Polar Equations
BC Calculus

Rectangular Coordinates (x, y)
Polar Coordinates (r, θ)

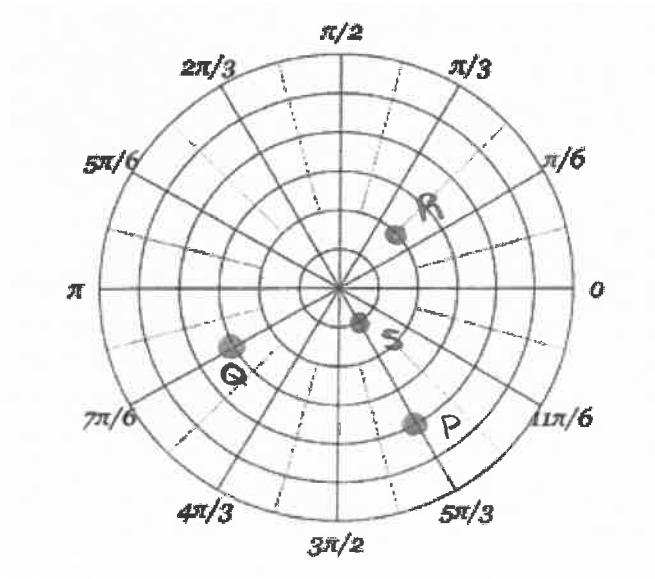
1. Graph the following polar coordinates:

$P\left(4, \frac{5\pi}{3}\right)$

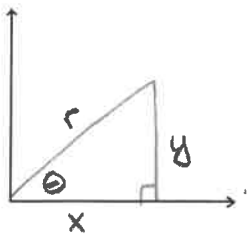
$Q\left(3, \frac{7\pi}{6}\right)$

$R\left(-2, \frac{5\pi}{4}\right)$

$S\left(1, -\frac{\pi}{3}\right)$



2. Fill in the information below:



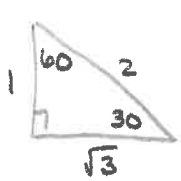
$\sin\theta = \frac{y}{r}$ so $y = r\sin\theta$

$\cos\theta = \frac{x}{r}$ so $x = r\cos\theta$

$\tan\theta = \frac{y}{x}$

$x^2 + y^2 = r^2$ so $r = \sqrt{x^2 + y^2}$

3. Convert $\left(2, \frac{5\pi}{6}\right)$ to rectangular coordinates.



$x = r\cos\theta$
$x = 2\cos(5\pi/6)$
$x = 2\left(-\frac{\sqrt{3}}{2}\right)$
$x = -\sqrt{3}$

$y = r\sin\theta$
$y = 2\sin(5\pi/6)$
$y = 2(1/2)$
$y = 1$

$(-\sqrt{3}, 1)$

4. Convert $(3,-3)$ to polar coordinates.

$r = \sqrt{3^2+(-3)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$x = r\cos\theta$

$3 = 3\sqrt{2}\cos\theta$

$\frac{1}{\sqrt{2}} = \cos\theta$

$\theta = \pi/4, 7\pi/4$

$(3,-3)$ in 4th quad

$\Rightarrow \theta = 7\pi/4$

$(3\sqrt{2}, 7\pi/4)$

5. Convert the following equations to polar form:

a. $y = 4$

$4 = r\sin\theta$

$r = \frac{4}{\sin\theta}$

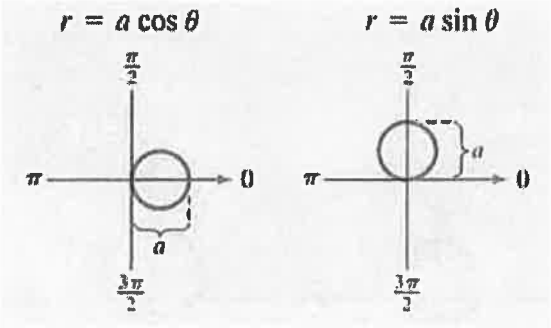
b. $x^2 + y^2 = 25$

$r^2 = 25$

$r = \pm 5$

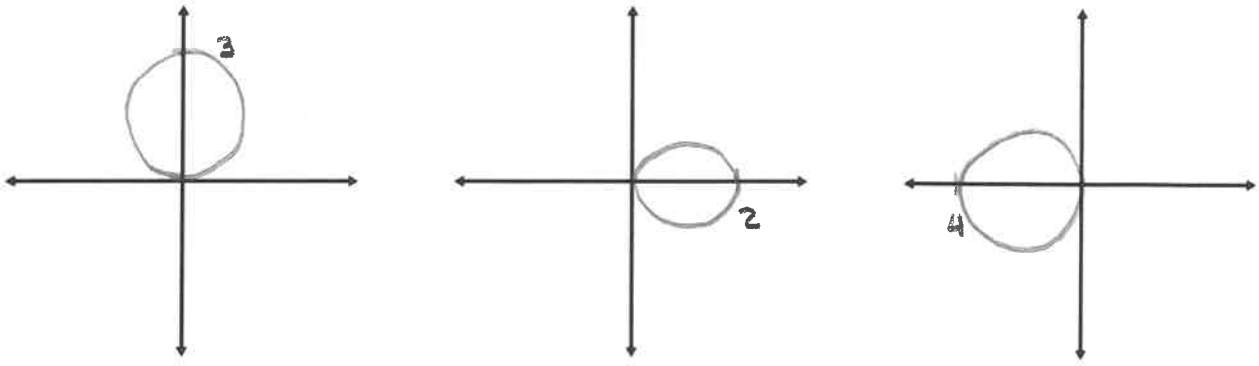
Circles

The graphs of $r = a\cos\theta$ and $r = a\sin\theta$ are circles



6. Sketch the following

a. $r = 3\sin\theta$ b. $r = 2\cos\theta$ c. $r = -4\cos\theta$

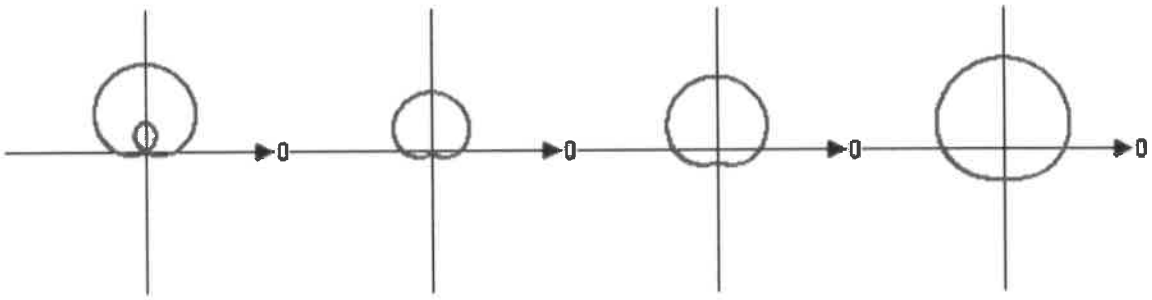


Limacons:

$$r = a \pm b\sin\theta \quad \text{or} \quad r = a \pm b\cos\theta$$

$\frac{a}{b} < 1$ | $\frac{a}{b} = 1$ | $1 < \frac{a}{b} < 2$ | $\frac{a}{b} \geq 2$

$r = a + b\sin\theta$



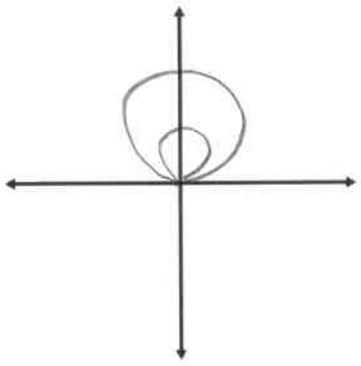
*If it were minus rather than plus then the graphs would be on the other side of the x-axis

*cosine has similar graphs but along the x-axis.

7. Sketch the following graphs:

a. $r = 2 + 3\sin\theta$

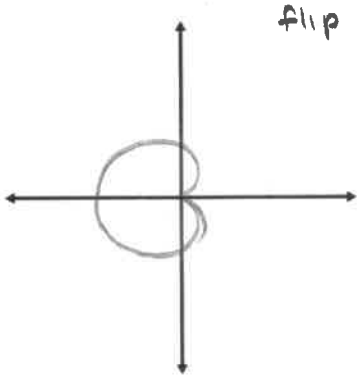
$\frac{2}{3} < 1$ so inner loop



b. $r = 4 - 4\cos\theta$

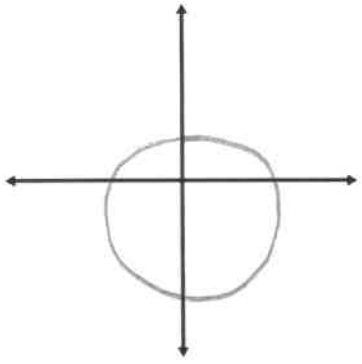
$\frac{4}{4} = 1$

minus so flip



c. $r = 5 - 2\sin\theta$

$\frac{5}{2} \geq 2$



$x = r\cos\theta$

$\frac{dx}{d\theta} = -r\sin\theta + \cos\theta \frac{dr}{d\theta}$

$y = r\sin\theta$

$\frac{dy}{d\theta} = r\cos\theta + \sin\theta \frac{dr}{d\theta}$

9. To find the slope of a tangent line to the polar graph $r = f(\theta)$, we can use the facts that $x = r\cos\theta$ and $y = r\sin\theta$, together with the product rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + \sin\theta \frac{dr}{d\theta}}{-r\sin\theta + \cos\theta \frac{dr}{d\theta}}$$

10. Find $\frac{dy}{dx}$ and the slope of the graph of the polar curve at the given value of θ.

$r = 3 + 2\sin\theta \qquad \theta = \frac{\pi}{6}$

$\frac{dr}{d\theta} = 2\cos\theta$

$$\frac{dy}{dx} = \frac{r\cos\theta + \sin\theta \, dr/d\theta}{-r\sin\theta + \cos\theta \, dr/d\theta}$$

$$= \frac{(3 + 2\sin\theta)\cos\theta + \sin\theta(2\cos\theta)}{-(3 + 2\sin\theta)\sin\theta + \cos\theta(2\cos\theta)}$$

$$= \frac{(3 + 2\sin(\pi/6))\cos(\pi/6) + \sin(\pi/6)(2\cos(\pi/6))}{-(3 + 2\sin(\pi/6))\sin(\pi/6) + \cos(\pi/6)(2\cos\pi/6)}$$

$$= \frac{(3 + 2(1/2))(\sqrt{3}/2) + (1/2)(2(\sqrt{3}/2))}{-(3 + 2(1/2))(1/2) + (\sqrt{3}/2)(2(\sqrt{3}/2))}$$

$$= \frac{2\sqrt{3} + \sqrt{3}/2}{-2 + 3/2}$$

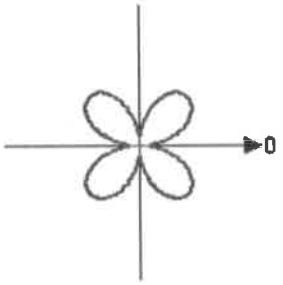
$$= \frac{5\sqrt{3}/2}{-1/2}$$

$$= -5\sqrt{3}$$

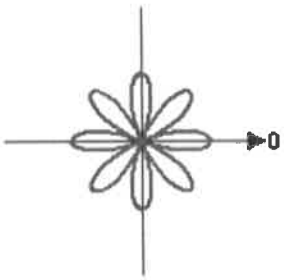
Rose Curves

$r = a \sin n\theta$ or $r = a \cos n\theta$

If n is an even integer, then the rose will have 2n petals.

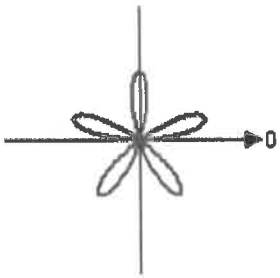


$r = a \sin 2\theta$

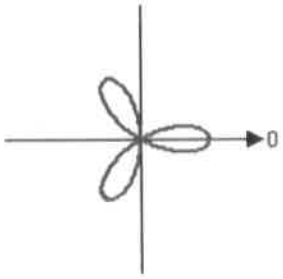


$r = a \cos 4\theta$

If n is an odd integer, then the rose will have n petals.



$r = a \sin 5\theta$



$r = a \cos 3\theta$

*Easiest way to graph is to find where sin or cos is equal to 0

8. Sketch the following graphs:

a. $r = 4 \sin 3\theta$ 3 petals

$\sin 3\theta = 0$

$3\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\theta = 0, \pi/3, 2\pi/3, \pi, 4\pi/3$

b. $r = 2 \cos 2\theta$ 4 petals

$\cos 2\theta = 0$

$2\theta = \pi/2, 3\pi/2$

$\theta = \pi/4, 3\pi/4$

c. $r = 2 \sin 5\theta$ 5 petals