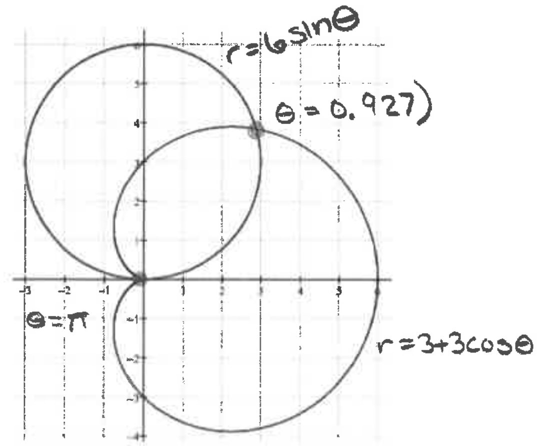
BC Calculus

Chapter 11 Take Home Problem Set

Name: Key Date: ____________ Period: ______

Show all of your work. Read the directions carefully. **Calculator.**

1. The figure to the left shows the graph of $r = 6\sin\theta$ and $r = 3 + 3\cos\theta$ for $0 \le \theta \le 2\pi$.



*calc

1) $|6\sin\theta - 3 - 3\cos\theta|$ $0 \le \theta \le 2\pi$ $\rightarrow G(\theta)$

2) Menu, #3, #1,

3) solve$(g(\theta) = 0, \theta)$

a. Set up an equation to find the value of θ for the intersection(s) of both graphs. Use your calculator to solve your equation and find the coordinates of the point(s) of intersection.

$6\sin\theta = 3 + 3\cos\theta$

$\theta = 0.927,\ \pi$

$(4.799, 0.927)$ and $(0, \pi)$

b. Set up an expression with two or more integrals to find the area common to both curves. Evaluate the integrals.

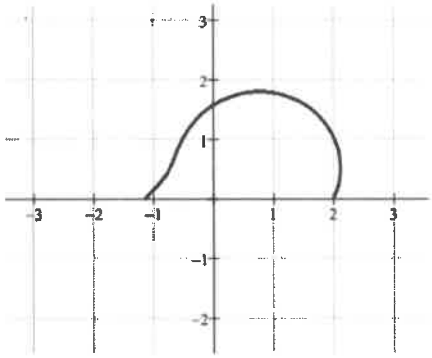
$$A = \int_0^{0.927} \tfrac{1}{2}(6\sin\theta)^2\,d\theta + \int_{0.927}^{\pi} \tfrac{1}{2}(3+3\cos\theta)^2\,d\theta$$

$$= 10.692$$

c. Set up an expression with two or more integrals to find the perimeter of the region common to both curves. Evaluate the integrals.

2. The figure to the right shows the graph of $r = \theta + 2\cos\theta$ for $0 \le \theta \le \pi$.

a. Find the area bounded by the curve and the x-axis



$$A = \int_0^{\pi} \tfrac{1}{2}(\theta + 2\cos\theta)^2 \, d\theta$$

$$= 4.309$$

b. Find the angle(s) θ that corresponds to the point(s) on the curve with y-coordinate 1.

c. Find an expression for $\frac{dr}{d\theta}$. Evaluate your expression for $\frac{dr}{d\theta}$ at $\theta = \frac{\pi}{3}$. Write a sentence interpreting your result.

$$\frac{dr}{d\theta} = 1 - 2\sin\theta$$

$$\left.\frac{dr}{d\theta}\right|_{\theta = \pi/3} = 1 - 2\sin\pi/3 = 1 - \sqrt{3} \approx -0.732$$

As we trace the curve the distance to the pole decreases

d. Find the value of θ for $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance to the origin. Justify your answer.

* max radius

$$\frac{dr}{d\theta} = 0$$

$$0 = 1 - 2\sin\theta$$

$$1/2 = \sin\theta$$

$$\theta = \pi/6$$

+ on $[0, \pi/6]$, − on $[\pi/6, 2\pi]$

θ	r
0	2
$\pi/6$	2.3
$\pi/2$	$\pi/2$

$\theta = \pi/6$ is where max radius of 2.256 occurs b/c r increasing from $[0, \pi/6]$ & dec from $[\pi/6, 2\pi]$

e. Find the slope of the point where $\theta = \frac{\pi}{3}$. Show all of your work.

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{(1 - 2\sin(\pi/3))(\sin\pi/3) + ((\pi/3 + 2\cos\pi/3)\cos\pi/3}{(1 - 2\sin\pi/3)(\cos\pi/3) - (\pi/3 + 2\cos\pi/3)\sin\pi/3}$$

$$= -0.182$$

3. A particle moves in the xy-plane with position vector $\langle x(t), y(t)\rangle$ such that $x(t) = t^3 - 6t^2 + 9t + 1$ and $y(t) = -t^2 + 6t + 2$ in the time interval $0 \le t \le 5$.

a. Find the velocity vector of the particle at $t = 5$.

$\frac{dx}{dt} = 3t^2 - 12t + 9$ $\quad$ $\frac{dy}{dt} = -2t + 6$

$\left.\frac{dx}{dt}\right|_{t=5} = 24$ $\quad$ $\left.\frac{dy}{dt}\right|_{t=5} = -4$ $\quad$ $\langle 24, -4\rangle$

b. Is the particle moving to the left or to the right when $t = 5$? Is the particle moving up or down when $t = 5$? Justify your answer.

right and down

b/c $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} < 0$

c. Find the equation of the tangent line to the path of the particle when $t = 5$.

$\frac{dy}{dx} = \frac{-2t+6}{3t^2 - 12t + 9}$ $\quad$ $x(5) = 21$ $\quad$ $y(5) = 7$ $\quad$ $y - 7 = -\frac{1}{6}(x - 21)$

$\left.dy/dx\right|_{t=5} = -\frac{1}{6}$

d. At what time is the particle at rest? Justify your answer.

$\frac{dx}{dt} = 0$ $\quad$ $t = 1$ or 3

$\frac{dy}{dt} = 0$ $\quad$ $t = 3$

The particle is at rest at $t = 3$ b/c $\frac{dx}{dt} = \frac{dy}{dt} = 0$

e. Find the acceleration vector at the time when the particle is at rest.

$x''(t) = 6t - 12$ $\quad$ $y''(t) = -2$

$x''(3) = 6$ $\quad$ $y''(3) = -2$ $\quad$ $\langle 6, -2\rangle$

f. How fast is the particle moving when $t = 5$?

speed $= \sqrt{24^2 + (-4)^2}$

$= 24.331$

g. Find the total distance traveled by the particle for the time interval $0 \le t \le 5$.

$$\int_0^5 \sqrt{(3t^2 - 12t + 9)^2 + (-2t + 6)^2}\, dt$$

$= 33.043$

h. If the path followed by the particle was graphed, would the graph be concave up or down at $t = 5$?

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{-2t+6}{3t^2-12t+9}\right)}{3t^2-12t+9} \quad \text{at} \quad t=5$$

$$= 0.002 > 0 \qquad \text{concave up}$$

4. What are the points of tangency of the graph of the parametric equations $x = t^2$ and $y = \frac{t}{1+t}$?

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = \frac{(1+t)(1) - t(1)}{(1+t)^2}$$

$$\frac{dy}{dt} \quad \frac{1}{(1+t)^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \begin{matrix} = 0 & \text{HA} \\ = 0 & \text{VA} \end{matrix}$$

$$\frac{1}{(1+t)^2} = 0$$

none

$$2t = 0$$

$$t = 0$$

$$x(0) = 0$$

$$y(0) = 0$$

$$\boxed{(0,0)}$$

The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

1998 BC 21

(A) $\int_0^1 \sqrt{t^2+1}\,dt$

(B) $\int_0^1 \sqrt{t^2+t}\,dt$

(C) $\int_0^1 \sqrt{t^4+t^2}\,dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4+t^4}\,dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

$\frac{dx}{dt} = t^2$ $\qquad$ $\frac{dy}{dt} = t$

$\int_0^1 \sqrt{(t^2)^2 + (t)^2}\,dt$

The length of the path described by the parametric equations $x=\cos^3 t$ and $y=\sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

1997 BC 15

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t+3\sin^2 t}\,dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t\,\sin t+3\sin^2 t\,\cos t}\,dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t+9\sin^4 t}\,dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t\,\sin^2 t+9\sin^4 t\,\cos^2 t}\,dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t+\sin^6 t}\,dt$

$\frac{dx}{dt} = -3\cos^2 t \sin t$

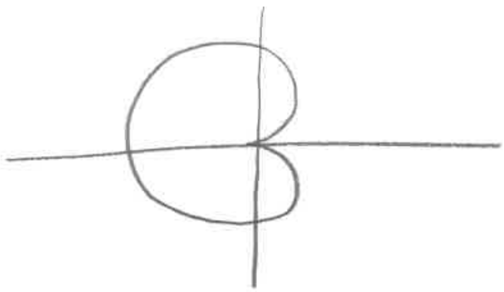
$\frac{dy}{dt} = 3\sin^2 t \cos t$

$\int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\,dt$

$= \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\,dt$

The area of the region enclosed by the polar curve $r = 1 - \cos\theta$ is 1973 BC 40

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π



$$\frac{1}{2}\int_0^{2\pi} (1-\cos\theta)^2 \, d\theta$$

$$\frac{1}{2}\int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta) \, d\theta$$

$$\frac{1}{2}\int_0^{2\pi} \left(1 - 2\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{2}\right) d\theta$$

$\cos 2x = 2\cos^2 x - 1$

$\frac{1}{2}\cos 2x + \frac{1}{2}$

$$= \frac{1}{2}\left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta \Big|_0^{2\pi}\right]$$

$$= \frac{3\pi}{2}$$

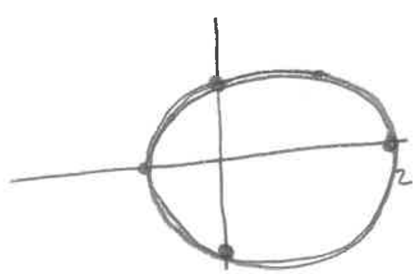
The area of the closed region bounded by the polar graph of $r = \sqrt{3+\cos\theta}$ is given by the integral 1969 BC 9

(A) $\int_0^{2\pi} \sqrt{3+\cos\theta}\, d\theta$ (B) $\int_0^{\pi} \sqrt{3+\cos\theta}\, d\theta$ (C) $2\int_0^{\pi/2} (3+\cos\theta)\, d\theta$

(D) $\int_0^{\pi} (3+\cos\theta)\, d\theta$ (E) $2\int_0^{\pi/2} \sqrt{3+\cos\theta}\, d\theta$

$r = \sqrt{3+\cos\theta}$

θ	0	$\pi/2$	π	$3\pi/2$
r	2	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{3}$



* play around w/ many ways to find area to see how they did it

θ	$\pi/4$	$3\pi/4$
r	pos	pos

$$2\left[\frac{1}{2}\int_0^{\pi} \left(\sqrt{3+\cos\theta}\right)^2 d\theta\right]$$