index, radical sign, radicand: $\sqrt[n]{b}$

Radical Notation
Let a be a real number, n be a positive integer, and $a^{1/n}$ be a real number

$$\sqrt[n]{a} = a^{1/n}$$

Let a be a real number, m be an integer, n be a positive integer, and $\sqrt[n]{a}$ be a real number.

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

216
2 108
54 2
2 27
3 9
3 3
$2^3\ 3^3$

343
7 49
7 7
$7^3 = 343$

1. Write each root using exponents and evaluate
   a. $\sqrt[3]{216} = 216^{1/3} = (2^3 \cdot 3^3)^{1/3} = 2 \cdot 3 = \boxed{6}$
   b. $\sqrt[4]{-256} = (-256)^{1/4}$ No Solution
   c. $-\sqrt[3]{-343} = -(-343)^{1/3} = -\sqrt[3]{-7^3} = -(-7) = \boxed{7}$

2. If the expression is in exponential form, write it in radical form. If it is in radical form, write it in exponential form. Assume all variables represent positive real numbers.
   a. $p^{5/4} = \sqrt[4]{p}^{\,5}$
   b. $(5r+3t)^{4/7} = \sqrt[7]{5r+3t}^{\,4}$
   c. $\sqrt[4]{z^5} = z^{5/4}$

3. Simplify the following:
   a. $\sqrt[6]{x^6} = (x^6)^{1/6} = \boxed{x}$
   b. $\sqrt[4]{81p^{12}q^4}$
   $= \sqrt[4]{81}\ \sqrt[4]{p^{12}}\ \sqrt[4]{q^4}$
   $= (3^4)^{1/4}\ (p^{12})^{1/4}\ (q^4)^{1/4}$
   $= 3^1\ p^{12/4}\ q^1$
   $= 3p^3q$
   OR
   $= \sqrt[4]{3^4\ p^4\ p^4\ p^4\ q^4}$
   $= 3pppq$
   $= \boxed{3p^3q}$

250
2 125
$2 \cdot 5^3$

4. Simplify each expression. Assume all variables represent positive real numbers.
   a. $\sqrt[3]{250} = \sqrt[3]{2 \cdot 5^3} = \boxed{5\sqrt[3]{2}}$
   b. $\sqrt{7} \cdot \sqrt{5xt} = \boxed{\sqrt{35xt}}$
   c. $-\sqrt{\frac{16}{49}} = -\frac{\sqrt{16}}{\sqrt{49}} = \boxed{-\frac{4}{7}}$
   d. $\sqrt[4]{\frac{m}{n^4}} = \frac{\sqrt[4]{m}}{\sqrt[4]{n^4}} = \boxed{\frac{\sqrt[4]{m}}{n}}$

e. $\sqrt{24m^6n^5}$

$= \sqrt{4 \cdot 6 m^6 n^4 n}$

$= \boxed{2m^3n^2\sqrt{6n}}$

f. $\sqrt[3]{25(-3)^4(5)^3}$

$= 5\sqrt[3]{25(-3)^3 3}$

$= 5(-3)\sqrt[3]{25 \cdot 3}$

$= \boxed{-15\sqrt[3]{75}}$

g. $\sqrt{\frac{g^3h^5}{r^3}} = \frac{\sqrt{g^2 g h^4 h}}{\sqrt{r^2 r}}$

$= \frac{gh^2\sqrt{gh}}{r\sqrt{r}} = \boxed{\frac{gh^2}{r}\sqrt{\frac{gh}{r}}}$

not simplified comp.

h. $\sqrt[5]{\sqrt[3]{9}}$

$= (9^{1/3})^{1/5}$

$= 9^{1/15} = \boxed{\sqrt[15]{9}}$

★ to add/sub must have same term in $\sqrt{\ }$

5. Perform the indicated operations. Assume all variables represent positive real numbers.

a. $4\sqrt{18k} - \sqrt{72k} + \sqrt{50k}$

$= 4\sqrt{9 \cdot 2k} - \sqrt{36 \cdot 2k} + \sqrt{25 \cdot 2k}$

$= 12\sqrt{2k} - 6\sqrt{2k} + 5\sqrt{2k}$

$= \boxed{11\sqrt{2k}}$

b. $\sqrt[4]{256x^5y^6} + \sqrt[4]{625x^9y^2}$

$= \sqrt[4]{4^4x^4xy^4y^2} + \sqrt[4]{5^4x^8xy^2}$

$= 4xy\sqrt[4]{xy^2} + 5x^2\sqrt[4]{xy^2}$

$= \boxed{(4xy + 5x^2)\sqrt[4]{xy^2}}$

★ c. $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$

$= \sqrt{25} - \sqrt{10} + \sqrt{10} - \sqrt{4}$

$= 5 - 2 = \boxed{3}$

d. $(\sqrt{5} + \sqrt{10})^2$

$= \sqrt{25} + \sqrt{50} + \sqrt{50} + \sqrt{100}$

$= 5 + 2\sqrt{50} + 10$

$= 15 + 2\sqrt{25 \cdot 2}$

$= 15 + 2(5)\sqrt{2}$

$= \boxed{15 + 10\sqrt{2}}$

e. $\frac{\sqrt[3]{8m^2n^3} \cdot \sqrt[3]{2m^2}}{\sqrt[3]{32m^4n^3}}$

$= \left(\frac{8m^2n^3 2m^2}{32m^4n^3}\right)^{1/3}$

$= \left(\frac{16m^4n^3}{32m^4n^3}\right)^{1/3}$

$= \left(\frac{1}{2}\right)^{1/3}$

$= \frac{1}{\sqrt[3]{2}}$ ★ rationalize

$= \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$= \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \boxed{\frac{\sqrt[3]{4}}{2}}$

No $\sqrt{\ }$ in den → multiply by conjugate

6. Rationalize each denominator. Assume all variables represent nonnegative numbers and that no denominators are 0.

a. $\frac{\sqrt{7}}{\sqrt{3}-\sqrt{7}}$

$= \frac{\sqrt{7}}{\sqrt{3}-\sqrt{7}} \cdot \frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

$= \frac{\sqrt{21} + 7}{3 - 7}$

$= \frac{\sqrt{21} + 7}{-4} = \boxed{-\frac{\sqrt{21} + 7}{4}}$

b. $\frac{9-r}{3-\sqrt{r}} \cdot \frac{3+\sqrt{r}}{3+\sqrt{r}}$

$= \frac{27 + 9\sqrt{r} - 3r + r\sqrt{r}}{9 - r}$

$= \boxed{\frac{27 - 3r + (9+r)\sqrt{r}}{9-r}}$

c. $\frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$= \frac{1+\sqrt{2}}{1-2}$

$\boxed{= -1 - \sqrt{2}}$

Homework:

Pg. 75
11, 17, 47, 57, 59, 63, 65, 77, 85, 93, 101, 102, 106

Most Difficult First:

Pg. 75
22, 70, 98, 108

Rationalize the Denom.

1) $\frac{4}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{2}$

$\boxed{= 2\sqrt{2}}$

2) $\frac{6}{\sqrt[4]{x^2}}$

$= \frac{6}{\sqrt[4]{x^2}} \cdot \frac{\sqrt[4]{x^2}}{\sqrt[4]{x^2}}$

$= \frac{6\sqrt[4]{x^2}}{\sqrt[4]{x^4}}$

$= \frac{6\sqrt[4]{x^2}}{x}$