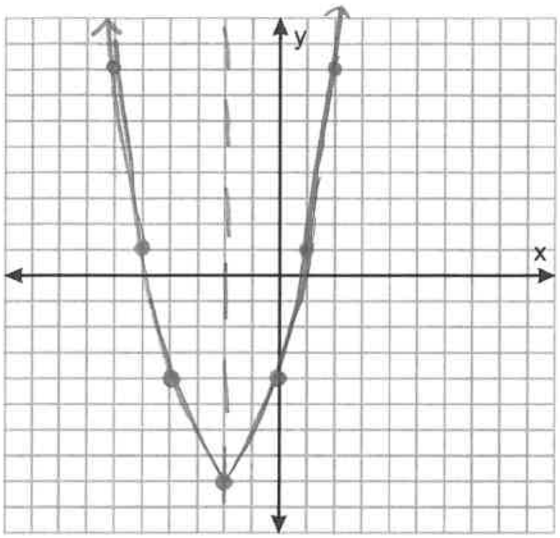
Review 3.1-3.3

1. Graph the following:

   a. $f(x) = x^2 + 4x - 4$

   $x = \frac{-4}{2(1)} = -2$ $f(-2) = 4 - 8 - 4 = -8$

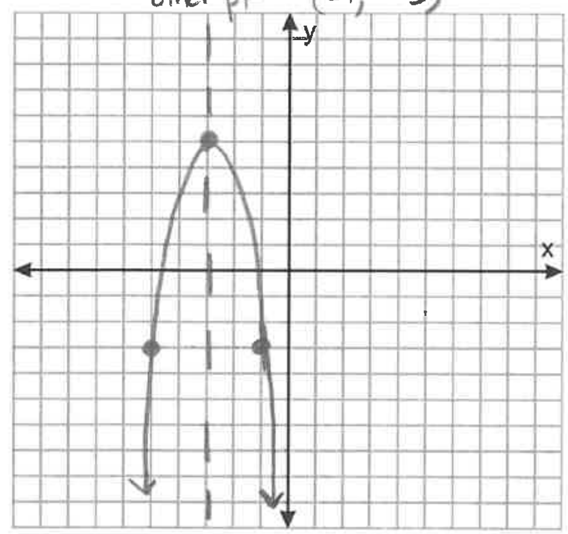
   points
   $(-2, -8)$
   $(0, -4)$



   b. $g(x) = -2(x+3)^2 + 5$

   upside down
   ↳ reflects over x-axis

   vertex $(-3, 5)$
   y-int $(0, -13)$
   other pt $(-1, -3)$



2. Graph the following by writing the equation in vertex form. Identify the vertex and axis of symmetry.

   a. $f(x) = x^2 + 2x - 5$

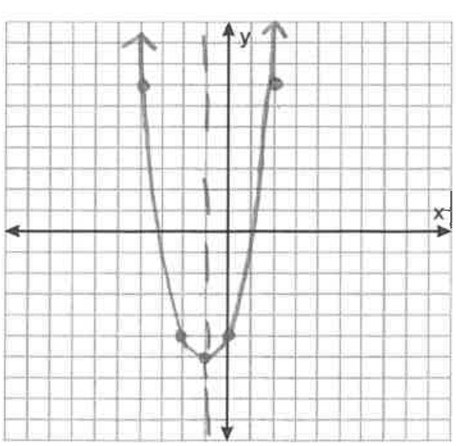
   $f(x) = (x^2 + 2x \quad) - 5$
   $= (x^2 + 2x + 1) - 5 - 1$
   $= (x+1)^2 - 6$
   vertex $(-1, -6)$
   AoS $x = -1$

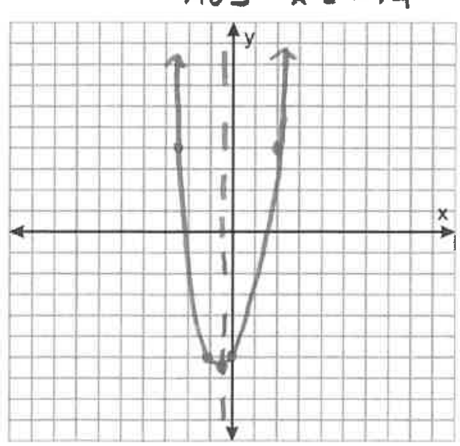
   b. $g(x) = 2x^2 + x - 6$

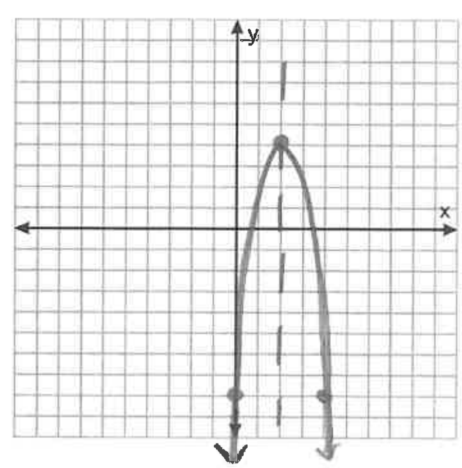
   $g(x) = 2(x^2 + \frac{1}{2}x \quad) - 6$
   $= 2(x^2 + \frac{1}{2}x + \frac{1}{16}) - 6 - \frac{1}{8}$
   $= 2(x + 1/4)^2 - 49/8$
   vertex $(-1/4, -49/8)$
   AoS $x = -1/4$

   c. $h(x) = -3x^2 + 12x - 8$

   $h(x) = -3(x^2 - 4x \quad) - 8$
   $= -3(x^2 - 4x + 4) - 8 + 12$
   $= -3(x-2)^2 + 4$
   vertex $(2, 4)$
   AoS $x = 2$







3. Divide the following using long division $\frac{12x^3-11x^2+9x+18}{4x+3}$

$$\begin{array}{r} 3x^2 - 5x + 6 \\ 4x+3 \overline{)\,12x^3 - 11x^2 + 9x + 18} \\ -(12x^3 + 9x^2) \\ \hline -20x^2 + 9x \\ -(-20x^2 - 15x) \\ \hline 24x + 18 \\ -\ 24x + 18 \\ \hline 0 \end{array}$$

$$\frac{12x^3 - 11x^2 + 9x + 18}{4x+3} = \boxed{3x^2 - 5x + 6}$$

4. Perform the following division using synthetic division:

a. $\frac{4x^3-15x^2+11x-10}{x-3}$

$$\begin{array}{r|rrrr} 3 & 4 & -15 & 11 & -10 \\ & & 12 & -9 & 6 \\ \hline & 4 & -3 & 2 & -4 \end{array}$$

$$\boxed{4x^2 - 3x + 2 - \frac{4}{x-3}}$$

b. $(2x^3 + 4x^2 - 5) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 2 & 4 & 0 & -5 \\ & & -6 & 6 & -18 \\ \hline & 2 & -2 & 6 & -23 \end{array}$$

$$\boxed{2x^2 - 2x + 6 - \frac{23}{x+3}}$$

5. Let $f(x) =- 3x^4 + 15x^2 - 50x + 25$. Use the remainder theorem to find $f(4)$.

$$\begin{array}{r|rrrrr} 4 & -3 & 0 & 15 & -50 & 25 \\ & & -12 & -48 & -132 & -728 \\ \hline & -3 & -12 & -33 & -182 & -703 \end{array}$$

$$\boxed{f(4) = -703}$$

6. Determine whether the given number k is a zero of $f(x)$.

a. $f(x) = 2x^3 - 3x^2 - 18;\ k = 2$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 0 & -18 \\ & & 4 & 2 & 4 \\ \hline & 2 & 1 & 2 & -14 \end{array}$$

$\boxed{\text{No not a zero}}$

b. $f(x) = x^4 - 4x^3 - 14x^2 + 36x + 45;\ k = -3$

3	1	-4	-14	36	45
		3	-3	-51	-45
	1	-1	-17	-15	0

$\boxed{\text{yes}}$ k=-3 is a zero

c. $f(x) = x^4 - x^3 + 6x^2 + 14x - 20;\ k = 1 + 3i$

$3i + 9i^2$

$(-3+3i)(1+3i)$

$-3 + 3i - 9i + 9i^2$

$-12 - 6i$

$1+3i$	1	-1	6	14	-20
		$1+3i$	$-9+3i$	$-12-6i$	20
	1	$3i$	$-3+3i$	$2-6i$	0

$(1+3i)(2-6i)$

$2 + 6i - 6i - 18i^2$

20

$\boxed{\text{yes}}$ 1+3i is a zero

7. Determine whether $x + 4$ is a factor of $f(x) = x^5 + 6x^4 + 11x^3 + 12x^2 + 5x - 20$.

-4	1	6	11	12	5	-20
		-4	-8	-12	0	-20
	1	2	3	0	5	-40

$\boxed{\text{no}}$ x+4 not a factor

8. Factor $f(x) = 6x^3 - 37x^2 + 32x + 15$ into linear factors given that 5 is a zero.

5	6	-37	32	15
		30	-35	-15
	6	-7	-3	0

$f(x) = (x-5)(6x^2 - 7x - 3)$

$= (x-5)(3x+1)(2x-3)$

zeros: $x = 5, -\frac{1}{3}, \frac{3}{2}$

9. Consider the polynomial function.

$$f(x) = 8x^4 - 26x^3 - 27x^2 + 11x + 4$$

a. List all possible rational zeros.

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 4, \pm 8} = \pm 1, \pm \tfrac{1}{2}, \pm \tfrac{1}{4}, \pm \tfrac{1}{8}, \pm 2, \pm 4$$

* tried f(1) so -26 & -27 too large so tried x = -1 b/c x^3 vs x^2 would reduce large coefficients

b. Find all the complex zeros and factor $f(x)$ into linear factors.

-1	8	-26	-27	11	4
		-8	34	-7	-4
	8	-34	7	4	0 ✓

4	8	-34	7	4
		32	-8	-4
	8	-2	-1	0 ✓

$f(x) = (x+1)(8x^3 - 34x^2 + 7x + 4)$

can't factor by grouping → synthetic div again

* need large pos # to reduce -34 w/ x^3 term so try 4

$f(x) = (x+1)(x-4)(8x^2 - 2x - 1)$

$= (x+1)(x-4)(4x+1)(2x-1)$

zeros: $x = -1, 4, -\frac{1}{4}, \frac{1}{2}$

10. Consider the polynomial function.

$$f(x) = 15x^4 + x^3 - 52x^2 + 20x + 16$$

a. List all possible rational zeros.

$$\frac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}{\pm 1, \pm 3, \pm 5, \pm 15} = \pm 1, \pm \tfrac{1}{3}, \pm \tfrac{1}{5}, \pm \tfrac{1}{15}, \pm 2, \pm \tfrac{2}{3}, \pm \tfrac{2}{5}, \pm \tfrac{2}{15}, \pm 4, \pm \tfrac{4}{3}, \pm \tfrac{4}{5}, \pm \tfrac{4}{15}, \pm 8, \pm \tfrac{8}{3}, \pm \tfrac{8}{5}, \pm \tfrac{8}{15}, \pm 16, \pm \tfrac{16}{3}, \pm \tfrac{16}{5}, \pm \tfrac{16}{15}$$

b. Find all the complex zeros and factor $f(x)$ into linear factors.

1	15	1	-52	20	16
		15	16	-36	-16
	15	16	-36	-16	0 ✓

-2	15	16	-36	-16
		-30	-28	16
	15	-14	-8	0

$f(x) = (x-1)(15x^3 + 16x^2 - 36x - 16)$

can't factor by grouping → syn. div. again

$= (x-1)(x+2)(15x^2 - 14x - 8)$

$= (x-1)(x+2)(3x-4)(5x+2)$ zeros: $x = 1, -2, \frac{4}{3}, -\frac{2}{5}$

11. Find a polynomial function $f(x)$ of degree 3 with real coefficients that satisfies the given conditions.
    a. Zeros of $-3, -2,$ and 5; $f(-1) = 6$

$f(x) = a(x+3)(x+2)(x-5)$

$6 = a(-1+3)(-1+2)(-1-5)$

$6 = -12a$

$-\frac{1}{2} = a$

$f(x) = -\frac{1}{2}(x+3)(x+2)(x-5)$

$= -\frac{1}{2}(x^2+5x+6)(x-5)$

$= -\frac{1}{2}(x^3 + \cancel{5x^2} + 6x - \cancel{5x^2} - 25x - 30)$

$= \boxed{-\frac{1}{2}x^3 + \frac{19}{2}x + 15}$

    b. 4 is a zero of multiplicity 3; $f(2) = -24$

$f(x) = a(x-4)^3$

$-24 = a(2-4)^3$

$-24 = -8a$

$3 = a$

$f(x) = 3(x-4)^3$

$= 3(x^2-8x+16)(x-4)$

$= 3(x^3 - 8x^2 + 16x - 4x^2 + 32x - 64)$

$= \boxed{3x^3 - 36x^2 + 144x - 192}$

12. Find a polynomial function $f(x)$ of least degree having only real coefficients and zeros -4 and $3-i$. $3+i$

$f(x) = (x+4)(x-(3-i))(x-(3+i))$

$= (x+4)[(x-3)^2 - i^2]$

$= (x+4)(x^2 - 6x + 9 + 1)$

$= (x+4)(x^2-6x+10)$

$= x^3 - 6x^2 + 10x + 4x^2 - 24x + 40$

$\boxed{= x^3 - 2x^2 - 14x + 40}$

13. Determine the different possibilities for the numbers of positive, negative, and nonreal complex zeros of $f(x) = -2x^4 + 3x^3 - 5x^2 + 4x - 1$.

4 changes

$f(-x) = -2x^4 - 3x^3 - 5x^2 - 4x - 1$

no changes

*4 zeros

4 pos	
2 pos	2 imaginary
0 pos	4 imaginary