R.5 Rational Expressions
Honors Algebra 2 with Trig

The quotient of two polynomials P and Q, with $Q \neq 0$ is a **rational expression** $\frac{P}{Q}$

Denominator of a fraction cannot be 0, the domain consists of all real numbers except those that make the denominator 0.

1. Find the domain of the rational expression

a. $\frac{2x-4}{x+7}$ $x \neq -7$

"all real #s x such that $x \neq -7$"

$\{x \mid x \neq -7\}$ or $(-\infty, -7) \cup (-7, \infty)$

b. $\frac{9x+12}{(2x+3)(x-5)}$

$\{x \mid x \neq -3/2, 5\}$

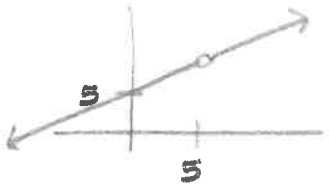
c. $\frac{3}{x^2-5x-6} = \frac{3}{(x-6)(x+1)}$ $x \neq -1, 6$

$\{x \mid x \neq -1, 6\}$

d. $\frac{x^2-25}{x-5} = \frac{(x-5)(x+5)}{x-5} = x+5$

$\{x \mid x \neq 5\}$

5 is not in domain ⇒ removable discontinuity



★ To determine the domain, find values of x that make the **original denominator** equal to 0 and exclude those.

2. Write each rational expression in lowest terms

a. $\frac{36y^2+72y}{9y^2}$ $y \neq 0$

$= \frac{9y(4y+8)}{9y^2}$

$= \frac{4y+8}{y}$, $(-\infty, 0) \cup (0, \infty)$

b. $\frac{-8(4-y)}{(y+2)(y-4)}$

$= \frac{8(y-4)}{(y+2)(y-4)}$

$= \frac{8}{y+2}$, $\{y \mid y \neq -2, 4\}$

c. $\frac{r^2-r-6}{r^2+r-12}$

$= \frac{(r-3)(r+2)}{(r+4)(r-3)}$

$= \frac{r+2}{r+4}$, $\{r \mid r \neq -4, 3\}$

d. $\frac{y^3-27}{y-3}$

$= \frac{(y-3)(y^2+3y+9)}{y-3}$

$= y^2+3y+9$, $\{y \mid y \neq 3\}$

Multiplying and Dividing

For fractions $\frac{a}{b}$ and $\frac{c}{d}$ $(b \neq 0,\ d \neq 0)$, the following hold.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \text{ and } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad (c \neq 0)$$

* factor 1st & cancel before multiply

3. Multiply or divide, as indicated.

a. $\frac{2y^2}{9} \cdot \frac{27}{8y^5}$

$= \frac{2y^2}{9} \cdot \frac{9 \cdot 3}{2y^2 \cdot 4y^3}$

$= \boxed{\frac{3}{4y^3}}$

b. $\frac{8r^3}{6r} \div \frac{5r^2}{9r^3}$

* flip before cancel terms

$= \frac{4r^2}{3} \cdot \frac{9r^3}{5r^2}$

$= \frac{4r^2}{3} \cdot \frac{3 \cdot 3 \cdot r^2}{5r^2} = \boxed{\frac{12r^2}{5}}$

c. $\frac{y^3+y^2}{7} \cdot \frac{49}{y^4+y^3}$

$= \frac{y^2(y+1)}{7} \cdot \frac{7 \cdot 7}{y^2 y(y+1)}$

$= \boxed{\frac{7}{y}}$

d. $\frac{6r-8}{9r^2+6r-24} \div \frac{4r-12}{12r-16}$

$= \frac{2(3r-4)}{(3r-4)(3r+6)} \cdot \frac{4(3r-4)}{4(r-3)}$

$= \boxed{\frac{6r-8}{r-3}}$

e. $\frac{x^2+2x-15}{x^2+11x+30} \cdot \frac{x^2+2x-24}{x^2-8x+15}$

$= \frac{(x+5)(x-3)}{(x+5)(x+6)} \cdot \frac{(x+6)(x-4)}{(x-5)(x-3)}$

$= \boxed{\frac{x-4}{x-5}}$

Addition and Subtraction

For fractions $\frac{a}{b}$ and $\frac{c}{d}$ $(b \neq 0,\ d \neq 0)$, the following hold

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd} \text{ and } \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

* common denominator

4. Add or subtract, as indicated:

a. $\frac{8}{5p} + \frac{3}{4p}$ common denominator $= 20p$

$= \frac{4}{4} \cdot \frac{8}{5p} + \frac{3}{4p} \cdot \frac{5}{5}$

$= \frac{32}{20p} + \frac{15}{20p}$

$= \boxed{\frac{47}{20p}}$

b. $\frac{3}{z} - \frac{x}{z^2}$

$= \frac{z}{z} \cdot \frac{3}{z} - \frac{x}{z^2}$

$= \frac{3z}{z^2} - \frac{x}{z^2} = \boxed{\frac{3z-x}{z^2}}$

c. $\frac{7}{18a^3b^2} - \frac{2}{9ab}$

$= \frac{7}{18a^3b^2} - \frac{2}{9ab} \cdot \frac{2a^2b}{2a^2b}$

$= \frac{7}{18a^3b^2} - \frac{4a^2b}{18a^3b^2} = \boxed{\frac{7-4a^2b}{18a^3b^2}}$

d. $\frac{p}{2p^2-9p-5} - \frac{2p}{6p^2-p-2}$

$= \frac{p}{(2p+1)(p-5)} - \frac{2p}{(3p-2)(2p+1)}$

$= \frac{p(3p-2)}{(2p+1)(p-5)(3p-2)} - \frac{2p(p-5)}{(2p+1)(p-5)(3p-2)}$

$= \frac{3p^2-2p-2p^2+10p}{(2p+1)(p-5)(3p-2)}$

$= \boxed{\frac{p^2+8p}{(2p+1)(p-5)(3p-2)}}$

e. $\frac{3}{(x-1)(x+3)} + \frac{1}{(x+3)(x-4)}$

$= \frac{3(x-4)}{(x-4)(x-1)(x+3)} + \frac{1(x-1)}{(x+3)(x-4)(x-1)}$

$= \boxed{\frac{4x-13}{(x-4)(x-1)(x+3)}}$

5. Simplify each complex fraction

a. $\frac{2-\frac{2}{y}}{2+\frac{2}{y}}$

$= \frac{\frac{y}{y} \cdot \frac{2}{1} - \frac{2}{y}}{\frac{y}{y} \cdot \frac{2}{1} + \frac{2}{y}}$

$= \frac{\frac{2y}{y} - \frac{2}{y}}{\frac{2y}{y} + \frac{2}{y}}$

$= \frac{\frac{2y-2}{y}}{\frac{2y+2}{y}}$

$= \frac{2y-2}{y} \cdot \frac{y}{2y+2}$

$= \frac{2y-2}{2y+2}$

$= \boxed{\frac{y-1}{y+1}}$

b. $\frac{\frac{1}{y+3} - \frac{1}{y}}{\frac{1}{y}}$

$= \frac{\frac{y}{y(y+3)} - \frac{y+3}{y(y+3)}}{\frac{1}{y}}$

$= \frac{\frac{y-y-3}{y(y+3)}}{\frac{1}{y}}$

$= \frac{-3}{y(y+3)} \cdot \frac{y}{1}$

$= \boxed{-3/y+3}$

c. $\frac{\frac{6}{x^2-25} + x}{\frac{1}{x-5}}$

$= \frac{\frac{6}{(x-5)(x+5)} + \frac{x}{1} \cdot \frac{(x-5)(x+5)}{(x-5)(x+5)}}{\frac{1}{x-5}}$

$= \frac{6+x(x-5)(x+5)}{(x-5)(x+5)} \div \frac{1}{x-5}$

$= \frac{6+x(x-5)(x+5)}{(x-5)(x+5)} \cdot \frac{x-5}{1}$

$= \boxed{\frac{x^3-25x+6}{x+5}}$

Homework:

Pg. 53
11, 15, 21, 27, 31, 33, 35, 37, 41, 47, 51, 57, 59, 69, 73, 75, 81

Most Difficult First:

Pg. 53
29, 45, 67, 86