No Calculator

1.

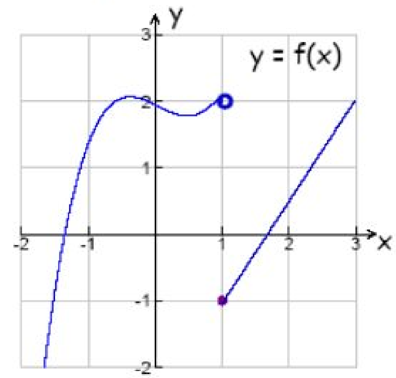
The graph of a function f is given to the right.

(I) $\lim_{x\to1^-} f(x) = 2$

(II) $\lim_{x\to1^+} f(x) = -1$

(III) $\lim_{x\to0} f(x) = 2$

(A) I only
(B) II only
(C) I and II
(D) III only
(E) I, II, and III



2.

$\lim_{x\to-2+} \dfrac{x^2-4x-12}{\sqrt{x+2}}$ is

(A) $= 0$

(B) not defined as the expression $\dfrac{x^2-4x-12}{\sqrt{x+2}}$ is not continuous at $x=-2$.

(C) $= \infty$

(D) $= -6$

(E) $= 6$

3.

Define the function

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Then

(I) f is continuous on the set $-\infty < x < \infty$.

(II) $\lim_{x\to\infty} f(x) = 0$.

(III) $\lim_{x\to0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) All are true
(E) None are true.

4.

The function $f(x) = \dfrac{x^2 + 5x - 6}{\sqrt{x-1}}$

(A) is continuous on the interval $[1, \infty)$.

(B) is continuous on the whole real line.

(C) is continuous on $[1, \infty)$ **provided that** we define $f(1) = 0$

(D) is continuous on $[1, \infty)$ **provided that** we define $f(1) = 7$

(E) cannot be extended to a continuous function on $[1, \infty)$

5.

Assume that we have a continuous function g defined on the interval $[-1, 3]$ such that $f(-1) = 3$, $f(0) = \frac{1}{2}$, $f(2) = -2$ and $f(3) = 1$. Using the **Intermediate Value Theorem** we may conclude that

(I) g has a zero on the interval $(-1, 0)$;

(II) g has a zero on the interval $(0, 2)$;

(III) g has a zero on the interval $(2, 3)$;

(A) I only

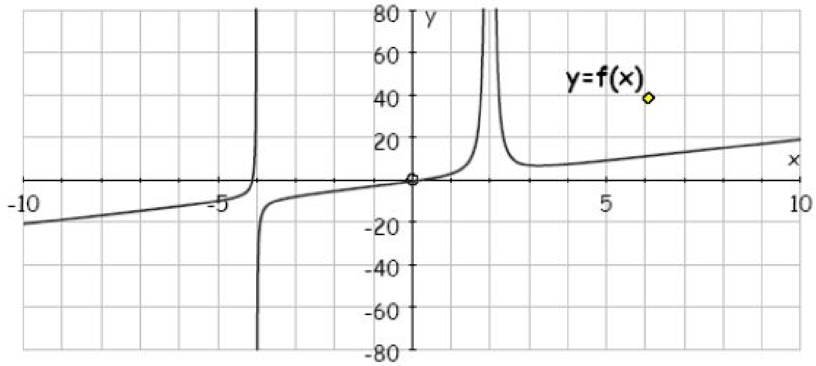
(B) II only

(C) III only

(D) I and II

(E) II and III

6.



The diagram above depicts the graph of a rational function f. Judging from the graph,

(I) $\lim_{x \to -4} f(x) = +\infty = \lim_{x \to 2} f(x)$

(II) $\lim_{x \to +\infty} f(x) = +\infty = \lim_{x \to 2} f(x)$

(III) $\lim_{x \to -4^+} f(x) = -\infty$ and $\lim_{x \to 0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

7. Compute the following limits:

(a) $\lim_{x \to 3} \frac{2x^2 - 7x + 3}{x - 3}$

(b) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

(c) $\lim_{x \to 4} \frac{x + \sqrt{x} - 6}{\sqrt{x} - 2}.$

8. Let:

$$f(x) = \begin{cases} \dfrac{x^2 + x - 12}{x + 4} & \text{if } x > -4 \\ -x - 11 & \text{if } x \leq -4, \end{cases}$$

and compute

(a) $\lim_{x \to -4^-} f(x)$,

(b) $\lim_{x \to -4^+} f(x)$,

(c) $\lim_{x \to -4} f(x)$

9. Sketch the graph of a function f that satisfies the following conditions.

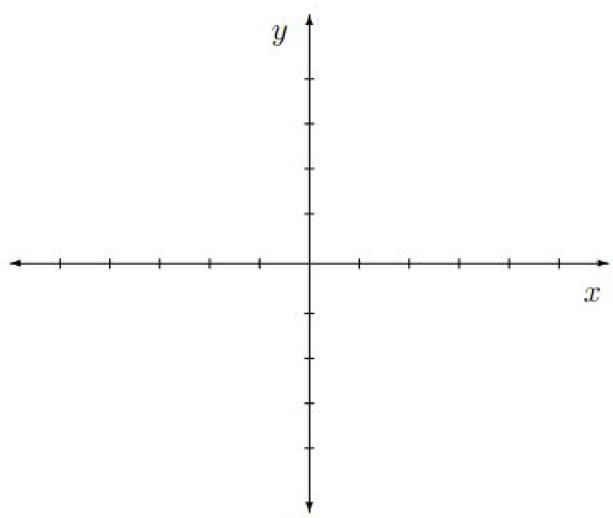
(a) The domain of f is $[-5, 5]$;

(b) $\lim_{x \to -2^-} f(x)$ exists;

(c) $\lim_{x \to -2^-} f(x) = f(-2)$

(d) $\lim_{x \to -2^+} f(x)$ exists;

(e) $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$.



10.

$\lim_{x \to 0} \frac{x^5 - 16x}{x^3 - 4x} =$ 02C

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

11.

Let $f(x) = \begin{cases} -2x + 6, & \text{if } x < 2 \\ x^2 - 1, & \text{if } x \geq 2 \end{cases}$. The $\lim_{x \to 2} f(x)$ is 94C-2

(A) 3 (B) 2 (C) 10 (D) 5
(E) non-existent

12.

The graph of $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4}$ has vertical asymptotes at 93C-4

(A) $x = 0$ only
(B) $x = -2$ only
(C) $x = 2$ only
(D) $x = 2$ and $x = -2$
(E) $x = 3$ only

13.

95 C-9

$\lim_{x\to 1}\frac{x^3-1}{x^2-1}=$

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 3 (E) ∞

14.

$\lim_{x\to\infty}\frac{3x^5-4}{x-2x^5}=$

(A) $-\frac{3}{2}$ (B) -1 (C) 0 (D) 1 (E) $\frac{3}{2}$

15.

$\lim_{x\to\infty}\frac{x^2-1}{1-2x^2}=$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1
(E) non-existent

16.

4. If $f(x)=\dfrac{x^2-4}{x^2-x-2}$, then which of the following is true?

(A) The lines $x=-1$ and $x=2$ are vertical asymptotes

(B) The lines $x=-2$ and $x=2$ are vertical asymptotes

(C) The line $x=1$ is the only vertical asymptote

(D) The line $y=1$ is the only vertical asymptote

(E) The line $x=-1$ is the only vertical asymptote

17.

If $f(x)=\begin{cases}3x+1, \text{ if } x<2\\ 9 \text{ , if } x=2\\ 6x-4, \text{ if } x>2\end{cases}$, then $\lim_{x\to 2} f(x)$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) undefined

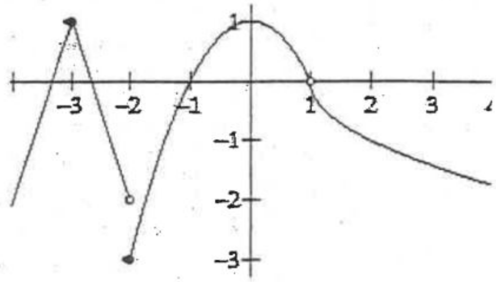
18.

Let $f(x)=\begin{cases}e^x & -\infty<x\le 0\\ |x-2|+k & 0<x<\infty\end{cases}$. Find k so that f is continuous everywhere.

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. e

19.

The graph of function f is shown below.



Which of the following statements are true?

I. $\lim_{x\to -3} f(x) = 1$

II. $\lim_{x\to -2} f(x) = -3$

III. $\lim_{x\to 1} f(x) = 0$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

20.

Given: $f(x) = 4 + \dfrac{1}{x-2}$. Which of the following statements is true?

A. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

B. $\lim_{x\to 2} f(x)$ exists

C. $\lim_{x\to -\infty} f(x) = \lim_{x\to +\infty} f(x)$

D. $\lim_{x\to 2} f(x) = -\infty$

E. $\lim_{x\to 2} f(x) = \infty$

21.

Let $h(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & if\ \ x \neq 2 \\ 0 & , if\ \ x = 2 \end{cases}$. Which of the following statements is true?

I. The function is continuous everywhere.

II. $\lim_{x\to 2^+} h(x) = \lim_{x\to 2^-} h(x)$.

III. The graph of $h(x)$ has a vertical asymptote at $x = 2$.

A. I only
B. II only
C. III only
D. II and III only
E. I and III only

22.

Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

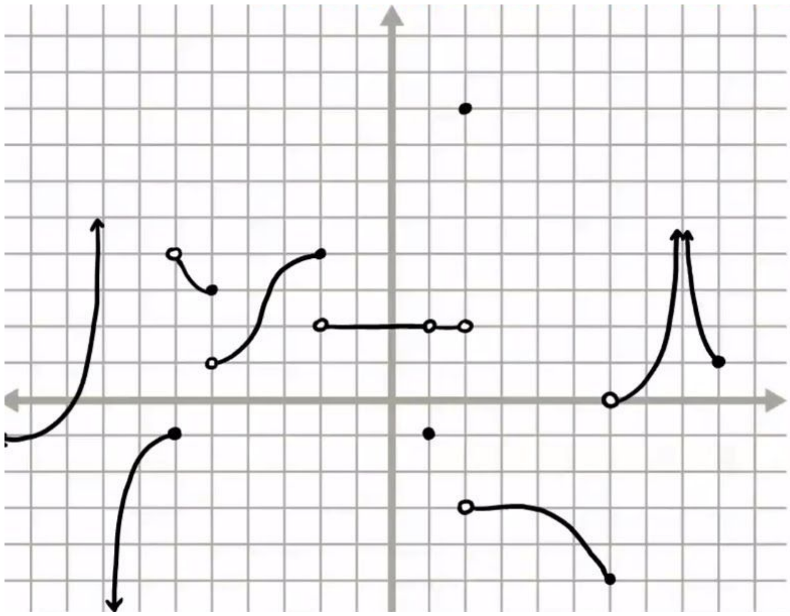
(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \leq f(x) \leq 3$ for all x between -3 and 6.

(D) $f(c) = 1$ for at least one c between -3 and 6

(E) $f(c) = 0$ for at least one c between -1 and 3

23.



1) $\lim_{x\to 1^+} f(x) =$

2) $\lim_{x\to 1^-} f(x) =$

3) $\lim_{x\to 1} f(x) =$

4) $f(1) =$

5) $\lim_{x\to 2^+} f(x) =$

6) $\lim_{x\to 2^-} f(x) =$

7) $\lim_{x\to 2} f(x) =$

8) $f(2) =$

9) $\lim_{x\to -2^+} f(x) =$

10) $\lim_{x\to -2^-} f(x) =$

11) $\lim_{x\to -2} f(x) =$

12) $f(-2) =$

13) $\lim_{x\to 6^+} f(x) =$

14) $\lim_{x\to 6^-} f(x) =$

15) $\lim_{x\to 6} f(x) =$

16) $f(6) =$

17) $\lim_{x\to 8^+} f(x) =$

18) $\lim_{x\to 8^-} f(x) =$

19) $\lim_{x\to 8} f(x) =$

20) $f(8) =$

21) $\lim_{x\to -5^+} f(x) =$

22) $\lim_{x\to -5^-} f(x) =$

23) $\lim_{x\to -5} f(x) =$

24) $f(-5) =$

25) $\lim_{x\to -6^+} f(x) =$

26) $\lim_{x\to -6^-} f(x) =$

27) $\lim_{x\to -6} f(x) =$

28) $f(-6) =$

29) $\lim_{x\to -8^+} f(x) =$

30) $\lim_{x\to -8^-} f(x) =$

31) $\lim_{x\to -8} f(x) =$

32) $f(-8) =$

10 / 13