 2$.

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

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32. The function g is continuous at all x except $x = 2$. If $\lim_{x \to 2} g(x) = \infty$, which of the following statements about g must be true?

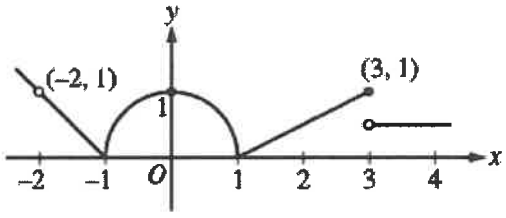
(A) $g(2) = \infty$

(B) The line $x = 2$ is a horizontal asymptote to the graph of g.

(C) The line $x = 2$ is a vertical asymptote to the graph of g.

(D) The line $y = 2$ is a vertical asymptote to the graph of g.

33.



Graph of f

The graph of a function f is shown above. For which of the following values of c does $\lim_{x \to c} f(x) = 1$?

(A) 0 only

(B) 0 and 3 only

(C) −2 and 0 only

(D) −2 and 3 only

(E) −2, 0, and 3

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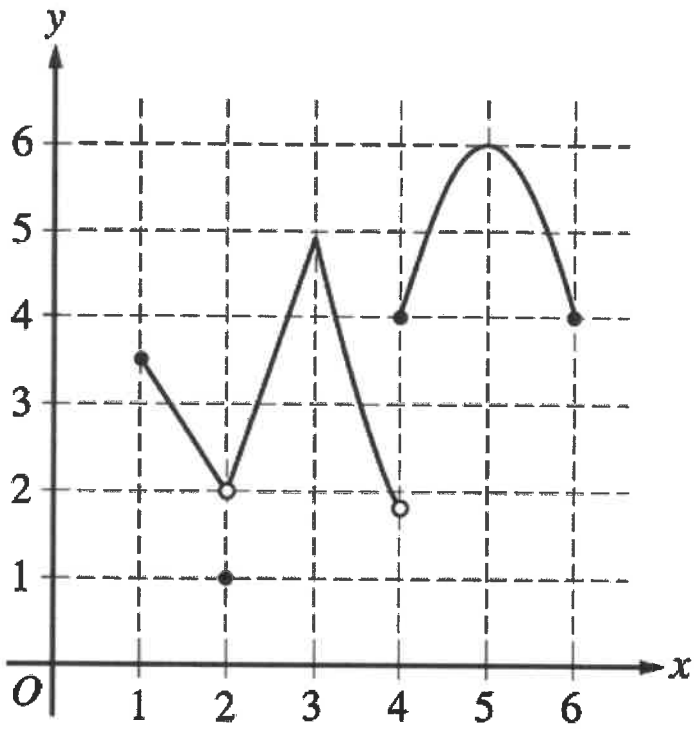
34.



Graph of f

The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

(C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

---

35. If f is a continuous function such that $f(2) = 6$, which of the following statements must be true?

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(A) $\lim_{x\to 1} f(2x) = 3$

(B) $\lim_{x\to 2} f(2x) = 12$

(C) $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 6$

(D) $\lim_{x\to 2} f(x^2) = 36$

(E) $\lim_{x\to 2} (f(x))^2 = 36$

36.

$\lim_{x\to -5} f(x) = 4$	$\lim_{x\to 5} f(x) = 2$	$\lim_{x\to 5} g(x) = 5$

The table above gives selected limits of the functions f and g. What is $\lim_{x\to 5} (f(-x) + 3g(x))$

(A) 19

(B) 17

(C) 13

(D) 9

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37. Which of the following limits are equal to −1 ?

1. $\lim_{x \to 0^-} \frac{|x|}{x}$
2. $\lim_{x \to 3} \frac{x^2 - 7x + 12}{3 - x}$
3. $\lim_{x \to \infty} \frac{1 - x}{1 + x}$

(A) I only

(B) I and III only

(C) II and III only

(D) I, II, and III only

---

38. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

(A) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to 2} f(x) = \infty$

(B) $\lim_{x \to 0^+} f(x) = 2$ and $\lim_{x \to \infty} f(x) = 0$

(C) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 2$

(D) $\lim_{x \to 2} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 2$

---

39. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

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(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

---

40. If $f(x) = \ln x$, then $\lim_{x \to 2} \frac{f(2) - f(x)}{x - 2} =$

(A) $-\ln 2$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\ln 2$

---

41. If $f(x) = \sin x$, then $\lim_{x \to 2\pi} \frac{f(2\pi) - f(x)}{x - 2\pi} =$

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(A) -2π

(B) -1

(C) 1

(D) 2π

---

42. $f(x) = \begin{cases} 5x-3 & \text{for } x < 2 \\ 9 & \text{for } x = 2 \\ 4x+3 & \text{for } x > 2 \end{cases}$

Let f be the piecewise function defined above. The value of $\lim_{x \to 2^+} f(x)$ is

(A) 7

(B) 9

(C) 11

(D) nonexistent

43.

$f(2) = 3$	$\lim_{x \to 2} f(x) = 4$
$g(2) = -6$	$\lim_{x \to 2} g(x) = -6$
$h(2) = -3$	$\lim_{x \to 2} h(x) = 2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim_{x \to 2} (h(x)(5f(x) + g(x)))$?

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(A) -27

(B) -20

(C) 28

(D) 34

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