1. Suppose a circular skin infection spreads with its radius increasing at a rate of 0.3 inches per day. Find the rate at which the area of the infection is increasing at the time when the radius reaches 1.8 inches.

$r = 1.8$ in

$\frac{dr}{dt} = 0.3$ in/day

$\frac{dA}{dr} = ?$

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dr} = 2\pi (1.8)(0.3)$

≈ 3.393 in²/day

2. The side of a square is increasing at a rate of 6 cm/sec. Find the rate at which the area of the square is changing at the time when the side length is 20 cm.

$\frac{ds}{dt} = 6$ cm/sec

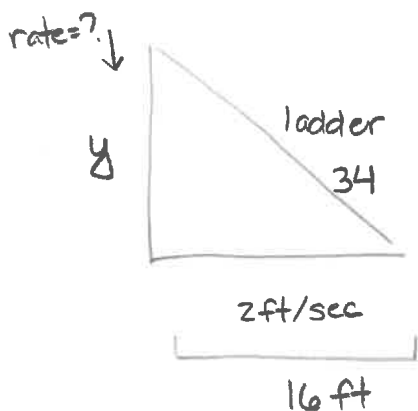$s = 20$ cm

$\frac{dA}{dt} = ?$

$A = s^2$

$\frac{dA}{dt} = 2s \frac{ds}{dt}$

$= 2(20)(6)$

$= 240$ cm²/sec

3. A 34 foot ladder was placed against the side of a house. The ladder began sliding away from the base of the house at a constant rate of 2 feet per second. At what rate was the top of the ladder moving toward the ground when the base of the ladder was 16 feet away from the house?

rate=?

y

ladder

34

2ft/sec

16 ft

$16^2 + y^2 = 34^2$

$256 + y^2 = 1156$

$y = 30$

$x^2 + y^2 = 34^2$

$2x \dfrac{dx}{dt} + 2y \dfrac{dy}{dt} = 0$

$2(16) 2 + 2(30) \dfrac{dy}{dt} = 0$

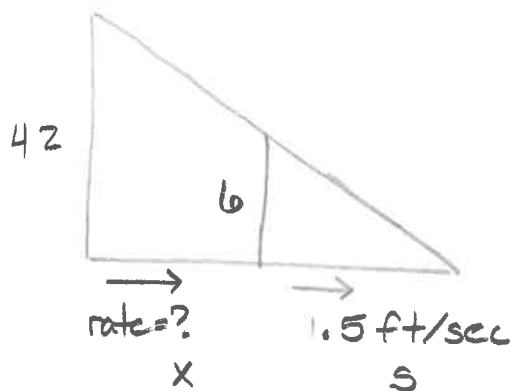$\dfrac{dy}{dt} = \dfrac{-64}{60} = \dfrac{-16}{15} \text{ ft/sec}$

$\approx 1.067 \text{ ft/sec}$

4. Pete is 6 feet tall. As he walks away from a 42 foot tall lamp post, the length of his shadow is increasing at a rate of 1.5 feet per second. At what rate is he walking away from the post?

42

6

rate=?

1.5ft/sec

x

s

$\dfrac{42}{6} = \dfrac{x+s}{s}$ *similar triangles

$42s = 6x + 6s$

$42 \dfrac{ds}{dt} = 6 \dfrac{dx}{dt} + 6 \dfrac{ds}{dt}$

$42(1.5) = 6 \dfrac{dx}{dt} + 6(1.5)$

$9 \text{ ft/sec} = \dfrac{dx}{dt}$

5. A snowman with a spherical head is uniformly melting at a rate of 3 $in.^3/hr$. At what rate is the radius shrinking when the volume of the head is 50 in^3 ?

$\frac{dV}{dt} = -3 \ in^3/hr$

$V = 50 \ in^3$

$\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$50 = \frac{4}{3}\pi r^3$

$r \approx 2.2854 \ in$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-3 = 4\pi r^2 (dr/dt)$

$-3 = 4\pi (2.2854)^2 (dr/dt)$

$-0.0457 \ in/hr \approx dr/dt$

* store full decimal in calc

6. The surface area of a cube in increasing at a rate of 18 in^2/sec. How fast is the volume of the cube increasing at the instant when the surface area is 54 in^2. ?

$\frac{dSA}{dt} = 18 \ in^2/sec$

$SA = 54 \ in^2$

$\frac{dV}{dt} = ?$

$SA = 6s^2$

$54 = 6s^2$

$9 = s^2$

$3 = s$

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3(3)^2 (1/2)$

$= 27/2 \ in^3/sec$

$SA = 6s^2$

$\frac{dSA}{dt} = 12s \frac{ds}{dt}$

$18 = 12(3)(ds/dt)$

$\frac{18}{36} = \frac{ds}{dt}$

$1/2 = ds/dt$

7. A cylindrical container is being filled with water at a rate of 18 cubic inches per minute. The container has a radius of 3 inches and a height of 20 inches. At what rate is the height changing at the moment when it is filled to the five inch level?
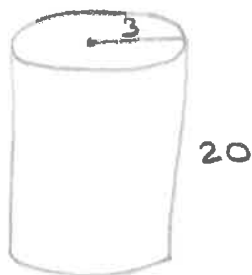
$V = \pi r^2 h$

$V = \pi (3)^2 h$

$V = 9\pi h$

$\dfrac{dV}{dt} = 18 \text{ in}^3/\text{min}$

$\dfrac{dV}{dt} = 9\pi \, dh/dt$

$r = 3 \text{ in}$

$h = 20 \text{ in}$

$\dfrac{dh}{dt} = ?$

$h = 5$

$18 = 9\pi \, dh/dt$

$0.63666 \text{ in}/\text{min} = dh/dt$

8. A conical water tank is leaking from the bottom (its point). The tank is 100 feet tall and has a diameter of 50 feet. The radius of the circle at the water's surface is changing at a rate of two feet per day. At what rate is the volume of the water changing at the time when the radius at the surface of the water is 15 feet?

$V = \dfrac{1}{3}\pi r^2 h$

25

15

50

100

60 = h

$V = \dfrac{1}{3}\pi r^2 (4r)$

$= \dfrac{4}{3}\pi r^3$

$r = 25$

$h = 100$

$\dfrac{dr}{dt} = 2 \text{ ft}/\text{day}$

$\dfrac{dV}{dt} = ?$

$r = 15$

$\dfrac{100}{h} = \dfrac{25}{15}$

$1500 = 25h$

$60 = h$

$h = 4r$
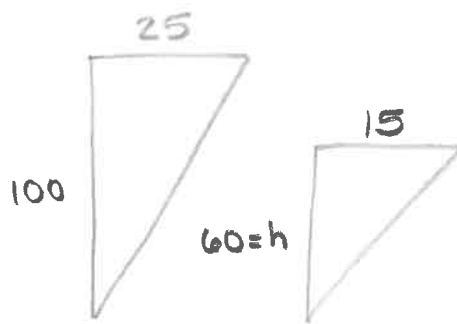
$\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$

$= 4\pi (15)^2 (2)$

$= 1800\pi \text{ ft}^3/\text{day}$

$\approx 5654.867 \text{ ft}^3/\text{day}$

9.   1993 AB 34

The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?
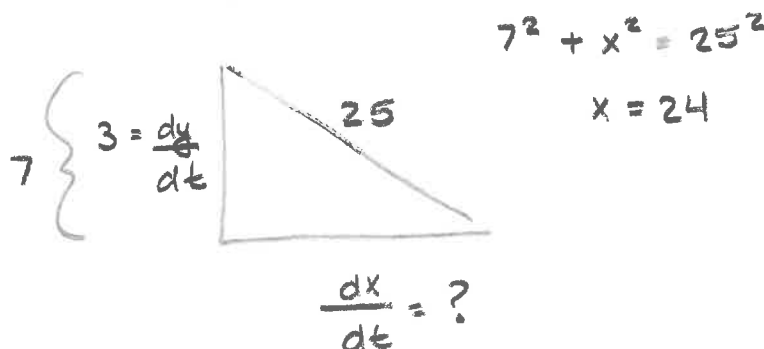
(A) $-\dfrac{7}{8}$ feet per minute

(B) $-\dfrac{7}{24}$ feet per minute

(C) $\dfrac{7}{24}$ feet per minute

(D) $\dfrac{7}{8}$ feet per minute

(E) $\dfrac{21}{25}$ feet per minute

$$7^2 + x^2 = 25^2$$
$$x = 24$$

$$3 = \frac{dy}{dt}$$

$$\frac{dx}{dt} = ?$$

$$x^2 + y^2 = 25$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(24)\frac{dx}{dt} + 2(7)(3) = 0$$

$$\frac{dx}{dt} = \frac{42}{48} = \frac{7}{8}$$

10.   1988 BC 37

A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\dfrac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?
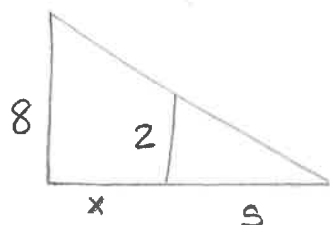
(A) $\dfrac{4}{27}$ (B) $\dfrac{4}{9}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{4}{3}$ (E) $\dfrac{16}{9}$

$$\frac{8}{2} = \frac{x+s}{s}$$

$$8s = 2x + 2s$$

$$6s = 2x$$

$$6\frac{ds}{dt} = 2\frac{dx}{dt}$$

$$6(4/9) = 2\frac{dx}{dt}$$

$$4/3 = dx/dt$$

$$\frac{dx}{dt} = ? \qquad \frac{ds}{dt} = 4/9 \; m/sec$$

11.

A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$