R.5 Day 1 Challenge
Honors Algebra 2 with Trig

1. Simplify the rational expressions:

a. $\frac{(x^{3y}+4x^{2y}-3x^{y}-12)}{x^{y}+4}$

$= \frac{x^{2y}(x^{y}+4) - 3(x^{y}+4)}{x^{y}+4}$

$= \frac{(x^{y}+4)(x^{2y}-3)}{x^{y}+4}$

$= \boxed{x^{2y} - 3}$

b. $\frac{5x^{3a}+5x^{2a}y^{a}}{x^{2a}-y^{2a}}$

$= \frac{5x^{2a}(x^{a}+y^{a})}{(x^{a}-y^{a})(x^{a}+y^{a})}$

$= \boxed{\frac{5x^{2a}}{x^{a}-y^{a}}}$

c. $\frac{(j^3k+j^3-k^4-k^3)}{(j^2k+j^2+jk^2+jk+k^3+k^2)}$

$= \frac{j^3(k+1) - k^3(k+1)}{j^2(k+1)+jk(k+1)+k^2(k+1)}$

$= \frac{(k+1)(j^3-k^3)}{(k+1)(j^2+jk+k^2)}$

$= \frac{j^3-k^3}{j^2+jk+k^2}$

$= \frac{(j-k)(j^2+jk+k^2)}{j^2+jk+k^2}$

$= \boxed{j-k}$

2. Simplify the following:

a. $\frac{4r^2+2rs+s^2}{2r+s} \cdot \frac{4r^2-s^2}{8r^3-s^3}$

$= \frac{4r^2+2rs+s^2}{2r+s} \cdot \frac{(2r+s)(2r-s)}{(2r-s)(4r^2+2rs+s^2)}$

$= \boxed{1}$

b. $\frac{2a^2-2a-12}{a^2-49} \cdot \frac{4a^2-1}{2a^2+5a+2} \cdot \frac{2a^2-13a-7}{2a^2-7a+3}$

$= \frac{(2a+4)(a-3)}{(a-7)(a+7)} \cdot \frac{(2a-1)(2a+1)}{(2a+1)(a+2)} \cdot \frac{(2a+1)(a-7)}{(2a-1)(a-3)}$

$= \frac{(2a+4)(a+1)}{(a+7)(a+2)}$

$= \frac{2(a+2)(2a+1)}{(a+7)(a+2)}$

$= \boxed{\frac{4a+2}{a+7}}$

c. $\frac{p^3-4p^2+p-4}{2p^3-8p^2+p-4} \cdot \frac{2p^3+2p^2+p+1}{p^4-p^3+p^2-p}$ on separate pg

3. Solve the following equation for y in terms of x, and write the resulting expression for y in simplest form. Identify any excluded values of x.

$$x^2(y+1) = 9(y+1) + 4x + 12$$

$x^2(y+1) - 9(y+1) = 4x + 12$

$(y+1)(x^2-9) = 4x+12$

$y+1 = \frac{4x+12}{x^2-9}$

$y = \frac{4x+12}{x^2-9} - 1$

$y = \frac{4(x+3)}{(x-3)(x+3)} - 1$

$y = \frac{4}{x-3} - 1$

extra challenge:

$y = \frac{4}{x-3} - \frac{x-3}{x-3}$

$y = \frac{4-x+3}{x-3}$

$\boxed{y = \frac{7-x}{x-3}}$

$x \neq \pm 3$

4. (NCTM March 2016 #11)
Find all the common factors for the following three expressions:

$(2x-1)(3x+4)$

$2x^2(2x-1) + 4(2x-1)$

$(2x^2+4)(2x-1)$

$2(x^2+2)(2x-1)$

$6x^2 + 5x - 4$

$(4x^3 - 2x^2) + (8x - 4)$

$4x^2 + 6x - 4$

$2(2x^2 + 3x - 2)$

$2(2x-1)(x+2)$

$\boxed{2x-1}$

5. (NCTM May 2015 #16)
If

$$x + \frac{1}{x} = \sqrt{22}$$

find the exact value of

$$x^2 + \frac{1}{x^2}$$

$\left(x + \frac{1}{x}\right)^2 = \left(\sqrt{22}\right)^2$

$x^2 + 2 + \frac{1}{x^2} = 22$

$x^2 + \frac{1}{x^2} = \boxed{20}$

$\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$x^2 + \frac{x}{x} + \frac{x}{x} + \frac{1}{x^2}$

$x^2 + 2 + \frac{1}{x^2}$

c) $\dfrac{(p^3-4p^2)+(p-4)}{(2p^3-8p^2)+(p-4)} \cdot \dfrac{(2p^3+2p^2)+(p+1)}{(p^4-p^3)+(p^2-p)}$

$\dfrac{p^2(p-4)+(p-4)}{2p^2(p-4)+(p-4)} \cdot \dfrac{2p^2(p+1)+(p+1)}{p^3(p-1)+p(p-1)}$

$\dfrac{\cancel{(p-4)}(p^2+1)}{\cancel{(p-4)}\cancel{(2p^2+1)}} \dfrac{(p+1)\cancel{(2p^2+1)}}{(p-1)(p^3+p)}$

$\dfrac{(p+1)\cancel{(p^2+1)}}{(p-1)\ p\ \cancel{(p^2+1)}}$

$\dfrac{p+1}{p^2-p}$ or $\dfrac{p+1}{p(p-1)}$

6. (NCTM April 2014 #6)Find the domain of the following function:

$$f(x) = \frac{2x-5-\frac{3}{x}}{5x-13-\frac{6}{x}}$$

$$f(x) = \frac{2x - 5 - \frac{3}{x}}{5x - 13 - \frac{6}{x}} \cdot \frac{x}{x}$$

$$= \frac{2x^2 - 5x - 3}{5x^2 - 13x - 6}$$

$$= \frac{(2x + 1)(x - 3)}{(5x - 3)(x - 2)}$$

$$x \neq \frac{3}{5}, 2$$