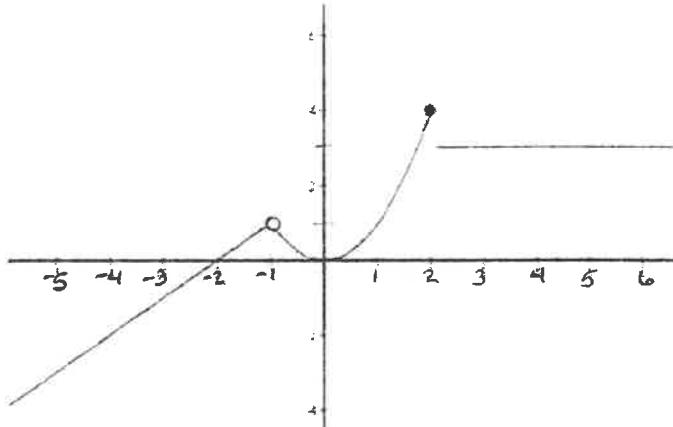


No Calculator

2018 - 19

1. Use the graph below to help answer the following questions:



a.  $f(-2) = \textcircled{O}$

e.  $\lim_{x \rightarrow -1^+} f(x) = 1$

i.  $\lim_{x \rightarrow 2^+} f(x) = 3$

b.  $\lim_{x \rightarrow -2} f(x) = \textcircled{O}$

f.  $\lim_{x \rightarrow -1} f(x) = 1$

j.  $\lim_{x \rightarrow 2} f(x) = \text{DNE}$

c.  $f(-1) = \text{DNE}$

g.  $f(2) = 4$

k. Name the type of discontinuities and where they occur.  
removable disc. at  $x = -1$

d.  $\lim_{x \rightarrow -1^-} f(x) = 1$

h.  $\lim_{x \rightarrow 2^-} f(x) = 4$

Jump disc. at  $x = 2$

41. If  $\lim_{x \rightarrow a} f(x) = L$ , where  $L$  is a real number, which of the following must be true? 1985 AB 41

(A)  $f'(a)$  exists.

(B)  $f(x)$  is continuous at  $x = a$ .

(C)  $f(x)$  is defined at  $x = a$ .

(D)  $f(a) = L$

(E) None of the above

} definition of continuity

AB Calculus  
Chapter 2 Review

4. Assume  $\lim_{x \rightarrow b} f(x) = 7$  and  $\lim_{x \rightarrow b} g(x) = -3$

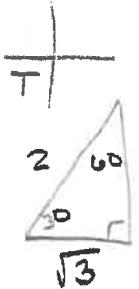
a.  $\lim_{x \rightarrow b} (f(x) + g(x)) =$

$$\lim_{x \rightarrow b} f(x) + \lim_{x \rightarrow b} g(x)  
7 + (-3) = \boxed{4}$$

b.  $\lim_{x \rightarrow b} 4g(x) =$

$$4 \lim_{x \rightarrow b} g(x) = 4(-3)  
= \boxed{-12}$$

5. Evaluate the following limits:



a.  $\lim_{x \rightarrow 7} \sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6}$   
 $= -\frac{2}{\sqrt{3}}$   
 $= \boxed{-\frac{2\sqrt{3}}{3}}$

b.  $\lim_{x \rightarrow 4} \sqrt[3]{x+4} =$   
 $= \sqrt[3]{8}$   
 $= \boxed{2}$

c.  $\lim_{x \rightarrow 2} 3x^2(2x-1) =$   
 $= 3\left(\frac{1}{2}\right)^2\left(2\left(\frac{1}{2}\right)-1\right)$   
 $= 3\left(\frac{1}{4}\right)(1-1)$   
 $= \boxed{0}$

6. Evaluate the following limits

a.  $\lim_{x \rightarrow 2^+} f(x)$ , if  $f(x) = \begin{cases} 3x+1, & x < 2 \\ \frac{5}{x+1}, & x \geq 2 \end{cases}$

$$\lim_{x \rightarrow 2^+} f(x) = \frac{5}{2+1}\\ = \boxed{\frac{5}{3}}$$

c.  $\lim_{x \rightarrow b} (f(x) * g(x)) =$

$$\lim_{x \rightarrow b} f(x) * \lim_{x \rightarrow b} g(x)  
= 7 * (-3) = \boxed{-21}$$

d.  $\lim_{x \rightarrow b} \left(\frac{f(x)}{g(x)}\right) =$

$$\frac{\lim_{x \rightarrow b} f(x)}{\lim_{x \rightarrow b} g(x)} = -\frac{7}{3}$$

d.  $\lim_{y \rightarrow 2} \frac{y^2+5y+6}{y+2} =$

$$= \frac{z^2+5(z)+6}{4} \quad z = 2  
= \frac{4+10+6}{4} \quad \boxed{= 5}$$

e.  $\lim_{x \rightarrow 2} (x-6)^{\frac{2}{3}} =$   
 $= \sqrt[3]{-8}^2$   
 $= (-2)^2$   
 $= \boxed{4}$

f.  $\lim_{x \rightarrow 2} \sqrt{x+3} = \boxed{\sqrt{5}}$

AB Calculus  
Chapter 2 Review

b.  $\lim_{x \rightarrow 1} \frac{x^2-4}{x-1}$

can't factor

VA at  $x=1$

DNE



c.  $\lim_{x \rightarrow 2} \frac{x+1}{x^2-4}$

can't factor

VA at  $x=2$

DNE



7. Evaluate, show your work:

a.  $\lim_{x \rightarrow 3} \frac{x^2-x-6}{x-3}$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{(x-3)}$$

= 5

d.  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$$= \lim_{x \rightarrow 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$$

$$= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x+5}+3}$$

$$= \frac{1}{6}$$

b.  $\lim_{x \rightarrow 0} \frac{\frac{z^2}{x^2} \cdot \frac{1}{x} - 1}{2z+x}$

$$= \lim_{x \rightarrow 0} \frac{\frac{2-z}{x}}{2(z+x)}$$

$$= \lim_{x \rightarrow 0} \frac{-x}{4+2x} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{4+2x} = \boxed{-\frac{1}{4}}$$

c.  $\lim_{x \rightarrow 0} \frac{\sqrt{2x+1}-1}{x} \cdot \frac{\sqrt{2x+1}+1}{\sqrt{2x+1}+1}$

$$= \lim_{x \rightarrow 0} \frac{2x+1-1}{x(\sqrt{2x+1}+1)}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{2x+1}+1)}$$

$$= \lim_{x \rightarrow 0} \frac{2}{\sqrt{2x+1}+1}$$

$$\gamma = \frac{2}{2}$$

$$= 1$$

e.  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

$$= \lim_{x \rightarrow 1} \frac{x-1}{(x+1)(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{1}{x+1}$$

$$= \frac{1}{2}$$

f.  $\lim_{x \rightarrow 0} \frac{(4+x)^2-16}{x}$

$$= \lim_{x \rightarrow 0} \frac{16+8x+x^2-16}{x}$$

$$= \lim_{x \rightarrow 0} 8+x$$

$$= 8$$

AB Calculus  
Chapter 2 Review

g.  $\lim_{t \rightarrow 2} \frac{t^2 - 3t + 2}{t^2 - 4}$

$$= \lim_{t \rightarrow 2} \frac{(t-2)(t-1)}{(t-2)(t+2)}$$

$$= \lim_{t \rightarrow 2} \frac{t-1}{t+2}$$

$= \frac{1}{4}$

h.  $\lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x}$

$$= \lim_{x \rightarrow 0} \frac{(4+4x+x^2)(2+x) - 8}{x}$$

$$= \lim_{x \rightarrow 0} \frac{8+8x+2x^2+4x^2+4x^3+x^4 - 8}{x}$$

$$= \lim_{x \rightarrow 0} 12 + 6x + x^2$$

$= 12$

8. Evaluate each of the following limits analytically. Be sure to show all steps in your evaluation.

a.  $\lim_{x \rightarrow 0} \frac{\sin x}{5x}$

$$= \lim_{x \rightarrow 0} \frac{1}{5} \frac{\sin x}{x}$$

$$= \frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$= \frac{1}{5} (1) = \boxed{\frac{1}{5}}$$

b.  $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

$$= \lim_{x \rightarrow 0} \frac{5 \sin 5x}{5x}$$

$$= 5 \left( \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \right) \rightarrow 1$$

$= 5$

9. Evaluate each of the following by combining properties of limits and your algebra skills.

a.  $\lim_{x \rightarrow 0} \frac{x + \sin x}{x}$

$$= \lim_{x \rightarrow 0} \left[ \frac{x}{x} + \frac{\sin x}{x} \right]$$

$$= 1 + 1$$

$= 2$

c.  $\lim_{x \rightarrow 0} \frac{\sin x}{2x^2 - x}$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x(2x-1)}$$

$$= \frac{1}{-1}$$

$= -1$

b.  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} \cdot \frac{1}{\cos x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\cos x} = \frac{1}{\cos 0} = \boxed{1}$$

d.  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$

$$= \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin x}{x}$$

$$= \sin 0$$

$= 0$

AB Calculus  
Chapter 2 Review

10. Evaluate each limit

a.  $\lim_{x \rightarrow 0} \frac{\sqrt[4]{3-x} - 3}{x}$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{4}(3-x)^{-\frac{3}{4}}(-1)}{1} = \lim_{x \rightarrow 0} \frac{-3}{4(4-x)} = \boxed{\frac{3}{16}}$$

b.  $\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$

$$= \lim_{x \rightarrow 0} \frac{x^2(5x+8)}{x^2(3x^2-16)}$$

$$= \lim_{x \rightarrow 0} \frac{5x+8}{3x^2-16}$$

$$= \boxed{-\frac{1}{2}}$$

c.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4}$

$$= \frac{\sqrt{3+1}}{3-4} = \boxed{-2}$$

d.  $\lim_{x \rightarrow 0} \frac{x^2 - 3x}{x}$

$$= \lim_{x \rightarrow 0} \frac{x(x-3)}{x} = \lim_{x \rightarrow 0} x - 3$$

$$= \boxed{-3}$$

---

If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ , then  $\lim_{x \rightarrow 2} f(x)$  is

- (A)  $\ln 2$       (B)  $\ln 8$       (C)  $\ln 16$       (D) 4      (E) nonexistent

$$\lim_{x \rightarrow 2^-} f(x) = \ln 2$$

$$\lim_{x \rightarrow 2^+} f(x) = 4 \ln 2$$

$$\ln 2 \neq 4 \ln 2$$

e.  $\lim_{x \rightarrow 1} \frac{x}{x^2 - x}$

$$= \lim_{x \rightarrow 1} \frac{x}{x(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x-1} = \boxed{\text{DNE}}$$

VA at  $x=1$

f.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

$$= \lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x}$$

$$= 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \rightarrow 1$$

$$= \boxed{2}$$

g.  $\lim_{x \rightarrow 0} \frac{\sin 7x}{3x}$

$$= \frac{1}{3} \lim_{x \rightarrow 0} \frac{7 \sin 7x}{7x}$$

$$= \frac{7}{3} \lim_{x \rightarrow 0} \frac{\sin 7x}{7x} \rightarrow 1$$

$$= \boxed{\frac{7}{3}}$$

h.  $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$

$$= \lim_{x \rightarrow 4} \frac{(x-4)(x-1)}{(x-4)(x+2)}$$

$$= \lim_{x \rightarrow 4} \frac{x-1}{x+2}$$

$$= \boxed{\frac{1}{2}}$$

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AB Calculus  
Chapter 2 Review

12. Suppose  $g(x) = \begin{cases} 2-x, & x \leq 1 \\ \frac{x}{2} + 1, & x > 1 \end{cases}$

a.  $\lim_{x \rightarrow 1^-} g(x) = 2 - (1)$   
 $= 1$

c.  $\lim_{x \rightarrow 1^+} g(x) = \text{DNE}$

b.  $\lim_{x \rightarrow 1^+} g(x) = \frac{1}{2} + 1$   
 $= \frac{3}{2}$

d.  $g(1) = 2 - (1)$   
 $= 1$

13. For each of the following find:

a.  $\lim_{x \rightarrow \infty} f(x)$

b.  $\lim_{x \rightarrow -\infty} f(x)$

c. Identify all horizontal asymptotes, if any

i.  $f(x) = \frac{x-2}{2x^2+3x-5} \approx \frac{1}{2x}$

$\lim_{x \rightarrow \infty} f(x) = 0$

$\lim_{x \rightarrow -\infty} f(x) = 0$

HA  $y=0$

iii.  $f(x) = \frac{3x^2-x+5}{x^2-4} \approx 3$

$\lim_{x \rightarrow \infty} f(x) = 3$

$\lim_{x \rightarrow -\infty} f(x) = 3$

HA  $y=3$

ii.  $f(x) = \frac{4x^3-2x+1}{x^2-2x+1} \approx x$

$\lim_{x \rightarrow \infty} f(x) = \infty$

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

no HA

iv.  $f(x) = \frac{e^{-x}}{x}$

$f(x) = \frac{1}{x e^x}$

$\lim_{x \rightarrow \infty} f(x) = 0$

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

HA  $y=0$

v.  $f(x) = \frac{\ln x}{x}$

$\lim_{x \rightarrow \infty} f(x) = 1$

$\lim_{x \rightarrow -\infty} f(x) = -1$

HA  $y=\pm 1$

vi.  $f(x) = \frac{\sin x}{2x^2+x}$

den control for larger  $x$

$\lim_{x \rightarrow \infty} f(x) = 0$

$\lim_{x \rightarrow -\infty} f(x) = 0$

HA  $y=0$

Calculus  
Chapter 2 Review

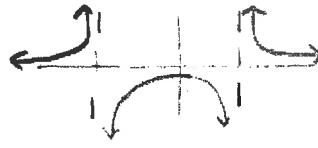
**14. For each of the following:**

- Find the vertical asymptotes of the graph of  $f(x)$
- Describe the behavior of the graph of  $f(x)$  to the left and right of each asymptote

i.  $f(x) = \frac{1}{x-3}$

ii.  $f(x) = \frac{1}{x^2-4}$

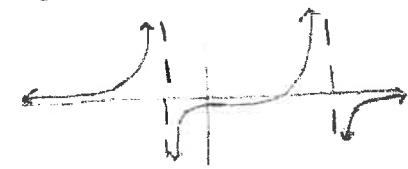
iii.  $f(x) = \frac{1-x}{2x^2-5x-3}$



i)  $f(x) = \frac{1}{x-3}$



ii)  $f(x) = \frac{1}{x^2-4}$



iii)  $f(x) = \frac{1-x}{2x^2-5x-3} = \frac{-1(x-1)}{(2x+1)(x-3)}$

a)  $x=3$ : VA

a) VA:  $x = \pm 2$

a) VA:  $x = -\frac{1}{2}, x = 3$

b)  $\lim_{x \rightarrow 3^-} f(x) = -\infty$

b)  $\lim_{x \rightarrow -2^-} f(x) = \infty$

$\lim_{x \rightarrow 2^-} f(x) = -\infty$

b)  $\lim_{x \rightarrow -\frac{1}{2}^-} f(x) = \infty$

$\lim_{x \rightarrow 3^+} f(x) = \infty$

$\lim_{x \rightarrow -2^+} f(x) = -\infty$

$\lim_{x \rightarrow 2^+} f(x) = \infty$

$\lim_{x \rightarrow -\frac{1}{2}^+} f(x) = -\infty$

$\lim_{x \rightarrow 3^-} f(x) = \infty$

$\lim_{x \rightarrow 3^+} f(x) = -\infty$

15. Find the limit of  $g(x)$  as

a.  $x \rightarrow \infty$

b.  $x \rightarrow -\infty$

c.  $x \rightarrow 0^-$

d.  $x \rightarrow 0^+$

i.  $g(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ \frac{2x-3}{x+1}, & x \geq 0 \end{cases}$

ii.  $g(x) = \begin{cases} \frac{3x}{x-1}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases}$

a)  $\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{2x-3}{x+1} = 2$

a)  $\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$

b)  $\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow -\infty} \frac{1}{x} = 0$

b)  $\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow -\infty} \frac{3x}{x-1} = 3$

c)  $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

c)  $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} \frac{3x}{x-1} = 0$

d)  $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} \frac{2x-3}{x+1} = -3$

d)  $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} \frac{1}{x^2} = \infty$

Calculus  
Chapter 2 Review

16. Sketch a function that satisfies the stated conditions:

$$\lim_{x \rightarrow 1^-} f(x) = 2$$

$$\lim_{x \rightarrow 5^-} f(x) = \infty$$

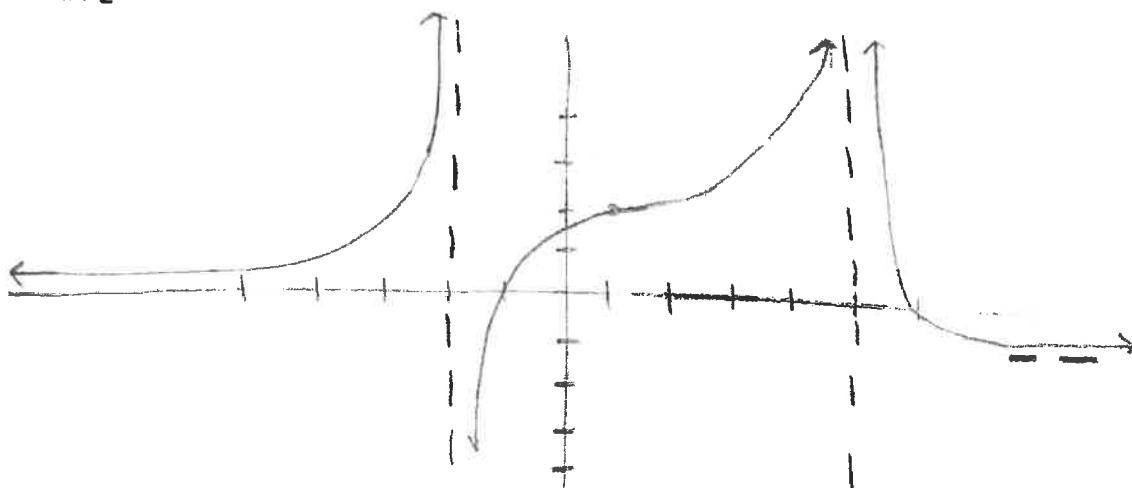
$$\lim_{x \rightarrow 5^+} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -1$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow 2^-} f(x) = \infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$



17. Sketch a function that satisfies the stated conditions. Include any asymptotes.

$$\lim_{x \rightarrow 2^-} f(x) = -1$$

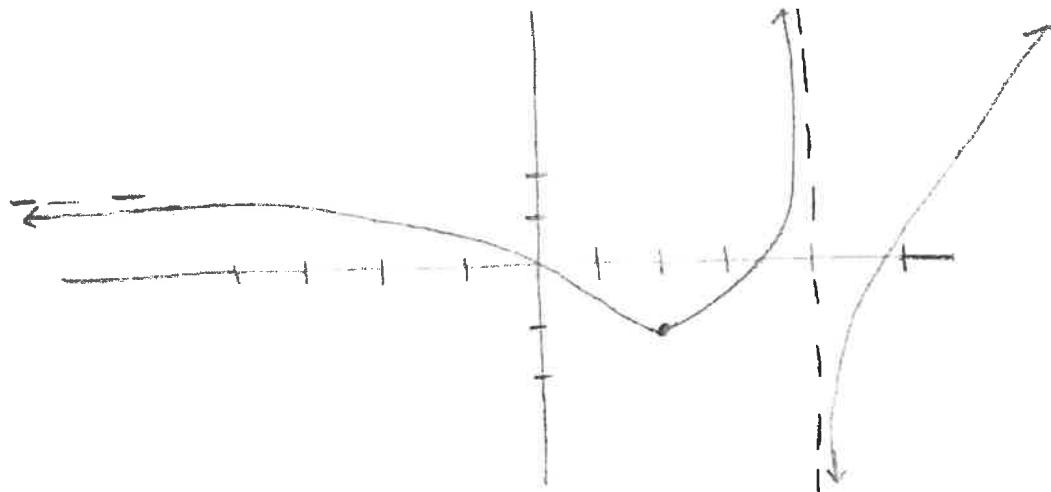
$$\lim_{x \rightarrow \infty} f(x) = 2$$

$$\lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$



Calculus  
Chapter 2 Review

18. Evaluate

$$\lim_{x \rightarrow \infty} \frac{3x-2}{\sqrt{2x^2+1}} \text{ and } \lim_{x \rightarrow -\infty} \frac{3x-2}{\sqrt{2x^2+1}}$$

$$\begin{aligned} & \stackrel{x \rightarrow \infty}{\frac{3x}{\sqrt{2x^2}}} \quad \stackrel{x \rightarrow -\infty}{\frac{-3x}{\sqrt{2x^2}}} \\ & = \frac{\frac{3}{\sqrt{2}}}{2} \quad = \frac{-\frac{3}{\sqrt{2}}}{2} \end{aligned}$$

19. Evaluate each limit

a.  $\lim_{x \rightarrow \infty} \frac{x}{|x|}$

$= 1$

c.  $\lim_{x \rightarrow \infty} \frac{2x^2+1}{(2-x)(2+x)} \approx \frac{2x^2+1}{-x^2 + \dots}$

$= -2$

e.  $\lim_{x \rightarrow \infty} \frac{2^{-x}}{2^x} = \frac{1}{2^{2x}}$

$= 0$

b.  $\lim_{x \rightarrow \infty} \frac{x}{|x|}$

$= 1$

d.  $\lim_{x \rightarrow \infty} \frac{2^{-x}}{2^x} = \frac{1}{2^{2x}}$

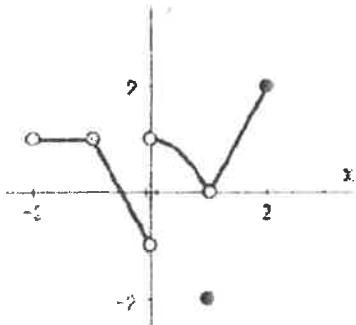
$= 0$

f.  $\lim_{x \rightarrow \infty} \frac{4-x^2}{x^2-1}$

$= -1$

Calculus  
Chapter 2 Review

20. Use the function  $g(x)$  defined and graphed below to answer the following questions:



$$g(x) = \begin{cases} 1 & \text{if } -2 < x < -1 \\ -2x-1 & \text{if } -1 < x < 0 \\ 1-x^2 & \text{if } 0 < x < 1 \\ -2 & \text{if } x=1 \\ 2x-2 & \text{if } 1 < x \leq 2 \end{cases}$$

- a. Does  $g(1)$  exist?

yes

$$g(1) = -2$$

- b. Does  $\lim_{x \rightarrow 1} g(x)$  exist?

yes

$$\lim_{x \rightarrow 1} g(x) = 0$$

- c. Does  $\lim_{x \rightarrow 1} g(x) = g(1)$ ?

no

- d. Is  $g$  continuous at  $x = 1$ ?

no

- e. Is  $g$  defined at  $x = -1$ ?

no

- f. Is  $g$  continuous at  $x = -1$ ?

no

- g. For what values <sup>(is)</sup> of  $g$  continuous?

$$(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$$

everywhere on domain except  
 $x = -2, -1, 0, 1$

- h. What value should be assigned to  $g(-1)$  to make the extended function continuous at  $x = -1$ ?

$$g(-1) = 1$$

- i. What new value should be assigned to  $g(1)$  to make the new function continuous at  $x = 1$ ?

$$g(1) = 0$$

- j. Is it possible to extend  $g$  to be continuous at  $x = 0$ ? If so, what value should the extended function have there? If not, why not?

no b/c discontinuity

is a jump/break at  $x=0$   
so no value can fill the gaps

Calculus  
Chapter 2 Review

21. Let  $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$  Find a value of  $a$  so that the function  $f$  is

continuous. Using the definition of continuity. Justify your response.

$$\lim_{x \rightarrow 3^-} f(x) = 8 \quad \lim_{x \rightarrow 3^+} f(x) = 6a$$

$$8 = 6a$$

$$\frac{8}{6} = a$$

$$\boxed{\frac{4}{3} = a}$$

22. Let  $f(x) = \begin{cases} x^2 - a^2x, & x < 2 \\ 4 - 2x^2, & x \geq 2 \end{cases}$  Find a value of  $a$  so that the function  $f$  is

continuous. Using the definition of continuity. Justify your response.

$$\lim_{x \rightarrow 2^-} f(x) = 4 - 2a^2 \quad \lim_{x \rightarrow 2^+} f(x) = 4 - 2(4) \\ = -4$$

$$4 - 2a^2 = -4$$

$$-2a^2 = -8$$

$$a^2 = 4$$

$$\boxed{a = \pm 2}$$

23. Let  $f$  be the function defined by the following:

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases} \quad \text{check: } x = 0, 1, 2$$

For what values of  $x$  is  $f$  NOT continuous? Use the definition of continuity to explain why.

$$\lim_{x \rightarrow 0^-} f(x) = \sin 0 = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = 1^2 = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = 2 - 2 = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = 0^2 = 0 \quad \checkmark$$

$$\lim_{x \rightarrow 1^+} f(x) = 2 - 1 = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = 2 - 3 = -1 \quad \times$$

$f(x)$  is not continuous at  $x = 2$  b/c

$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$  thus the limit does not exist at  $x = 2$

Calculus  
Chapter 2 Review

24. Without using a picture, give a **written** explanation of why the function

$$f(x) = x^2 - 4x + 3 \text{ has a zero in the interval } [2, 4].$$

$$\begin{aligned} f(2) &= 2^2 - 4(2) + 3 & f(4) &= 4^2 - 4(4) + 3 \\ &= 4 - 8 + 3 & &= 3 \\ &= -1 & \end{aligned}$$

since  $f(x)$  is continuous on  $[2, 4]$   
and  $f(2) = -1$  and  $f(4) = 3$  then by IVT

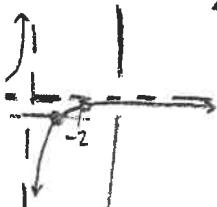
there must exist a  $c$  where  $2 < c < 4$  such that  $f(c) = 0$  since

25. Without using a picture, give a **written** explanation of why the function  $f(x) = x^2 + 2x - 3$  must equal 3 at least once in the interval  $[0, 2]$ .  $-1 < 0 < 3$

$$\begin{aligned} f(0) &= 0^2 + 2(0) - 3 & \text{since } f(x) \text{ is continuous on } [0, 2] \\ &= -3 & \text{and } f(0) = -3 \text{ and } f(2) = 5 \text{ then by} \end{aligned}$$

$$\begin{aligned} f(2) &= 2^2 + 2(2) - 3 & \text{IVT there must exist a } c \text{ where} \\ &= 5 & 0 < c < 2 \text{ such that } f(c) = 3 \text{ since} \end{aligned}$$

$$26. \text{ Let } g(x) = \frac{x^2+5x+6}{x^2+7x+10} = \frac{(x+3)(x+2)}{(x+5)(x+2)} \quad \begin{matrix} \text{hole at} \\ x = -2 \end{matrix} \quad -3 < 3 < 5$$



a. Find the domain of  $g(x)$

$$(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$$

b. Find the  $\lim_{x \rightarrow c} g(x)$  for all values of  $c$  where  $g(x)$  is not defined.

$$\lim_{x \rightarrow -5} g(x) = \text{DNE} \quad \lim_{x \rightarrow -2} g(x) = \frac{-2+3}{-2+5} = \frac{1}{3}$$

c. Find any horizontal asymptotes and justify your response.

$$\text{HA: } y = 1 \quad \lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow -\infty} g(x) = 1$$

d. Find any vertical asymptotes and justify your response.

$$\text{VA: } x = -5 \quad \lim_{x \rightarrow -5^-} g(x) = \infty \quad \lim_{x \rightarrow -5^+} g(x) = -\infty$$

e. Write an extension to the function so that  $g(x)$  is continuous at  $x = -2$ . Use the definition of continuity to justify your response.

$$f(x) = \begin{cases} \frac{x^2+5x+6}{x^2+7x+10}, & x \neq -2 \\ \frac{1}{3}, & x = -2 \end{cases}$$

justify:

$$\lim_{x \rightarrow -2} g(x) = \frac{1}{3} \Delta f(-2) = \frac{1}{3} \Rightarrow \lim_{x \rightarrow -2} g(x) = f(-2) = \frac{1}{3}$$

Calculus  
Chapter 2 Review

27. Let  $h(x) = \begin{cases} 3x^2 - 4, & x \leq 2 \\ 5 + 4x, & x > 2 \end{cases}$

a. What is  $h(0)$ ?

$$h(0) = 3(0)^2 - 4 = \boxed{-4}$$

b. What is  $h(4)$ ?

$$h(4) = 5 + 4(4) = \boxed{21}$$

c. On the interval  $[0, 4]$  there is no value of  $x$  such that  $h(x) = 10$  even though  $h(0) < 10$  and  $h(4) > 10$ . Explain why this result does not contradict the IVT.

$$\lim_{x \rightarrow 2^-} h(x) = 3(2)^2 - 4 \quad \lim_{x \rightarrow 2^+} h(x) = \text{DNE}$$

$$= 8$$

$$\lim_{x \rightarrow 2^+} h(x) = 5 + 4(2)$$

$$= 13$$

since  $h(x)$  is not continuous  
the results above do not  
contradict the INT

28. At what points is the tangent to  $f(x) = x^2 + 4x - 1$  horizontal?  $m = 0$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 4(x+h) - 1 - [x^2 + 4x - 1]}{h}$$

$$m = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 1 - x^2 - 4x + 1}{h}$$

$$m = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 4h}{h}$$

$$m = \lim_{h \rightarrow 0} \frac{2x + h + 4}{1}$$

$$m = 2x + 4$$

$$0 = 2x + 4$$

$$-2 = x$$

$$f(-2) = (-2)^2 + 4(-2) - 1$$

$$= -5$$

$$(-2, -5)$$

29. Find the average rate of change of  $f(x) = 1 + \sin x$  over the interval  $[0, \frac{\pi}{2}]$

$$\frac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0}$$

$$= \frac{1 + \sin \frac{\pi}{2} - [1 + \sin 0]}{\frac{\pi}{2}}$$

$$= \frac{1 + 1 - 1 - 0}{\frac{\pi}{2}}$$

$$= \frac{1}{\frac{\pi}{2}}$$

$$= \boxed{\frac{2}{\pi}}$$

Calculus  
Chapter 2 Review

30. Let  $f(x) = 2x - x^2$

Find

a.  $f(3)$

$$= 2(3) - 3^2$$

$= -3$

b.  $f(3+h)$

$$\begin{aligned} &= 2(3+h) - (3+h)^2 \\ &= 6 + 2h - 9 - 6h - h^2 \\ &= -h^2 - 4h - 3 \end{aligned}$$

c.  $\frac{f(3+h) - f(3)}{h}$

$$\begin{aligned} &= \frac{-h^2 - 4h}{h} \\ &= -h - 4 \end{aligned}$$

$= -h - 4$

d. Find the instantaneous rate of change of  $f$  at  $x = 3$ .

$$= \lim_{h \rightarrow 0} -h - 4$$

$= -4$

$$\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

31. Let  $f(x) = x^2 - 3x$ , and point  $P = (1, f(1))$ . Find:

(1, -2)

a. The slope of the curve at point P.

b. The equation of the tangent line at the point P.

c. An equation of the normal at point P.

a)  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(1+h)^2 - 3(1+h) - (1^2 - 3(1))}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 3 - 3h + 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h + h^2 - 3h}{h}$$

$$= \lim_{h \rightarrow 0} -1 + h$$

$= -1$

b)  $y + 2 = -1(x - 1)$

c)  $y + 2 = 1(x - 1)$

AB Calculus  
Chapter 2 Review

32. An object is dropped from the top of a 130m tower. Its height above ground after  $t$  seconds is  $130 - 4.9t^2$  m. How fast is the object falling 2 seconds after it is dropped?

$$\begin{aligned}
 & \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{130 - 4.9(2+h)^2 - [130 - 4.9(2)^2]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{130 - 4.9(4 + 4h + h^2) - 110.4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-19.6h - 4.9h^2}{h} \\
 &= \lim_{h \rightarrow 0} -19.6 - 4.9h \\
 &= -19.6 \text{ m/s}
 \end{aligned}$$

The object is falling

at a rate of  
19.6 m/s

$$\begin{aligned}
 & \lim_{t \rightarrow 2} \frac{f(t) - f(2)}{t - 2} \\
 &= \lim_{t \rightarrow 2} \frac{130 - 4.9t^2 - 110.4}{t - 2} \\
 &= \lim_{t \rightarrow 2} \frac{19.6 - 4.9t^2}{t - 2} \\
 &= \lim_{t \rightarrow 2} \frac{-4.9(t^2 - 4)}{t - 2} \\
 &= \lim_{t \rightarrow 2} \frac{-4.9(t-2)(t+2)}{t-2} \\
 &= -4.9(4) = -19.6 \text{ m/s}
 \end{aligned}$$

$x$	0	1	2
$f(x)$	1	$k$	2

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The function  $f$  is continuous on the closed interval  $[0, 2]$  and has values that are given in the table above.

The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval  $[0, 2]$  if  $k =$

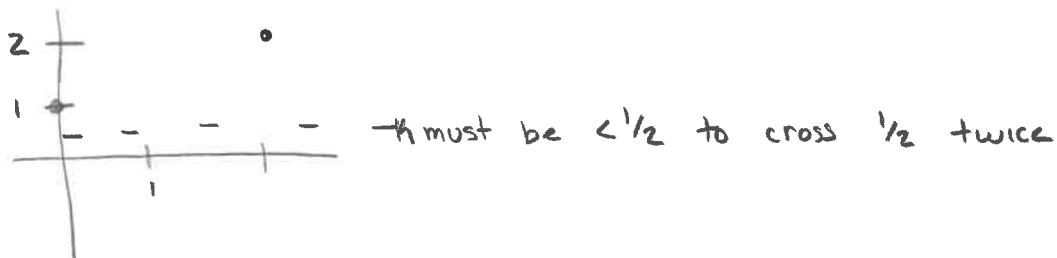
(A) 0

(B)  $\frac{1}{2}$

(C) 1

(D) 2

(E) 3



$$\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)} \text{ is } = \frac{2x^2, \dots}{x^2, \dots}$$

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(A) -3

(B) -2

(C) 2

(D) 3

(E) nonexistent

$$\lim_{n \rightarrow \infty} \frac{4n^2}{n^2 + 10,000n} \text{ is }$$

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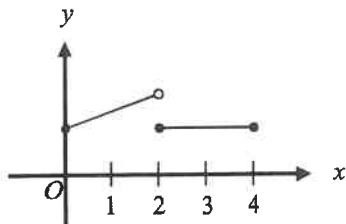
(A) 0

(B)  $\frac{1}{2,500}$

(C) 1

(D) 4

(E) nonexistent



2008 AB 77 calc

Graph of  $f$

The figure above shows the graph of a function  $f$  with domain  $0 \leq x \leq 4$ . Which of the following statements are true?

I.  $\lim_{x \rightarrow 2^-} f(x)$  exists. ✓

II.  $\lim_{x \rightarrow 2^+} f(x)$  exists. ✓

III.  $\lim_{x \rightarrow 2} f(x)$  exists. ✗

- (A) I only      (B) II only      (C) I and II only      (D) I and III only      (E) I, II, and III

$$\lim_{x \rightarrow 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$$

is

2008 AB 5

- (A)  $-\frac{1}{2}$       (B) 0      (C) 1      (D)  $\frac{5}{3} + 1$       (E) nonexistent

$$\lim_{x \rightarrow 0} \frac{x^2(5x^2 + 8)}{x^2(3x^2 - 16)}$$

$$\lim_{x \rightarrow 0} \frac{5x^2 + 8}{3x^2 - 16}$$

$$= -\frac{1}{2}$$

If  $\begin{cases} f(x) = \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ , and if  $f$  is continuous at  $x=2$ , then  $k =$

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(A) 0

(B)  $\frac{1}{6}$

(C)  $\frac{1}{3}$

(D) 1

(E)  $\frac{7}{5}$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} f(x) = \frac{1}{\sqrt{2(2)+5} + \sqrt{2+7}} = \frac{1}{6}$$

$$\frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \cdot \frac{(\sqrt{2x+5} + \sqrt{x+7})}{(\sqrt{2x+5} + \sqrt{x+7})}$$

$$= \frac{2x+5 - x-7}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})} = \frac{x-2}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})} = \frac{1}{\sqrt{2x+5} + \sqrt{x+7}}$$

If  $a \neq 0$ , then  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$  is

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Calc

(A)  $\frac{1}{a^2}$

(B)  $\frac{1}{2a^2}$

(C)  $\frac{1}{6a^2}$

(D) 0

(E) nonexistent

$$= \frac{(x^2 - a^2)}{(x^2 - a^2)(x^2 + a^2)}$$

$$= \frac{1}{x^2 + a^2}$$

$$\lim_{x \rightarrow a} \frac{1}{x^2 + a^2} = \frac{1}{a^2 + a^2} = \frac{1}{2a^2}$$