Midterm Review Key

No Calculator

1) The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$0 = -\sin x - \cos x$

$0 = \sin x + \cos x$

$\frac{-\cos x}{\sin x} = 1$

$-\cot x = 1$

$\cot x = -1$

S A
T C

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

f'': $+$ | $\frac{3\pi}{4}$ | $-$ | $\frac{7\pi}{4}$ | $+$

2) The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

y value

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

$g'(x) = 12x^2 + 6x - 6$

$0 = 12x^2 + 6x - 6$

$0 = 2x^2 + x - 1$

$0 = (2x - 1)(x + 1)$

$x = \frac{1}{2}, -1$

x	$g(x)$
-2	min
-1	$-4+3+6+1=6$
$\frac{1}{2}$	min
1	$4+3-6+1=2$

$[\ +\ \ -1\ \ -\ \ \frac{1}{2}\ \ +\ ]$

min, -1 max, $\frac{1}{2}$ min, max

3) If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$

$g'(x) = x^2 + 3x - 70$

$0 = (x+10)(x-7)$

$x = 7, -10$

$+ \quad - \quad +$

$-10 \quad 7$

always pos

4) If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$ ✓

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

$x = 2, 3, 4$

$+ \quad - \quad - \quad +$ f'

$2 \quad 3 \quad 4$

5) Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

$f'(x) = 3x^2 - 12x$

$f''(x) = 6x - 12$

$0 = 6x - 12$

$x = 2$

$-$ $+$ f''

2

6) The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

$f'(x) = 6x^2 - 6x - 12$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = -1, 2$

$\nearrow$ $\searrow$ $\nearrow$

$+$ $-$ $+$ f'

-1 2

7) For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

$f'(1) = 3$

$y - 4 = 3(x-1)$

$y(1.2) - 4 \approx 3(1.2-1)$

$y(1.2) \approx 0.6 + 4$

≈ 4.6

8) If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

$y = 5x(x^2+1)^{1/2}$

$y' = 5\sqrt{x^2+1} + 5x(1/2)(x^2+1)^{-1/2}(2x)$

$y' = 5\sqrt{x^2+1} + \frac{5x^2}{\sqrt{x^2+1}}$

$y'(3) = 5\sqrt{10} + 45/\sqrt{10}$

9) If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

$2xy + x^2\, dy/dx - 3 = 3y^2\, dy/dx$

$2(-1)(2) + (-1)^2\, dy/dx - 3 = 3(2)^2\, dy/dx$

$-4 + dy/dx - 3 = 12\, dy/dx$

$-7 = 11\, dy/dx$

$-7/11 =$

10) If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

$\frac{1}{\sqrt{1-(5x)^2}}(5)$

$\frac{5}{\sqrt{1-25x^2}}$

11) What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

$$y' = \frac{(x+1)(-e^{-x}) - e^{-x}(1)}{(x+1)^2}$$

$$y'(1) = \frac{2(-e^{-1}) - e^{-1}}{4} = \frac{-2/e - 1/e}{4}$$

$$= \frac{-3}{e} \cdot \frac{1}{4}$$

12) Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

$$f'(x) = 5(2x-1)^4(2)(x+1) + (2x-1)^5$$

$$f'(1) = 5(1)^4(2)(2) + (1)^5$$

$$= 21$$

$$f(1) = 2$$

$$y - 2 = 21(x-1)$$

$$y = 21x - 19$$

13) If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

$$3\cos^2(4x)(-\sin(4x))(4)$$

$$-12\cos^2(4x)\sin(4x)$$

14) If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x+y-2$ (C) $2x+y$ (D) $4x+2y-2$ (E) $y-\frac{y}{x}$

$$\frac{1}{2x+y}(2+dy/dx) = 1$$

$$2 + dy/dx = 2x+y$$

$$dy/dx = 2x+y-2$$

15) The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\frac{3}{2}$. For $6 < c < 9$, there is no value of c such that $f'(c) = -\frac{3}{2}$. Of the following, which must be true?

(A) $\frac{1}{3}\int_6^9 f(x)\,dx = -\frac{3}{2}$

(B) $\int_6^9 f(x)\,dx$ does not exist.

(C) $\frac{f'(6)+f'(9)}{2} = -\frac{3}{2}$

(D) $f'(x) < 0$ for all x in the open interval $(6, 9)$.

(E) f is not differentiable on the open interval $(6, 9)$.

$$\frac{f(9)-f(6)}{9-6} = -\frac{3}{2}$$

$f'(c) \neq -3/2$

fails MVT

is cont $[6,9]$

so not diff $(6,9)$

16) If $\lim_{h\to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2

$f(x) = \sin^{-1} x$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = 2$$

$\sqrt{1-x^2} = 1/2$

$1-x^2 = 1/4$

$-x^2 = -3/4$

$x^2 = 3/4$

$x = \pm\sqrt{3}/2$

17) Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

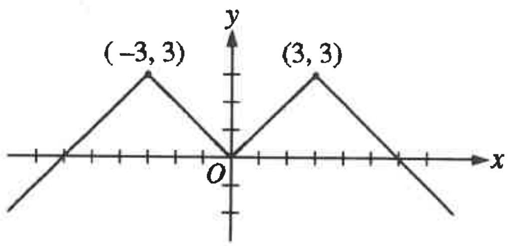
(A) -5 (B) $-\frac{15}{4}$ (C) -1 (D) 6 (E) 17

$f'(x) = 3x^2 - 12x + 8$

$f'(3) = 3(3)^2 - 12(3) + 8$

$= 27 - 36 + 8$

$= -1$



18) : The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x\to 0}(f(x) - f(0)) = 0$ $\lim_{x\to 0} f(x) - \lim_{x\to 0} f(0)$

(B) $\lim_{x\to 0} \frac{f(x) - f(0)}{x} = \cancel{0}$ und f' at 0

(C) $\lim_{x\to 0} \frac{f(x) - f(-x)}{2x} = 0$ even function so $f(-x) = f(x)$

def of a derivative

(D) $\lim_{x\to 2} \frac{f(x) - f(2)}{x - 2} = 1$ f' at 2

(E) $\lim_{x\to 3} \frac{f(x) - f(3)}{x - 3}$ does not exist.

f' at 3

19) Let f be the function given by $f(x)=\dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

$x = 2, -1$

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

20) $\lim\limits_{x \to 0} \dfrac{2x^6+6x^3}{4x^5+3x^3}$ is

$$\frac{x^3(2x^3+6)}{x^3(4x^2+3)} = \frac{2x^3+6}{4x^2+3}$$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$$\lim_{x \to 0} \frac{2x^3+6}{4x^2+3} = \frac{6}{3} = 2$$

* calculator *

21) The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

EVT if cont on $[a,b]$ guaranteed max & min

22) The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

$\frac{dV}{dt} = -3$

$\frac{dr}{dt} = -0.25$

$r = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-3 = 4\pi r^2 (-1/4)$

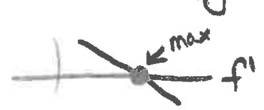
$3/\pi = r^2$

23) Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

- graph
- find zeros
- pos f' to neg f'

1.4646



24) Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six

- graph
- zeros

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

25) The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

~~I.~~ The minimum value of f on $[3, 7]$ is 12. not guaranteed

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$. —— Rolles Thm

↳ corollary of MVT

~~III.~~ $f'(x) > 0$ for $5 < x < 7$. not guaranteed

(A) I only

(B) II only

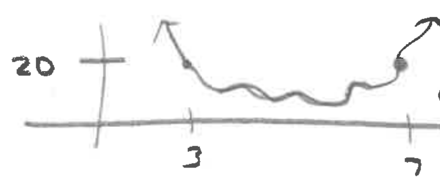
(C) III only

(D) I and III only

(E) I, II, and III

$f(3) = 20$ and $f(7) = 20$

so must have $f'(c) = 0$



can do anything between given pts but cont & diff so has a extrema

26) A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin(\sqrt{t+1})$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

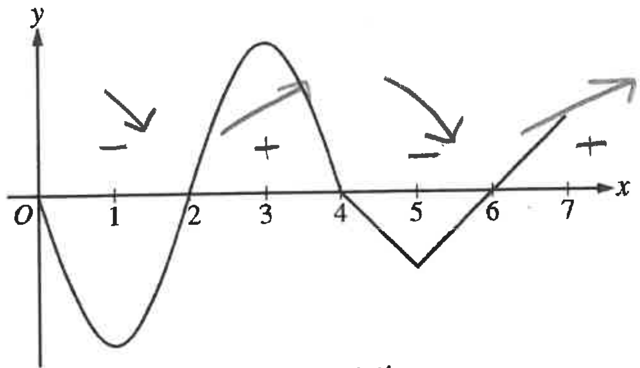
(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

$V = 4/3 \pi r^3$

$81.637 = 4/3 \pi r^3$

$r = 2.691118$

Integral



Graph of f'

27) The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) $[2, 4]$ only

(B) $[3, 5]$ only

(C) $[0, 1]$ and $[3, 5]$

(D) $[2, 4]$ and $[6, 7]$

(E) $[0, 2]$ and $[4, 6]$

28) If f is a continuous function on the closed interval $[a, b]$, which of the following must be true? EVT so has max

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$. f(a) could = f(b) not guaranteed derivative

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$. ✓ max

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. MVT

29) The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

(A) -1.970 and 0

(B) -1.467 and 1.075

(C) -0.475, 0.542, and 1.396

(D) -0.475 and 1.396 only

(E) 0.542 only

- graph
- zeros
- pos to neg

0.5423

30) The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

$g(8) = 3$

$$g'(8) = \frac{1}{f'(g(8))} = \frac{1}{f'(3)} = 1/9$$

31) A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t + 2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

der @ a pt

26.5 29 30.5

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

32) The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

pos slope

– can't be 29 or 30.5
must be ↑ slope b/c

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

$$\frac{f(3.1) - f(2.8)}{3.1 - 2.8} = 29.5$$

brings to C or D

Mid Term Review Extra Problems

Name

1. At $x = 3$, the function given by

$$f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases} \text{ is}$$

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

$\lim_{x \to 3^-} f(x) = 9$ $\lim_{x \to 3^+} f(x) = 9$

$f(3) = 9$ cont ✓

$f'(x) = \begin{cases} 2x, & x < 3 \\ 6, & x \geq 3 \end{cases}$

$\lim_{x \to 3^-} f'(x) = 6$ $\lim_{x \to 3^+} f'(x) = 6$

diff ✓

2. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$

(B) $-\frac{3}{4e}$

(C) $-\frac{1}{4e}$

(D) $\frac{1}{4e}$

(E) $\frac{1}{e}$

$y' = \frac{(x+1)(-e^{-x}) - e^{-x}}{(x+1)^2}$

$y'(1) = \frac{-2e^{-1} - e^{-1}}{4}$

$= \frac{-\frac{2}{e} - \frac{1}{e}}{4}$

$= -\frac{3}{e} \cdot \frac{1}{4} = -\frac{3}{4e}$

3. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

(A) 0.246

(B) 0.430

(C) 0.447

(D) 0.790

$f'(x) = \frac{2x}{x^2+1}$ $\quad g'(x) = 5x^4 + 3x^2$

$f'(2) = \frac{4}{5}$

$\frac{4}{5} = 5x^4 + 3x^2$

calc

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4. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is decreasing only when $b<h$.

(D) A is decreasing only when $b>h$.

(E) A remains constant.

Δ $\quad \frac{db}{dt} = 3 \quad \frac{dh}{dt} = -3$

$A = \frac{1}{2}bh$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt}h + \frac{dh}{dt}b\right)$

$= \frac{3}{2}(h-b)$

A ↑ when $h>b$

A ↓ when $h<b$

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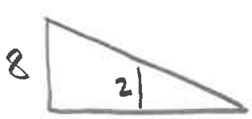
5. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of 4/9 meter per second, at what rate, in meters per second, is the person walking?



$\frac{dx}{dt} \quad \frac{ds}{dt} = \frac{4}{9}$

$\frac{8}{x+s} = \frac{2}{s}$

$8s = 2x + 2s$

$6s = 2x$

$3s = x$

$3\frac{ds}{dt} = \frac{dx}{dt}$

$3(4/9) = \frac{dx}{dt}$

$\frac{4}{3} = \frac{dx}{dt}$

(A) 4/27

(B) 4/9

(C) 3/4

(D) 4/3

(E) 16/9

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6. Sand is deposited into a pile with a circular base. The volume V of the pile is given by $V = \frac{r^3}{3}$, where r is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of 5π feet per hour. When the circumference of the base is 8π feet, what is the rate of change of the volume of the pile, in cubic feet per hour?

(A) $\frac{8}{\pi}$

(B) 16

(C) 40

(D) 40π

(E) 80π

$V = \frac{r^3}{3}$ $\quad \frac{dC}{dt} = 5\pi$ $\quad C = 8\pi$

$C = 2\pi r$

$8\pi = 2\pi r$

$4 = r$

$\frac{dV}{dt} = \frac{3r^2}{3}\frac{dr}{dt}$

$= r^2 \frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$5\pi = 2\pi \frac{dr}{dt}$

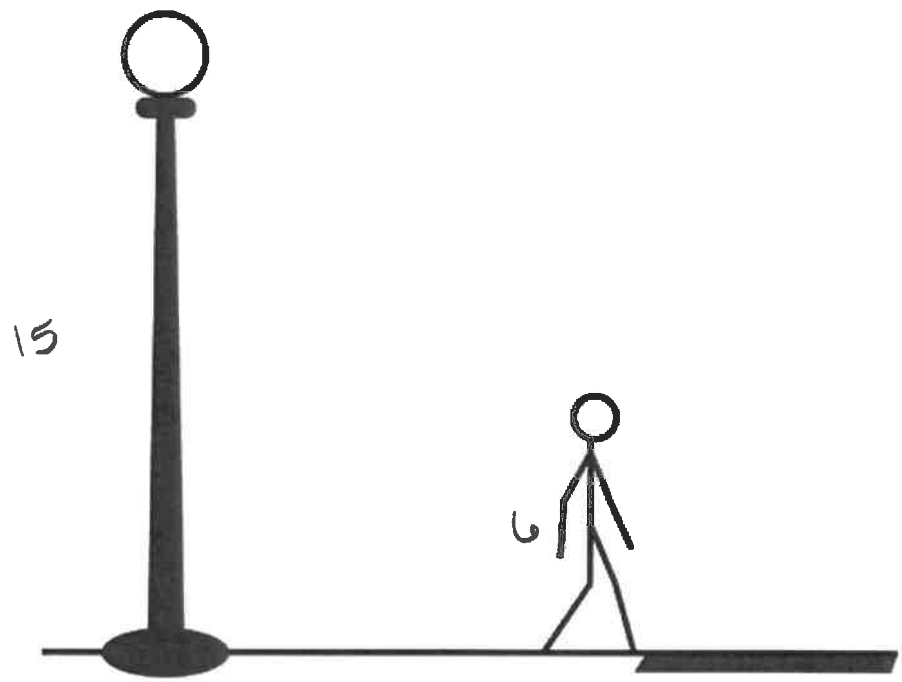
$5/2 = dr/dt$

$= 4^2 (5/2)$

$= 40$

Mid Term Review Extra Problems

7.



A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec

(B) 2.667 ft/sec

(C) 3.75 ft/sec

(D) 6 ft/sec

(E) 10 ft/sec

$\frac{dx}{dt} = 4$

$\frac{ds}{dt} = ?$

$\frac{15}{x+s} = \frac{6}{s}$

$15s = 6x + 6s$

$9s = 6x$

$3s = 2x$

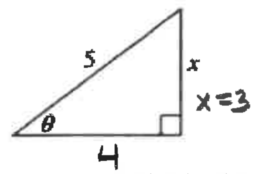
$3\frac{ds}{dt} = 2\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{8}{3}$

Mid Term Review Extra Problems

8.

$\cos\theta = 4/5$



$\frac{d\theta}{dt} = 3$

In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is *x* increasing in units per minute when *x* equals 3 units?

(A) 3

(B) $\frac{15}{4}$

(C) 4

(D) 9

(E) 12

$\sin\theta = \frac{x}{5}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

$\frac{4}{5}(3) = \frac{1}{5}\frac{dx}{dt}$

$12 = \frac{dx}{dt}$

9. The function *f* is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of *f* at $x = 2$?

(A) 0.4

(B) 0.6

(C) 0.7

(D) 1.3

(E) 1.4

$y - 1 = 4(x-2)$

$y = 4x - 7$

$y(1.9) = 4(1.9) - 7$

$= 0.6$

Mid Term Review Extra Problems

10. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of *f* at $x = 1$?

$f'(1) = 2(1)+1$
$= 3$

(A) 0.6

$y - 4 = 3(x-1)$

(B) 3.4

$y = 3x + 1$

(C) 4.2

$y(1.2) = 3(1.2) + 1$

(D) 4.6

$= 4.6$

(E) 4.64

---

11. Let $y = f(x)$ be a differentiable function such that $\frac{dy}{dx} = \frac{x}{y}$ and $f(8) = 2$. What is the approximation of $f\ (8.1)$ using the line tangent to the graph of f at $x = 8$?

$\left.\frac{dy}{dx}\right|_{(8,2)} = \frac{8}{2} = 4$

(A) 0.4

$y - 2 = 4(x-8)$

(B) 2.025

$y = 4x - 30$

(C) 2.4

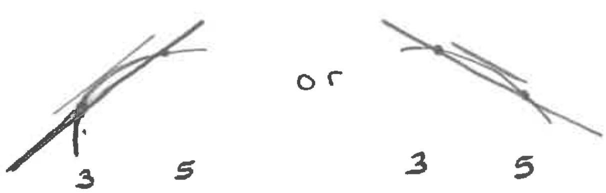
$y(8.1) = 4(8.1) - 30$

(D) 6

$= 2.4$

---

12. Let *f* be a twice-differentiable function such that $f''(x) < 0$ for all *x*. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of *f* at $x = 4$. Which of the following is true?



$T(4.2) > S(4.2)$

$T(4.2) > f(4.2) > S(4.2)$

(A) $f(4.2) < S(4.2) < T(4.2)$

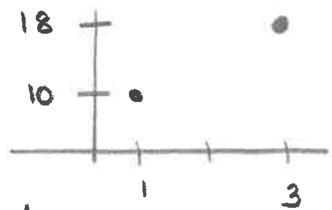
(B) $f(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f(4.2) < T(4.2)$

(D) $T(4.2) < f(4.2) < S(4.2)$

---

13. Let f be a function that is continuous on the closed interval [1,3] with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?



(A) $10 \leq f(2) \leq 18$

f(x) yes not necessarly x=2

(B) f is increasing on the interval [1, 3].

not necessarily entire interval

(C) $f(x) = 17$ has at least one solution in the interval [1, 3].

yes by IVT

(D) $f'(x) = 8$ has at least one solution in the interval (1, 3). MVT check $\frac{f(3) - f(1)}{3-1} = \frac{18-10}{2} = 4$

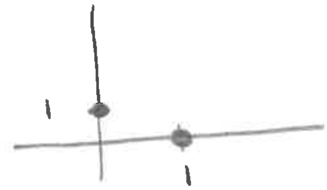
$f'(x) = 4$ yes

(E) $\int_1^3 f(x)dx > 20$

nice looking answer but graph could be decreasing for most of interval & could go below x-axis = neg area

---

14. Let g be a continuous function on the closed interval [0,1]. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?



Not True

T (A) There exists a number h in [0,1] such that $g(h) \geq g(x)$ for all x in [0,1].

EVT cont & closed interval so yes there is a max

T (B) For all a and b in [0,1], if $a=b$, then $g(a)=g(b)$

def of function

↳ saying this does NOT happen

so passes vertical line test

T (C) There exists a number h in [0,1] such that $g(h) = \frac{1}{2}$

IVT

F (D) There exists a number h in [0,1] such that $g(h) = \frac{3}{2}$

IVT

T (E) For all h in the open interval (0,1) , $\lim\limits_{x \to h} g(x) = g(h)$

def of continuity

15.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval [0,2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0,2] if k=

IVT

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 3

16.

x	$f(x)$
–1	–30
0	–2
3	10
5	18

The table above gives selected values for a twice-differentiable function f. Which of the following must be true?

(A) f has no critical points in the interval -1< x < 5.

T (B) f′ (x) = 8 for some value of x in the interval -1<x < 5. MVT $\frac{f(5)-f(-1)}{5-(-1)} = \frac{18+30}{6} = 8$

(C) f ' (x) > 0 for all values of x in the interval -1<x<5
not necessarily true

(D) f " (x) < 0 for for all values of x in the interval -1 < x < 5
not necessarily true

appears so but dont know whats happening in between given points

(E) The graph of f has no points of inflection in the interval -1< x < 5
can't determine

17.

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

The table above shows selected values of a continuous function *f*. For 0 ≤ *x* ≤ 13, what is the fewest possible number of times *f* (*x*) = 4 ?

Mid Term Review Extra Problems

(A) one

(B) two

(C) three

(D) four

---

18. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases} = \begin{cases} x+2 & x \neq 2 \\ 1 & x = 2 \end{cases}$

$(x+2)(x-2)$

Let *f* be the function defined above. Which of the following statements about *f* are true?

✓ I. *f* has a limit at x=2.

~~II.~~ *f* is continuous at x=2.

~~III.~~ *f* is differentiable at x=2.

↳ must be continuous

$\lim_{x \to 2^-} f(x) = 4$ $\lim_{x \to 2^+} f(x) = 4$ $f(2) = 1$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

---

19. $f(x) = \begin{cases} 3x+5 & \text{when } x < -1 \\ -x^2+3 & \text{when } x \geq -1 \end{cases}$

If *f* is the function defined above, then $f'(-1)$ is

$f'(x) = \begin{cases} 3 & x < -1 \\ -2x & x \geq -1 \end{cases}$

check cont. 1st

$\lim_{x \to -1^-} f(x) = 2$ $\lim_{x \to -1^+} f(x) = 2$

$f(-1) = 2$

continuous ✓

$\lim_{x \to -1^-} f'(x) = 3$ $\lim_{x \to -1^+} f'(x) = 2$

not diff.

$f'(-1)$ DNE

Mid Term Review Extra Problems

(A) -3

(B) -2

(C) 2

(D) 3

(E) nonexistent

---

20. $f(x) = \begin{cases} 2x+5 & \text{for } x < -1 \\ -x^2+6 & \text{for } x \geq -1 \end{cases}$

If f is the function defined above, then $f'(-1)$ is

check cont

$\lim\limits_{x \to -1^-} f(x) = 3$ $\lim\limits_{x \to -1^+} f(x) = 5$

not cont.

(A) -2

(B) 2

(C) 3

(D) 5

(E) nonexistent

21.

$$f(x) = \begin{cases} x+b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

$f'(x) = \begin{cases} 1 & x \leq 1 \\ 2ax & x > 1 \end{cases}$

Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

$\lim\limits_{x \to 1^-} f(x) = 1+b$ $\lim\limits_{x \to 1^+} f(x) = a$ $\lim\limits_{x \to 1^-} f'(x) = 1$ $\lim\limits_{x \to 1^+} f'(x) = 2a$

$1+b = a$

$1 = 2a$

$a = 1/2$

$1+b = 1/2$ $b = -1/2$

Mid Term Review Extra Problems

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$

(C) $a = \frac{1}{2}$ and and b is any real number

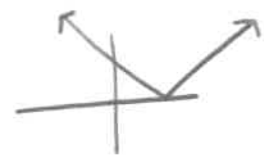
(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b.

---

22. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

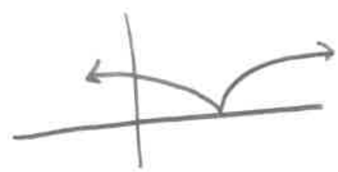
$|x-2|$



T (A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

$\sqrt{|x-2|}$



(C) f is not continuous at $x = 2$.

(D) $\lim\limits_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a is a vertical asymptote of the graph of f.

---

23. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

shifted right 3

corner @ $x=3$

(A) -1

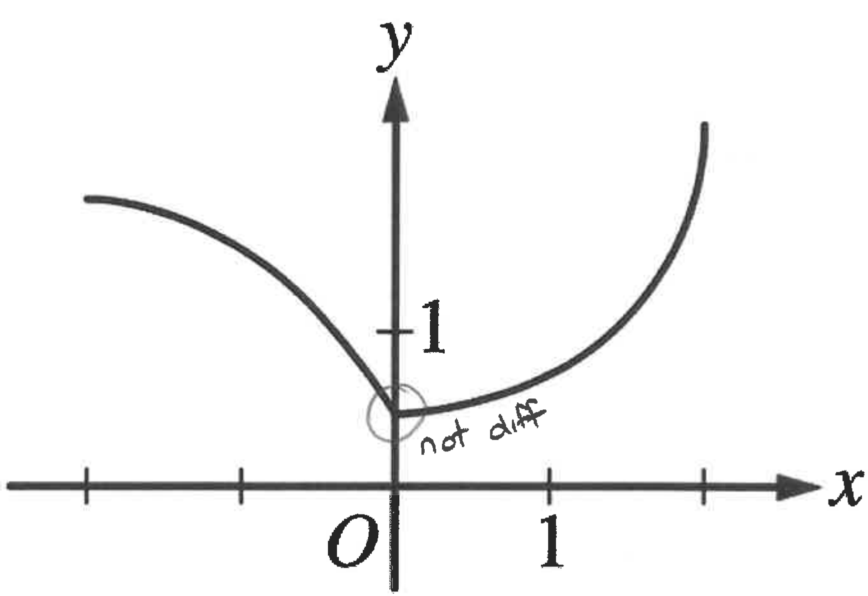
(B) 0

(C) 1

(D) 2

(E) Nonexistent

24.



Graph of f

The function f, whose graph is shown above, is defined on the interval $-2 \leq x \leq 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

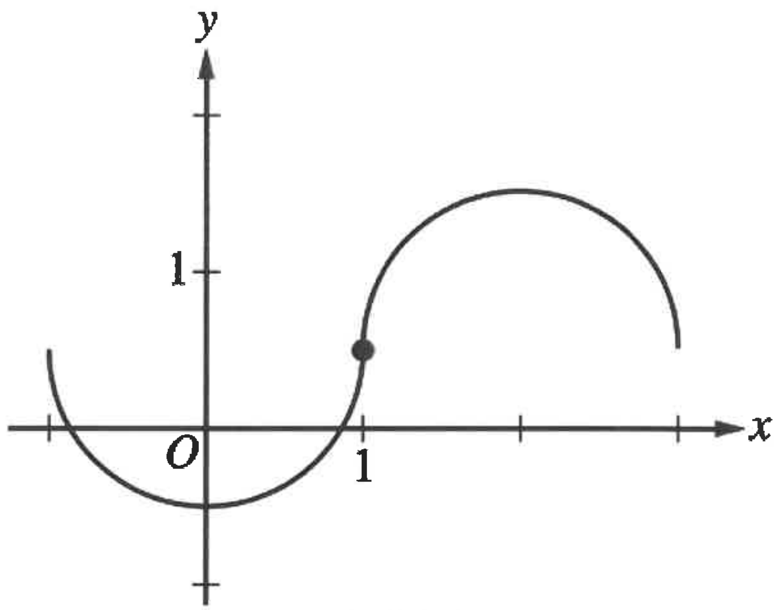
(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

Mid Term Review Extra Problems

25.



Graph of h'

The function h is defined on the closed interval $[-1, 3]$. The graph of h', the derivative of h, is shown above. The graph consists of two semicircles with a common endpoint at $x = 1$. Which of the following statements about h must be true?

1.
$h(-1) = h(3)$ don't know would need initial condition and FTC to find $h(-1)$ and $h(3)$

2.
h is continuous at $x = 1$.

3.
The graph of h has a vertical asymptote at $x = 1$. no b/c h' would be discontinuous @ $x = 1$

(A) None

(B) II only

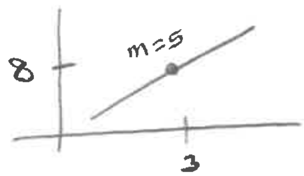
(C) I and II only

(D) I and III only

Mid Term Review Extra Problems

implies cont

26. If f is a differentiable function such thatf(3)=8 and $f'(3) = 5$, which of the following statements could be false?



T (A) $\lim_{x \to 3} f(x) = 8$ cont

T (B) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$ cont

T (C) $\lim_{x \to 3} \frac{f(x)-8}{x-3} = 5$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

T (D) $\lim_{h \to 0} \frac{f(3+h)-8}{h} = 5$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

(E) $\lim_{h \to 3} f'(x) = 5$

$$\lim_{x \to 3} f'(x) = 5$$

---

27. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

$$f'(x) = 2(x-1)\sin x + (x-1)^2 \cos x$$

$$f'(0) = 2(-1)\sin 0 + (-1)^2 \cos 0$$

$$= 0 + 1$$

$$= 1$$

---

28. Let f be the function defined by $f(x) = \frac{x^3-2x^2-3x}{x^3-3x^2+4}$. Which of the following statements about f at $x = 2$ and $x = -1$ is true?

$$f(x) = \frac{x(x^2-2x-3)}{x^3-3x^2+4} = \frac{x(x-3)(x+1)}{x^3-3x^2+4}$$

2	1	-3	0	4
		2	-2	-4
	1	-1	-2	0

$(x-2)(x^2-x-2)$

$(x-2)(x+1)(x-2)$

$$f(2) = \frac{2(-1)(3)}{8-12+4} = \frac{-6}{0} \qquad f(-1) = \frac{-1(-4)(0)}{-1-3+4} = \frac{0}{0}$$

$$f(x) = \frac{x(x-3)(x+1)}{(x-2)^2(x+1)} = \frac{x(x-3)}{(x-2)^2}, \quad x \neq -1$$

-1 removable

2 VA

Mid Term Review Extra Problems

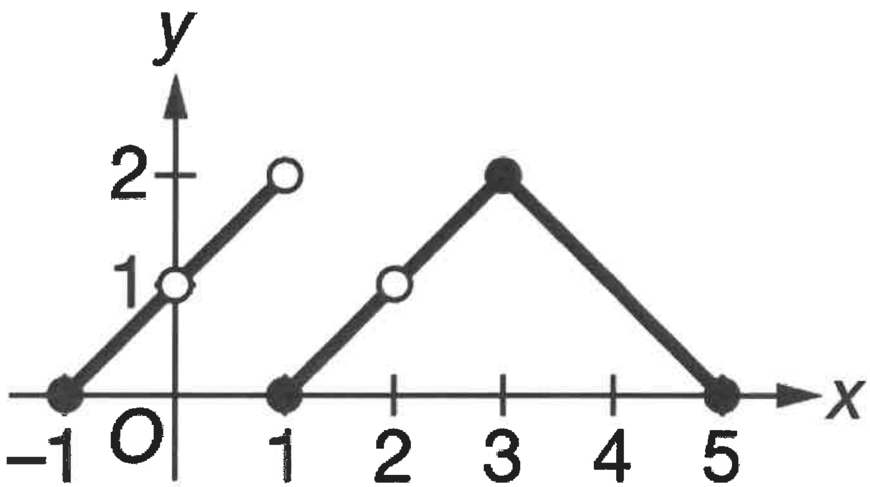
(A) f has a jump discontinuity at $x = 2$, and f is continuous at $x = -1$.

(B) f has a jump discontinuity at $x = 2$, and f has a removable discontinuity at $x = -1$.

(C) f has a discontinuity due to a vertical asymptote at $x = 2$, and f is continuous at $x = -1$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$, and f has a removable discontinuity at $x = -1$.

29.



Graph of f

The graph of the function f is shown above. What are all values of x for which f has a removable discontinuity?

(A) 0 only

(B) 1 only

(C) 0 and 2 only

(D) 0, 1, and 2

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$(x+2)(x^2-x-6)$

$(x+2)(x-3)(x+2)$

Jump disc occurs w/ piecewise

30. Let f be the function defined by $f(x) = \frac{x^3-9x}{(x^3+x^2-8x-12)}$. Which of the following statements about f at $x=-2$ and $x=3$ is true?

$x^2(x+1)$ doesn't factor by grouping

$$\frac{x(x-3)(x+3)}{(x+2)^2(x-3)} = \frac{x(x+3)}{(x+2)^2}, x \neq 3$$

3 removable

−2 VA

(A) f has a jump discontinuity at $x=-2$, and f is continuous at $x=3$.

(B) f has a jump discontinuity at $x=-2$, and f has a removable discontinuity at $x=3$.

(C) f has a discontinuity due to a vertical asymptote at $x=-2$, and f is continuous at $x=3$.

(D) f has a discontinuity due to a vertical asymptote at $x=-2$, and f has a removable discontinuity at $x=3$.

31. The values $f(x)$ of a function f can be made arbitrarily large by taking x sufficiently close to 2 but not equal to 2. Which of the following statements must be true?

$\lim_{x \to 2} f(x) = \infty$

(A) $f(2)$ does not exist.

(B) f is continuous at $x=2$.

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

Mid Term Review Extra Problems

32. The function g is continuous at all x except $x = 2$. If $\lim_{x \to 2} g(x) = \infty$, which of the following statements about g must be true?

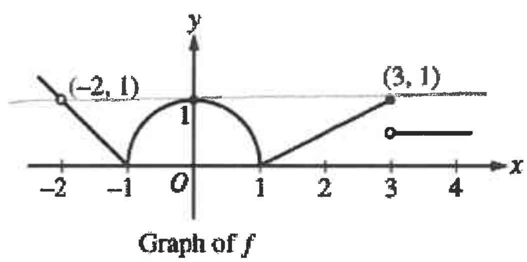
(A) $g(2) = \infty$

(B) The line $x = 2$ is a horizontal asymptote to the graph of g.

(C) The line $x = 2$ is a vertical asymptote to the graph of g.

(D) The line $y = 2$ is a vertical asymptote to the graph of g.

33.



Graph of f

The graph of a function f is shown above. For which of the following values of c does $\lim_{x \to c} f(x) = 1$?

* not one sided

(A) 0 only

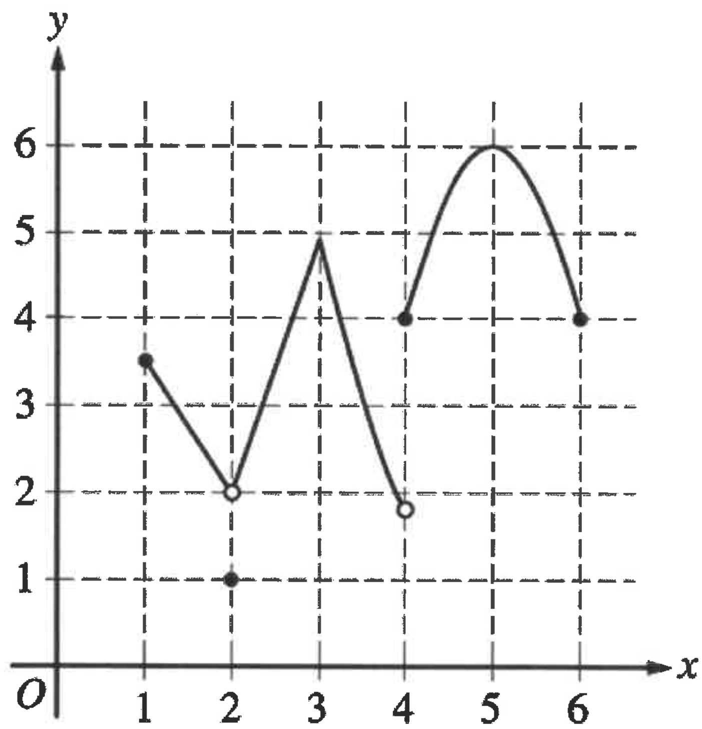
(B) 0 and 3 only

(C) −2 and 0 only

(D) −2 and 3 only

(E) −2, 0, and 3

34.



Graph of f

The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

f (C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

35. If f is a continuous function such that $f(2) = 6$, which of the following statements must be true?

Mid Term Review Extra Problems

(A) $\lim_{x \to 1} f(2x) = 3$

$f(2) = 6$

(B) $\lim_{x \to 2} f(2x) = 12$

(C) $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = 6$

(D) $\lim_{x \to 2} f(x^2) = 36$

property of limits

$\lim_{x \to c} [f(x)]^n = [\lim_{x \to c} f(x)]^n$

(E) $\lim_{x \to 2} (f(x))^2 = 36$

$\lim_{x \to 2} (f(x))^2 = (\lim_{x \to 2} f(x))^2 = 6^2 = 36$

36.

$\lim_{x \to -5} f(x) = 4$	$\lim_{x \to 5} f(x) = 2$	$\lim_{x \to 5} g(x) = 5$

The table above gives selected limits of the functions f and g. What is $\lim_{x \to 5} (f(-x) + 3g(x))$

$\lim_{x \to 5} f(-x) + 3 \lim_{x \to 5} g(x)$

$4 + 3(5)$

19

(A) 19

(B) 17

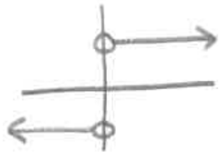
(C) 13

(D) 9

Mid Term Review Extra Problems

37. Which of the following limits are equal to −1 ?

1.
$\lim\limits_{x \to 0^-} \frac{|x|}{x}$



2.
F $\lim\limits_{x \to 3} \frac{x^2-7x+12}{3-x}$

$\frac{(x-3)(x-4)}{-(x-3)} = -x+4$

$\lim\limits_{x \to 3} -x+4 = 1$

3.
$\lim\limits_{x \to \infty} \frac{1-x}{1+x} = -1$

(A) I only

(B) I and III only

(C) II and III only

(D) I, II, and III only

---

38. The continuous function *f* is positive and has domain *x* > *0*. If the asymptotes of the graph of *f* are *x* = *0* and *y* = *2*, which of the following statements must be true?

(A) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to 2} f(x) = \infty$

(B) $\lim\limits_{x \to 0^+} f(x) = 2$ and $\lim\limits_{x \to \infty} f(x) = 0$

(C) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

(D) $\lim\limits_{x \to 2} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

$x = 0$ VA $\lim\limits_{x \to 0} f(x) = \infty$

$y = 2$ HA $\lim\limits_{x \to \infty} f(x) = 2$

---

39. The vertical line $x = 2$ is an asymptote for the graph of the function *f*. Which of the following statements must be false?

Mid Term Review Extra Problems

(A) $\lim_{x \to 2} f(x) = 0$ $x = 2$ VA False

(B) $\lim_{x \to 2} f(x) = -\infty$ $\lim_{x \to 2} f(x) = \pm\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

don't have any info on

---

40. If $f(x) = \ln x$, then $\underbrace{\lim_{x \to 2} \frac{f(2) - f(x)}{x - 2}}_{f'(2)} =$

(A) $-\ln 2$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\ln 2$

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

---

41. If $f(x) = \sin x$, then $\underbrace{\lim_{x \to 2\pi} \frac{f(2\pi) - f(x)}{x - 2\pi}}_{f'(2\pi)} =$

$f'(x) = \cos x$

$f'(2\pi) = \cos 2\pi$

$= 1$

(A) -2π

(B) -1

(C) 1

(D) 2π

---

42. $f(x) = \begin{cases} 5x - 3 & \text{for } x < 2 \\ 9 & \text{for } x = 2 \\ 4x + 3 & \text{for } x > 2 \end{cases}$

Let f be the piecewise function defined above. The value of $\lim\limits_{x \to 2^+} f(x)$ is

$4(2)+3$

11

(A) 7

(B) 9

(C) 11

(D) nonexistent

43.

$f(2) = 3$	$\lim\limits_{x \to 2} f(x) = 4$
$g(2) = -6$	$\lim\limits_{x \to 2} g(x) = -6$
$h(2) = -3$	$\lim\limits_{x \to 2} h(x) = 2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim\limits_{x \to 2} (h(x)(5f(x) + g(x)))$?

$5 \lim\limits_{x \to 2} h(x) f(x) + \lim\limits_{x \to 2} g(x)$

$5 \lim\limits_{x \to 2} h(x) \lim\limits_{x \to 2} f(x) + \lim\limits_{x \to 2} g(x)$

$5(2)(4) + (-6)$

$40 - 6$

34

(A) -27

(B) -20

(C) 28

(D) 34