1. State the end behavior of each of the following:

a. $f(x) = 5x^3 - 2x + 9$

$x \to \infty$ $f(x) \to$ ∞

$x \to -\infty$ $f(x) \to$ $-\infty$

b. $f(x) = -x^{100} + 4x^6 - 3x + 8$

$x \to \infty$ $f(x) \to$ $-\infty$

$x \to -\infty$ $f(x) \to$ $-\infty$

c. $f(x) = -x^9 + x^8 - 2x^5 + 10$

$x \to \infty$ $f(x) \to$ $-\infty$

$x \to -\infty$ $f(x) \to$ ∞

d. $f(x) = x^8 - 4x^2 - 7x^{15}$

$x \to \infty$ $f(x) \to$ $-\infty$

$x \to -\infty$ $f(x) \to$ ∞

e. $f(x) = (x+3)(2x-5)(x+6) = 2x^3 + \ldots$

$x \to \infty$ $f(x) \to$ ∞

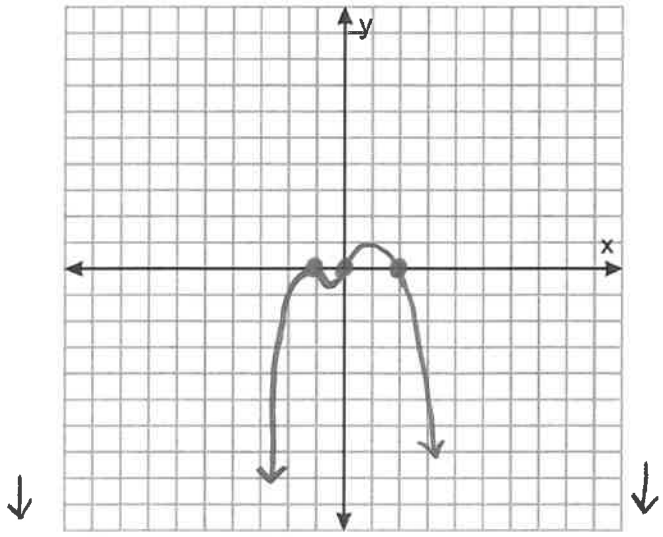
$x \to -\infty$ $f(x) \to$ $-\infty$

2. Graph the following functions and state the key characteristics below:

a. $f(x) = -x^4 + x^3 + 2x^2$

$= -x^2(x^2 - x - 2)$

$= -x^2(x-2)(x+1)$

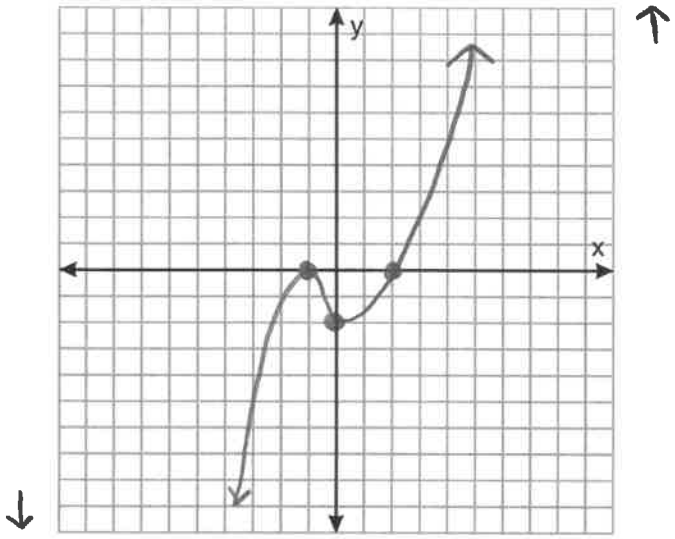


End Behavior: $x \to \infty$ $f(x) \to -\infty$; $x \to -\infty$ $f(x) \to -\infty$

X-intercepts: $(0,0)$ $(2,0)$ $(-1,0)$

Y-intercepts: $(0,0)$

b. $f(x) = (x-2)(x+1)^2$



$x \to \infty \quad f(x) \to \infty$

End Behavior: $x \to -\infty \quad f(x) \to -\infty$

X-intercepts: $(2,0) \quad (-1,0)$

Y-intercepts: $(0,-2)$

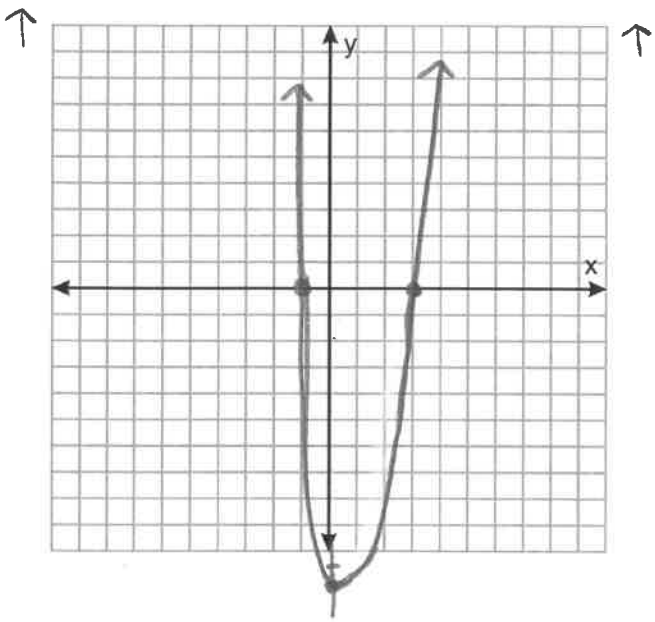
c. $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$ $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

-1	1	-2	1	-8	-12
		-1	3	-4	12
	1	-3	4	-12	0

$f(x) = (x+1)(x^3 - 3x^2 + 4x - 12)$

$= (x+1)\left[x^2(x-3) + 4(x-3)\right]$

$= (x+1)(x^2+4)(x-3)$



End Behavior: $x \to \infty \quad f(x) \to \infty$
$x \to -\infty \quad f(x) \to \infty$

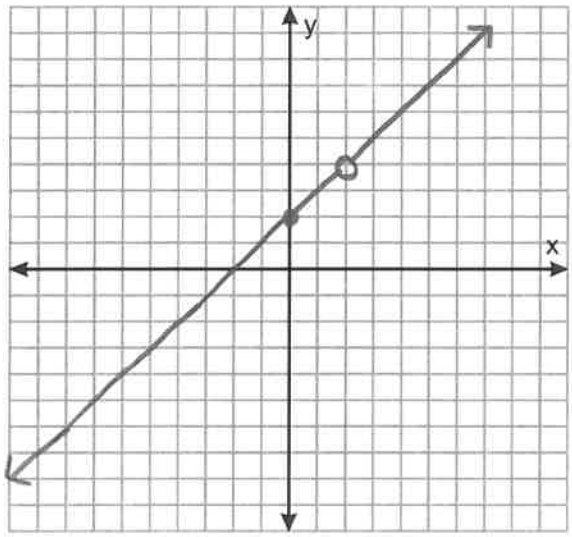
X-intercepts: $(-1,0) \quad (3,0)$

Y-intercepts: $(0,-12)$

Nonreal x-int $(2i,0) \quad (-2i,0)$

3. Graph the following functions and state the key characteristics below:

a. $f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2,\ x \neq 2$



Hole(s): (2, 4)

Vertical Asymptote(s) EQUATION(S): none

Horizontal Asymptote(s) EQUATION(S): none

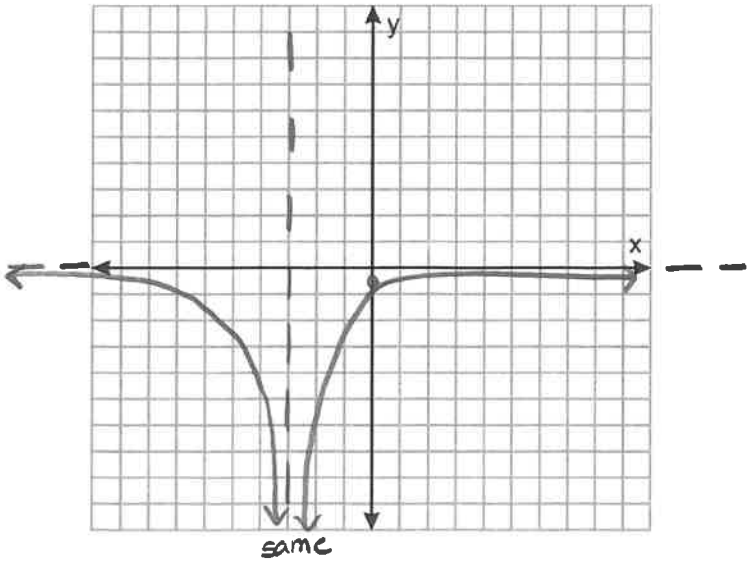
Slant Asymptote EQUATION(S): none

X-intercepts: (-2, 0)

Y-intercepts: (0, 2)

Domain: $(-\infty, 2) \cup (2, \infty)$

b. $f(x) = \frac{-2}{(x+3)^2}$



Hole(s): none

Vertical Asymptote(s) EQUATION(S): $x = -3$

Horizontal Asymptote(s) EQUATION(S): $y = 0$

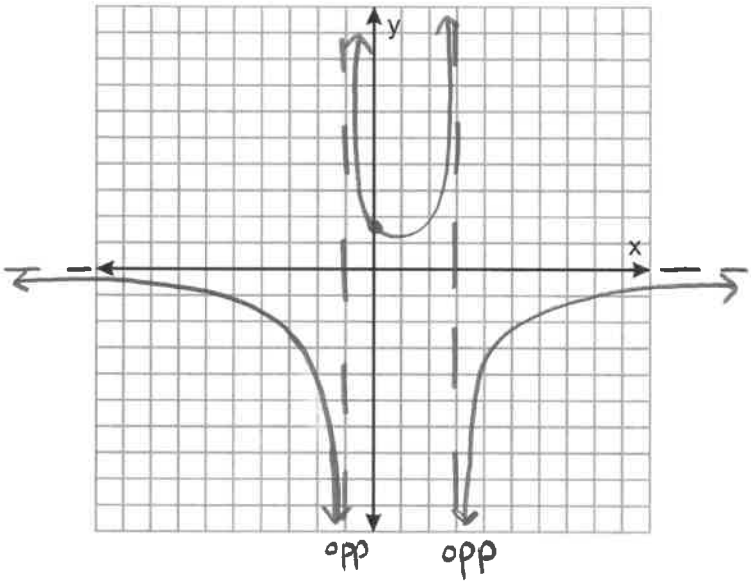
Slant Asymptote EQUATION(S): none

X-intercepts: none

Y-intercepts: $(0, -2/9)$

Domain: $(-\infty, -3) \cup (-3, \infty)$

c. $f(x) = \frac{-5}{x^2-2x-3} = \frac{-5}{(x-3)(x+1)}$



Hole(s): none

Vertical Asymptote(s) EQUATION(S): $x=3$ $x=-1$

Horizontal Asymptote(s) EQUATION(S): $y=0$

Slant Asymptote EQUATION(S): none

X-intercepts: none

Y-intercepts: $(0, 5/3)$

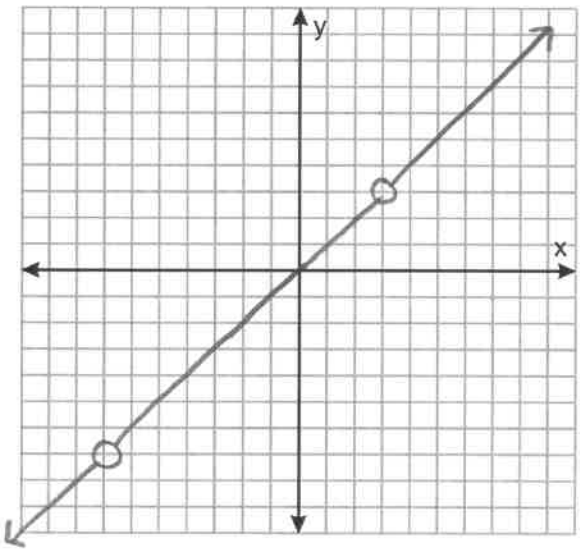
Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

or

$\{x \mid x \neq -1 \text{ or } 3\}$

d. $f(x) = \frac{x^3+4x^2-21x}{x^2+4x-21} = \frac{x(x^2+4x-21)}{(x+7)(x-3)}$

$= \frac{x(x+7)(x-3)}{(x+7)(x-3)} = x,\ x \neq -7, 3$



Hole(s): $(-7, -7)$ $(3, 3)$

Vertical Asymptote(s) EQUATION(S): none

Horizontal Asymptote(s) EQUATION(S): none

Slant Asymptote EQUATION(S): none

X-intercepts: $(0, 0)$

Y-intercepts: $(0, 0)$

Domain: $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

or

$\{x \mid x \neq -7 \text{ or } 3\}$

$x = \frac{-5 \pm \sqrt{25-4(1)(8)}}{2}$

$= \frac{-5 \pm \sqrt{-7}}{2}$

e. $f(x) = \frac{x^2+5x+8}{x+3}$

$$x+3 \overline{)\, x^2+5x+8}$$

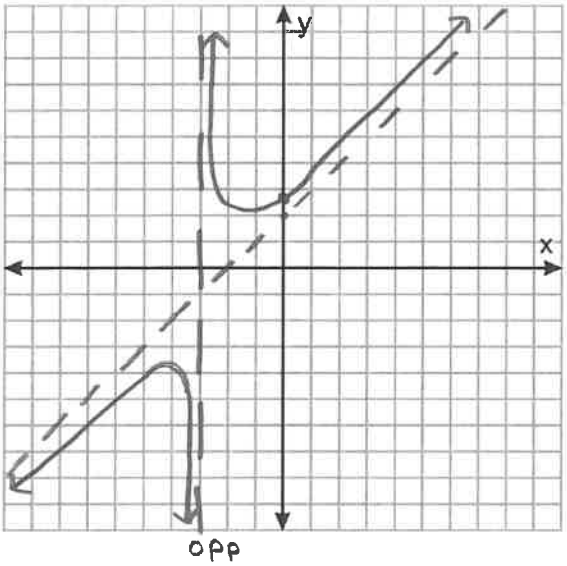
$x + 2 + \frac{2}{x+3}$

$-(x^2+3x)$

$2x+8$

$-(2x+6)$

2



Hole(s): none

Vertical Asymptote(s) EQUATION(S): $x = -3$

Horizontal Asymptote(s) EQUATION(S): none

Slant Asymptote EQUATION(S): $y = x+2$

X-intercepts: no real x-int

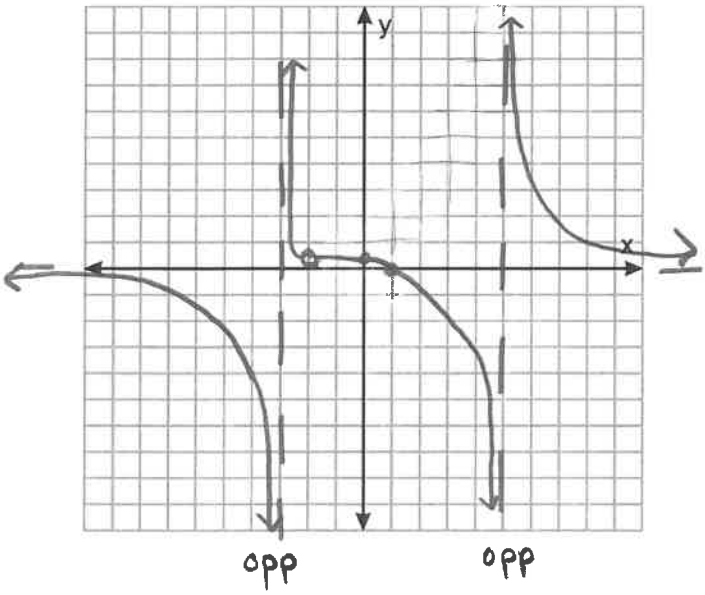
Y-intercepts: $(0, 8/3)$

Domain: $(-\infty, -3) \cup (-3, \infty)$

nonreal x-int: $\left(\frac{-5+\sqrt{7}i}{2}, 0\right)$ and $\left(\frac{-5-\sqrt{7}i}{2}, 0\right)$

f. $f(x) = \frac{x^2+x-2}{(x+2)(x^2-2x-15)}$

$= \frac{(x+2)(x-1)}{(x+2)(x-5)(x+3)} = \frac{x-1}{(x-5)(x+3)}, x \neq -2$



Hole(s): $(-2, 3/7)$

Vertical Asymptote(s) EQUATION(S): $x = -3, x = 5$

Horizontal Asymptote(s) EQUATION(S): $y = 0$

Slant Asymptote EQUATION(S): none

X-intercepts: $(1, 0)$

Y-intercepts: $(0, 1/15)$

Domain: $(-\infty, -3) \cup (-3, -2) \cup (-2, 5) \cup (5, \infty)$

or

$\{x \mid x \neq -3, -2, \text{ or } 5\}$

$x + 3 \quad -4/x$

$x \overline{)\ x^2 + 3x - 4}$

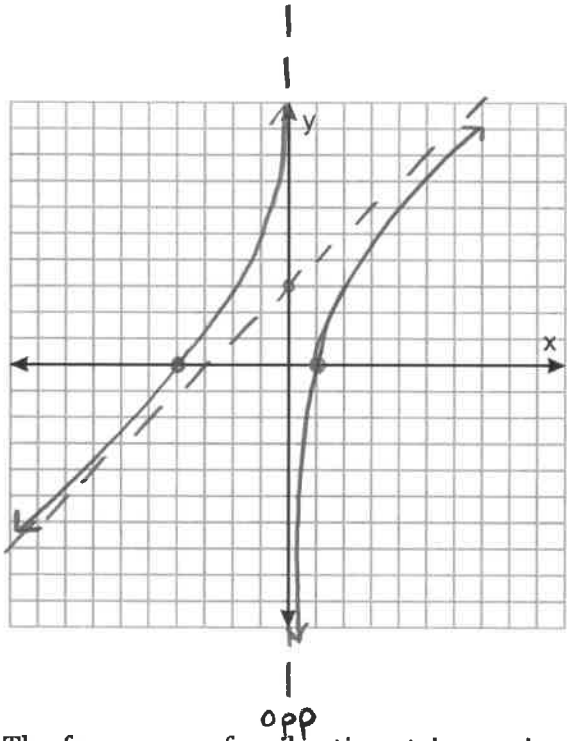
$-(x^2)$

$3x$

$-3x$

-4

g. $f(x) = \frac{x^2+3x-4}{x} = \frac{(x+4)(x-1)}{x}$



Hole(s): none

Vertical Asymptote(s) EQUATION(S): $x = 0$

Horizontal Asymptote(s) EQUATION(S): none

Slant Asymptote EQUATION(S): $y = x + 3$

X-intercepts: $(-4, 0)$ $(1, 0)$

Y-intercepts: none

Domain: $(-\infty, 0) \cup (0, \infty)$

4. The frequency of a vibrating string varies inversely as its length. A string 3 feet long vibrates 175 cycles per second. Find the frequency of a 5 foot string.

$f = \frac{k}{l}$ $\quad f = \frac{525}{l}$ $\quad f = \frac{525}{5}$

$175 = \frac{k}{3}$

$525 = k$

$f = 105$ cycles per second

5. The force of the wind blowing on a vertical surface varies jointly as the area of the surface and the square of the velocity. If a wind blowing at 50mph exerts a force of 75 pounds on a surface of 500 ft^2, how much force will a wind of 75 mph place on a surface of 10 ft^2?

$f = kav^2$ $\quad f = \frac{3}{50,000} av^2$ $\quad f = \frac{3}{50,000}(10)(75)^2$

$75 = k(500)(50)^2$

$k = \frac{3}{50,000}$

$f = \frac{9}{100}$ pounds

6. The time required to process a shipment of goods at Wal-Mart varies directly with the number of items in the shipment and inversely with the number of workers assigned. If 15,000 items can be processed by 8 workers in 10 hours, then how long would it take 12 workers to process 20,000 items?

$t = \frac{ki}{w}$

$10 = \frac{k(15{,}000)}{8}$

$\frac{2}{375} = k$

$t = \frac{2}{375}\frac{i}{w}$

$t = \frac{2}{375}\frac{20{,}000}{12}$

$t = \frac{80}{9} \approx 8.889$ hours

7. A Body Mass Index, or BMI is a measure of a person's weight relative to their height and gives an approximation of total body fat. A BMI (rounded to the nearest whole number) in low 20's is desirable. BMI varies directly as a person's weight in pounds and inversely as the square of the person's height in inches. A person who weighs 140 points and is 70 inches tall has a BMI of 20. Find the BMI of a person who weighs 165 pounds and is 71 inches tall.

$B = k\frac{w}{h^2}$

$20 = k\frac{140}{70^2}$

$700 = k$

$B = 700\frac{w}{h^2}$

$B = 700\frac{165}{71^2}$

$= \frac{115{,}500}{5041}$

≈ 22.912