No calculator unless says "calc" in top right corner of question

1) The coefficient of x^6 in the Taylor series expansion about $x=0$ for $f(x)=\sin\left(x^2\right)$ is 1993 BC 43 calc

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

---

2) A series expansion of $\frac{\sin t}{t}$ is 1973 BC 16

(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

3) If $f(x)=x\sin(2x)$, which of the following is the Taylor series for f about $x=0$? 2008 BC 23

(A) $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$

(B) $x-\frac{4x^3}{2!}+\frac{16x^5}{4!}-\frac{64x^7}{6!}+\cdots$

(C) $2x-\frac{8x^3}{3!}+\frac{32x^5}{5!}-\frac{128x^7}{7!}+\cdots$

(D) $2x^2-\frac{2x^4}{3!}+\frac{2x^6}{5!}-\frac{2x^8}{7!}+\cdots$

(E) $2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}-\frac{128x^8}{7!}+\cdots$

---

4) What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

1998 BC 14

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

5) The graph of the function represented by the Maclaurin series $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots+\frac{(-1)^n x^n}{n!}+\ldots$ intersects the graph of $y=x^3$ at $x=$

1998 BC 89 Calc

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

6) What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x=0$?

2003 BC 28

(A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) 1
(D) 3
(E) 6

7) Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

1997 BC 17

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

8) The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty}x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

2003 BC 11

(A) $1+x^2+x^4+x^6+x^8+\cdots$

(B) $x^2+x^3+x^4+x^5+\cdots$

(C) $x^2+2x^3+3x^4+4x^5+\cdots$

(D) $x^2+x^4+x^6+x^8+\cdots$

(E) $x^2-x^4+x^6-x^8+\cdots$

9) The Taylor series for $\ln x$, centered at $x=1$, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

1998 BC 83 calc

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

---

10) If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

1998 BC 27

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n{}^{n-1}$

---

11) The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1,024}+\ldots$ is

1997 BC 14

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

12) What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges? 2008 BC 16

(A) $-1<x<1$

(B) $x>1$ only

(C) $x\geq 1$ only

(D) $x<-1$ and $x>1$ only

(E) $x\leq -1$ and $x\geq 1$

---

13) What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n}$ converges? 1985 BC 31

(A) $-1\leq x<1$ (B) $-1\leq x\leq 1$ (C) $0<x<2$ (D) $0\leq x<2$ (E) $0\leq x\leq 2$

14) The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is 1969 BC 45

(A) $0 < x < 2$ (B) $0 \le x \le 2$ (C) $-2 < x \le 0$

(D) $-2 \le x < 0$ (E) $-2 \le x \le 0$

---

15) What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges? 1998 BC 84 Calc

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

16) Which of the following series converge?

1988 BC 44

I. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^{n}$

III. $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

17) Which of the following series are convergent?

1983 BC 14

I. $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\ldots$

II. $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$

III. $1-\frac{1}{3}+\frac{1}{3^2}-\ldots+\frac{(-1)^{n+1}}{3^{n-1}}+\ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

18) For what values of x does the series $1+2^x+3^x+4^x+\cdots+n^x+\cdots$ converge? 1969 BC 32

(A) No values of x (B) $x<-1$ (C) $x\geq -1$ (D) $x>-1$ (E) All values of x

19) Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

2008 BC 79 Calc

(A) $\lim_{n\to\infty} a_n = k$

(B) $\int_1^n f(x)\,dx = k$

(C) $\int_1^\infty f(x)\,dx$ diverges.

(D) $\int_1^\infty f(x)\,dx$ converges.

(E) $\int_1^\infty f(x)\,dx = k$

20) Which of the following sequences converge?

1997 BC 76 Calc

I. $\left\{\frac{5n}{2n-1}\right\}$

II. $\left\{\frac{e^n}{n}\right\}$

III. $\left\{\frac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

21) For what integer k, $k>1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

1998 BC 76 calc

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

22) What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}dx$ converges? 2003 BC 6

(A) $p<-1$

(B) $p>0$

(C) $p>\frac{1}{2}$

(D) $p>1$

(E) There are no values of p for which this integral converges.

23) Which of the following series diverge? 2003 BC 24

$I.\ \sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$

$II.\ \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$

$III.\ \sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

(A) III only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

24) Which of the following series converges for all real numbers x?

2008 BC 12

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$