$\boxed{\dfrac{1}{x+3} \text{ and } \dfrac{1}{x+2}}$

1. (NCTM May 2018#23)
   Find two fractions with different denominators such that their sum is

$$\frac{2x-1}{x^2-x-6}$$

$$\frac{2x-1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$2x-1 = A(x+2) + B(x-3)$$

2 options

① let $x=-2$ let $x=3$

$-5 = -5B$ $5 = 5A$

$1 = B$ $1 = A$

② $2x-1 = Ax + 2A + Bx - 3B$

$2x-1 = Ax + Bx + 2A - 3B$

$2x-1 = (A+B)x + 2A - 3B$

$2 = A+B$ $-1 = 2A - 3B$

$6 = 3A + 3B$

$-1 = 2A - 3B$

$5 = 5A$

$A = 1$

$B = 1$

2. (NCTM May 2018 #10)
   A student incorrectly adds 2 unit fractions together by putting the sum of the 2 numerators over the sum of the 2 denominators. If the result coincidentally is the correct answer, what is the ratio of the two denominators?

   Definition → Unit Fraction: a rational number in which the numerator is 1 and the denominator is a positive integer.

   **Your answer will have a negative under a square root, its okay, just leave it as it is.

$\frac{1}{x} + \frac{1}{y}$ incorrect → $\frac{2}{x+y}$

correct → $\frac{y}{xy} + \frac{x}{xy} = \frac{y+x}{xy}$

coincidentally be equal

$$\frac{2}{x+y} = \frac{x+y}{xy}$$

$$2xy = (x+y)^2$$

$$2xy = x^2 + 2xy + y^2$$

$$0 = x^2 + y^2$$

$$x^2 = -y^2$$

$$x = \pm\sqrt{-y^2} = \pm y\sqrt{-1}$$

$x = \pm y\sqrt{-1}$

$$\boxed{\frac{x}{y} = \pm\sqrt{-1}}$$

3. (NCTM March 2018 #3)
Find all real solutions to the equation below, given that $x \neq 1, 2$.

$$\frac{1}{x-1} + \frac{2}{x-2} = A_2$$

$\star A_2 = 1$

$$\frac{x-2+2(x-1)}{(x-1)(x-2)} = 1$$

$$\frac{3x-4}{(x-1)(x-2)} = 1$$

$$3x-4 = (x-1)(x-2)$$

$$3x-4 = x^2-3x+2$$

$$0 = x^2-6x+6$$

$$x = \frac{6 \pm \sqrt{36-4(1)(6)}}{2}$$

$$= \frac{6 \pm \sqrt{12}}{2}$$

$$= \frac{6 \pm 2\sqrt{3}}{2}$$

$$= \boxed{3 \pm \sqrt{3}}$$

4. In the seventeenth century, Lord Crouncker wrote down a most peculiar mathematical equation:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \ddots}}}}$$

This is an example of an infinite continued fraction. Simplify the infinite continued fraction:

$$n + \cfrac{1}{n + \cfrac{1}{n + \cfrac{1}{n}}}. = n + \cfrac{1}{n + \cfrac{1}{\frac{n^2+1}{n}}}$$

$$n + \frac{1}{n} = \frac{n^2+1}{n}$$

$$= n + \cfrac{1}{n + \cfrac{n}{n^2+1}}$$

$$n + \frac{n}{n^2+1} = \frac{n(n^2+1)+n}{n^2+1}$$

$$= \frac{n^3+n+n}{n^2+1}$$

$$= n + \cfrac{1}{\frac{n^3+2n}{n^2+1}}$$

$$= \frac{n^3+2n}{n^2+1}$$

$$= n + \frac{n^2+1}{n^3+2n}$$

continues →

$$= n + \frac{n^2+1}{n^3+2n}$$

$$= \frac{n(n^3+2n)+n^2+1}{n^3+2n}$$

$$= \frac{n^4+2n^2+n^2+1}{n^3+2n}$$

$$= \boxed{\frac{n^4+3n^2+1}{n^3+2n}}$$