No Calculator

1.

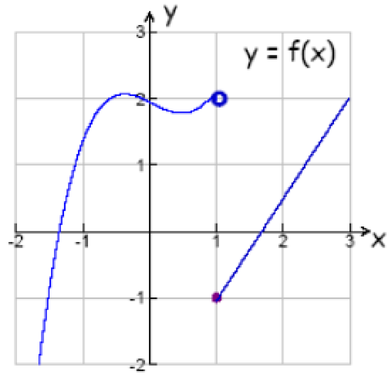
The graph of a function f is given to the right.

(I) $\lim_{x\to 1^-} f(x) = 2$

(II) $\lim_{x\to 1^+} f(x) = -1$

(III) $\lim_{x\to 0} f(x) = 2$

(A) I only
(B) II only
(C) I and II
(D) III only
(E) I, II, and III



2. Let f be the function defined by $f(x) = \frac{x^4-4x^2}{x^2-4x}$. Which of the following statements are true?

   a. f has a discontinuity due to a vertical asymptote at $x = 0$ and $x = 4$.
   b. f has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 4$
   c. f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 4$
   d. f is continuous at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 4$.

3.

Define the function

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Then

(I) f is continuous on the set $-\infty < x < \infty$.

(II) $\lim_{x\to\infty} f(x) = 0$.

(III) $\lim_{x\to 0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) All are true
(E) None are true.

4.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if k =?

a. 0
b. 1/2
c. 1
d. 2
e. 3

5.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

The function f has the properties indicated in the table above. Which of the following must be true?

a. f is continuous at $x = -1$
b. f is continuous at $x = 0$
c. f is continuous at $x = 1$
d. f is differentiable at $x = 0$
e. f is differentiable at $x = 1$

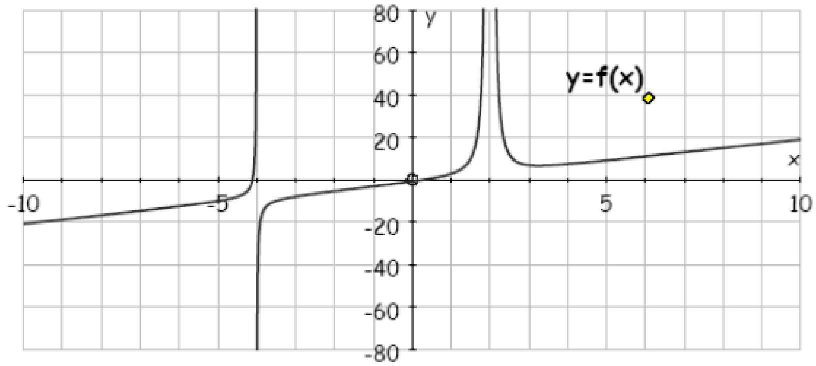
6.

Assume that we have a continuous function g defined on the interval $[-1, 3]$ such that $f(-1) = 3$, $f(0) = \frac{1}{2}$, $f(2) = -2$ and $f(3) = 1$. Using the **Intermediate Value Theorem** we may conclude that

(I) g has a zero on the interval $(-1, 0)$;
(II) g has a zero on the interval $(0, 2)$;
(III) g has a zero on the interval $(2, 3)$;

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

7.



The diagram above depicts the graph of a rational function f. Judging from the graph,

(I) $\lim_{x\to -4} f(x) = +\infty = \lim_{x\to 2} f(x)$

(II) $\lim_{x\to +\infty} f(x) = +\infty = \lim_{x\to 2} f(x)$

(III) $\lim_{x\to -4^+} f(x) = -\infty$ and $\lim_{x\to 0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

8. Compute the following limits:

(a) $\lim_{x\to 3} \dfrac{2x^2 - 7x + 3}{x - 3}$

(b) $\lim_{x\to 0} \dfrac{\sqrt{x+1} - 1}{x}$

(c) $\lim_{x\to 4} \dfrac{x + \sqrt{x} - 6}{\sqrt{x} - 2}$.

9. Let:

$$f(x) = \begin{cases} \dfrac{x^2 + x - 12}{x + 4} & \text{if } x > -4 \\ -x - 11 & \text{if } x \leq -4, \end{cases}$$

and compute

(a) $\lim_{x \to -4^-} f(x)$,

(b) $\lim_{x \to -4^+} f(x)$,

(c) $\lim_{x \to -4} f(x)$

10. Sketch the graph of a function f that satisfies the following conditions.

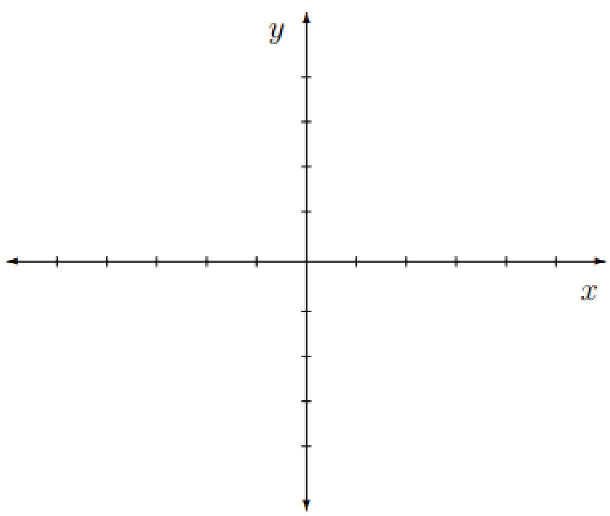
(a) The domain of f is $[-5, 5]$;

(b) $\lim_{x \to -2^-} f(x)$ exists;

(c) $\lim_{x \to -2^-} f(x) = f(-2)$

(d) $\lim_{x \to -2^+} f(x)$ exists;

(e) $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$.



11.

$$\lim_{x \to 0} \frac{x^5 - 16x}{x^3 - 4x} =$$

02C

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

12.

Let $f(x) = \begin{cases} -2x+6, & \text{if } x < 2 \\ x^2 - 1, & \text{if } x \geq 2 \end{cases}$. The $\lim_{x \to 2} f(x)$ is

94C-2

(A) 3 (B) 2 (C) 10 (D) 5
(E) non-existent

13.

The graph of $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 4}$ has vertical asymptotes at

93C-4

(A) $x = 0$ only
(B) $x = -2$ only
(C) $x = 2$ only
(D) $x = 2$ and $x = -2$
(E) $x = 3$ only

14.

95 C-9

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} =$$

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 3 (E) ∞

15.

$$\lim_{x \to \infty} \frac{3x^5 - 4}{x - 2x^5} =$$

(A) $-\frac{3}{2}$ (B) -1 (C) 0 (D) 1 (E) $\frac{3}{2}$

16.

$$\lim_{x \to \infty} \frac{x^2 - 1}{1 - 2x^2} =$$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1
(E) non-existent

17.

4. If $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2}$, then which of the following is true?

(A) The lines $x = -1$ and $x = 2$ are vertical asymptotes

(B) The lines $x = -2$ and $x = 2$ are vertical asymptotes

(C) The line $x = 1$ is the only vertical asymptote

(D) The line $y = 1$ is the only vertical asymptote

(E) The line $x = -1$ is the only vertical asymptote

18.

If $f(x) = \begin{cases} 3x+1, \text{ if } x<2 \\ 9, \text{ if } x=2 \\ 6x-4, \text{ if } x>2 \end{cases}$, then $\lim\limits_{x \to 2} f(x)$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) undefined

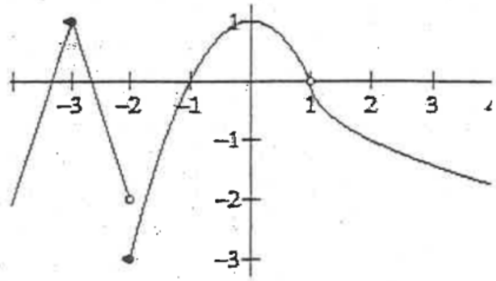
19.

Let $f(x) = \begin{cases} e^x & -\infty < x \le 0 \\ |x-2| + k & 0 < x < \infty \end{cases}$. Find k so that f is continuous everywhere.

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. e

20.

The graph of function f is shown below.



Which of the following statements are true?

I. $\lim_{x\to -3} f(x) = 1$

II. $\lim_{x\to -2} f(x) = -3$

III. $\lim_{x\to 1} f(x) = 0$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

21.

Given: $f(x) = 4 + \dfrac{1}{x-2}$. Which of the following statements is true?

A. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

B. $\lim_{x\to 2} f(x)$ exists

C. $\lim_{x\to -\infty} f(x) = \lim_{x\to +\infty} f(x)$

D. $\lim_{x\to 2} f(x) = -\infty$

E. $\lim_{x\to 2} f(x) = \infty$

22.

Let $h(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & \text{if } x\neq 2\\ 0, & \text{if } x=2\end{cases}$. Which of the following statements is true?

I. The function is continuous everywhere.

II. $\lim_{x\to 2^+} h(x)=\lim_{x\to 2^-} h(x)$.

III. The graph of $h(x)$ has a vertical asymptote at $x=2$.

A. I only
B. II only
C. III only
D. II and III only
E. I and III only

23.

Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$

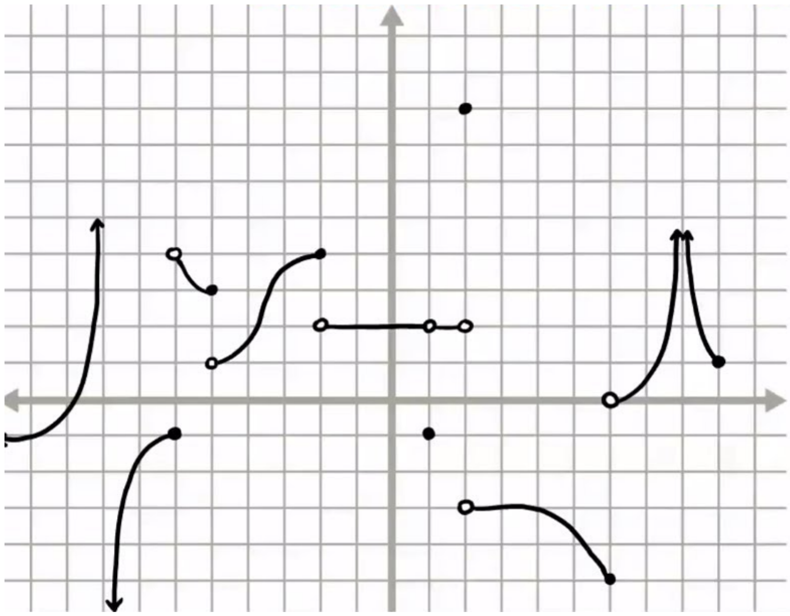
(B) $f'(c)=\frac{4}{9}$ for at least one c between -3 and 6

(C) $-1\le f(x)\le 3$ for all x between -3 and 6.

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

24.



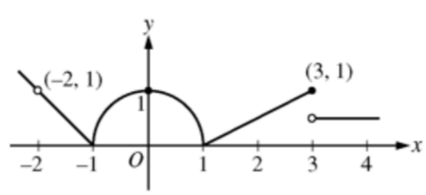
1) $\lim_{x\to 1^+} f(x) =$
2) $\lim_{x\to 1^-} f(x) =$
3) $\lim_{x\to 1} f(x) =$
4) $f(1) =$
5) $\lim_{x\to 2^+} f(x) =$
6) $\lim_{x\to 2^-} f(x) =$
7) $\lim_{x\to 2} f(x) =$
8) $f(2) =$
9) $\lim_{x\to -2^+} f(x) =$
10) $\lim_{x\to -2^-} f(x) =$
11) $\lim_{x\to -2} f(x) =$
12) $f(-2) =$
13) $\lim_{x\to 6^+} f(x) =$
14) $\lim_{x\to 6^-} f(x) =$
15) $\lim_{x\to 6} f(x) =$
16) $f(6) =$
17) $\lim_{x\to 8^+} f(x) =$
18) $\lim_{x\to 8^-} f(x) =$
19) $\lim_{x\to 8} f(x) =$
20) $f(8) =$
21) $\lim_{x\to -5^+} f(x) =$
22) $\lim_{x\to -5^-} f(x) =$
23) $\lim_{x\to -5} f(x) =$
24) $f(-5) =$
25) $\lim_{x\to -6^+} f(x) =$
26) $\lim_{x\to -6^-} f(x) =$
27) $\lim_{x\to -6} f(x) =$
28) $f(-6) =$
29) $\lim_{x\to -8^+} f(x) =$
30) $\lim_{x\to -8^-} f(x) =$
31) $\lim_{x\to -8} f(x) =$
32) $f(-8) =$

25. Let g and h be the function defined by $g(x) = \sin\left(\frac{\pi}{2}(x+2)\right) + 3$ and $h(x) = -\frac{1}{4}x^3 - \frac{3}{2}x^2 - \frac{9}{4}x + 3$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-2 < x < 0$, what is $\lim_{x\to -1} f(x)$?

a. 3
b. 3.5
c. 4
d. The limit cannot be determined from the information given.

26. The graph of the function f is shown below. For which of the following values of c does $\lim_{x\to c} f(x) = 1$?

a. 0 only
b. 0 and 3 only
c. -2 and 0 only
d. -2 and 3 only



Graph of f

e. $-2,\ 0,$ and 3 only