No Calculator

1.

The graph of a function f is given to the right.

✓(I) $\lim_{x\to 1^-} f(x) = 2$

✓(II) $\lim_{x\to 1+} f(x) = -1$

✓(III) $\lim_{x\to 0} f(x) = 2$

(A) I only

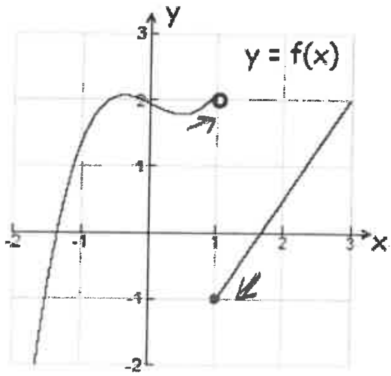
(B) II only

(C) I and II

(D) III only

(E) I, II, and III

debatable
unclear
graph



2.

$$\lim_{x\to -2+} \frac{x^2-4x-12}{\sqrt{x+2}} \text{ is}$$

$$\frac{(x-6)(x+2)}{\sqrt{x+2}}$$

$x = -1$ $y = -7$

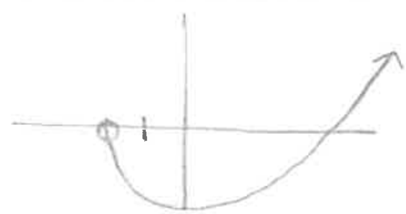
(A) $= 0$

~~(B)~~ not defined as the expression $\frac{x^2-4x-12}{\sqrt{x+2}}$ is not continuous at $x = -2$.

~~(C)~~ $= \infty$

(D) $= -6$

(E) $= 6$



3.

Define the function

$$f(x) = \begin{cases} \frac{1-\cos x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Then

✓(I) f is continuous on the set $-\infty < x < \infty$.

✓(II) $\lim_{x\to\infty} f(x) = 0$.

✓(III) $\lim_{x\to 0} f(x) = 0$

(A) I only

(B) II only

(C) III only

(D) All are true

(E) None are true.

$\lim_{x\to\infty} \frac{1-\cos x}{x}$

$\lim_{x\to\infty} \left(\frac{1}{x} - \frac{\cos x}{x}\right)$

sandwich thm

$0 - 0$

$= 0$

$f(0) = 0$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

↑
know

4.

The function $f(x) = \dfrac{x^2+5x-6}{\sqrt{x-1}}$

~~(A)~~ is continuous on the interval $[1, \infty)$.
~~(B)~~ is continuous on the whole real line.
(C) is continuous on $[1, \infty)$ **provided that** we define $f(1) = 0$
(D) is continuous on $[1, \infty)$ **provided that** we define $f(1) = 7$
~~(E)~~ cannot be extended to a continuous function on $[1, \infty)$

$$\frac{(x+6)(x-1)}{\sqrt{x-1}} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}}$$

$$\frac{(x+6)(x-1)^{3/2}}{x-1} = (x+6)(x-1)^{1/2} \qquad x \neq 1$$

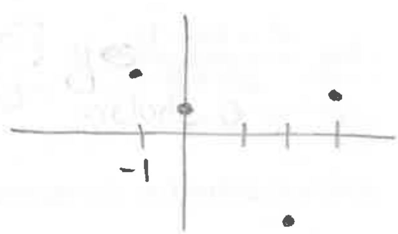
$$f(x) = (x+6)\sqrt{x-1}$$

$$\left.\begin{aligned} f(1) &= 7\sqrt{1-1} \\ &= 0 \end{aligned}\right\} \text{to fill in hole}$$

5.

Assume that we have a continuous function g defined on the interval $[-1, 3]$ such that $g(-1) = 3$, $g(0) = \frac{1}{2}$, $g(2) = -2$ and $g(3) = 1$. Using the **Intermediate Value Theorem** we may conclude that

~~(I)~~ g has a zero on the interval $(-1, 0)$;
✓(II) g has a zero on the interval $(0, 2)$;
✓(III) g has a zero on the interval $(2, 3)$;



(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

6.

Consider the function defined by $f(x) = x \sin \frac{1}{x}$. Then

~~(A)~~ $\lim_{x \to 0} f(x)$ does not exist, but $\lim_{x \to +\infty} f(x) = 0$
~~(B)~~ $\lim_{x \to 0} f(x) = +\infty$ and $\lim_{x \to +\infty} f(x) = 0$
~~(C)~~ $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to +\infty} f(x)$ does not exist.
(D) $\lim_{x \to 0} f(x) = 0$ and $\lim_{x \to +\infty} f(x) = +\infty$
(E) $\lim_{x \to 0} f(x) = 0$ and $\lim_{x \to +\infty} f(x) = 1$

$$\lim_{x \to 0} x \sin \tfrac{1}{x}$$

$$-1 \le \sin \tfrac{1}{x} \le 1$$

$$-x \le x \sin \tfrac{1}{x} \le x$$

$$\lim_{x \to 0} -x \le \lim_{x \to 0} x \sin \tfrac{1}{x} \le \lim_{x \to 0} x$$

$$0 \le \lim_{x \to 0} x \sin \tfrac{1}{x} \le 0$$

$$\downarrow$$

$$0$$

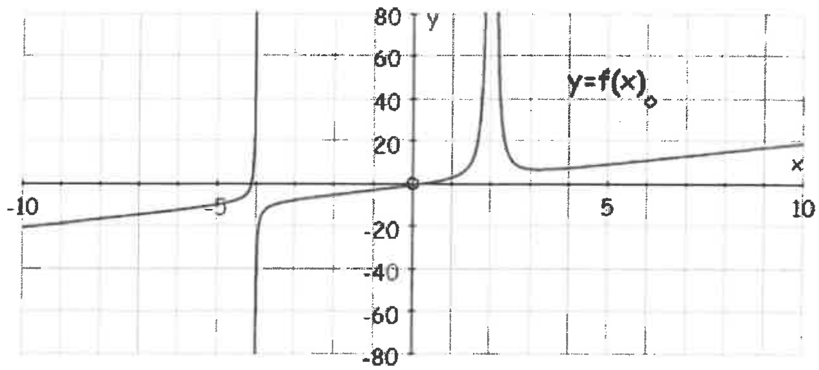
$$\lim_{x \to \infty} x \sin \tfrac{1}{x}$$

$$\lim_{x \to \infty} -x \le \lim_{x \to \infty} x \sin \tfrac{1}{x} \le \lim_{x \to \infty} x$$

$$-\infty \le \qquad \le \infty$$

doesn't work

7.



The diagram above depicts the graph of a rational function f. Judging from the graph,

(I) $\lim_{x \to -4} f(x) = +\infty = \lim_{x \to 2} f(x)$

✓(II) $\lim_{x \to +\infty} f(x) = +\infty = \lim_{x \to 2} f(x)$

✓(III) $\lim_{x \to -4^+} f(x) = -\infty$ and $\lim_{x \to 0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

8. Compute the following limits:

(a) $\lim_{x \to 3} \frac{2x^2 - 7x + 3}{x - 3} = \lim_{x \to 3} \frac{(2x-1)(x-3)}{x-3}$

$= \lim_{x \to 3} 2x - 1$

$= 5$

(b) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

option 1

$f(x) = \sqrt{x}$

$f'(x) = 1/2\, x^{-1/2}$

$f'(1) = 1/2$

option 2

$\frac{0}{0}$

$\lim_{x \to 0} \frac{1/2(x+1)^{-1/2}}{1}$

$= 1/2$

(c) $\lim_{x \to 4} \frac{x + \sqrt{x} - 6}{\sqrt{x} - 2}$

$\lim_{x \to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 3)}{\sqrt{x} - 2}$

$= \lim_{x \to 4} \sqrt{x} + 3$

$= 5$

9. Let:

$$\frac{(x+4)(x-3)}{x+4} = x-3$$

$$f(x) = \begin{cases} \dfrac{x^2+x-12}{x+4} & \text{if } x > -4 \\ -x-11 & \text{if } x \le -4, \end{cases}$$

and compute

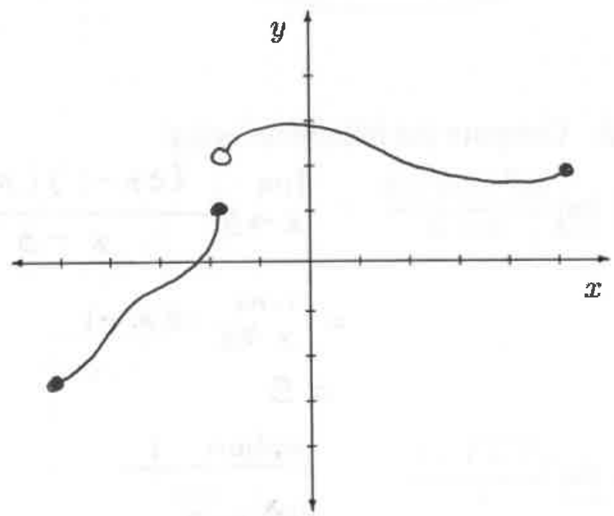
(a) $\lim_{x \to -4^-} f(x)$, $= -(-4)-11$
$= -7$

(b) $\lim_{x \to -4^+} f(x)$, $= -4-3$
$= -7$

(c) $\lim_{x \to -4} f(x) = -7$

10. Sketch the graph of a function f that satisfies the following conditions.

(a) The domain of f is $[-5, 5]$;
(b) $\lim_{x \to -2^-} f(x)$ exists;
(c) $\lim_{x \to -2^-} f(x) = f(-2)$
(d) $\lim_{x \to -2^+} f(x)$ exists;
(e) $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$.



11.

$\lim_{x\to 0}\frac{x^5-16x}{x^3-4x}=$ $\lim_{x\to 0}\frac{x(x^4-16)}{x(x^2-4)}$

02C

(A) −4 (B) −2 (C) 0 (D) 2 (E) 4

$= \lim_{x\to 0}\frac{(x^2-4)(x^2+4)}{x^2-4}$

$= \lim_{x\to 0} x^2+4 = 4$

12.

Let $f(x)=\begin{cases}-2x+6, & \text{if } x<2\\ x^2-1, & \text{if } x\geq 2\end{cases}$. The $\lim_{x\to 2} f(x)$ is

94C-2

(A) 3 (B) 2 (C) 10 (D) 5
(E) non-existent

$\lim_{x\to 2^-} f(x) = -2(2)+6$
$= 2$

$\lim_{x\to 2^+} f(x) = 2^3-1$
$= 7$

13.

The graph of $f(x)=\frac{x^2-5x+6}{x^2-4}$ has vertical asymptotes at

93C-4

(A) $x=0$ only
(B) $x=-2$ only
(C) $x=2$ only
(D) $x=2$ and $x=-2$
(E) $x=3$ only

$f(x) = \frac{(x-2)(x-3)}{(x-2)(x+2)}$ $x\neq 2$ removable

$= \frac{x-3}{x+2}$

$x\neq -2$

14.

95 C-9

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}= \quad \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$$

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 3 (E) ∞

$$\lim_{x\to 1}\frac{x^2+x+1}{x+1}=\frac{3}{2}$$

15.

$$\lim_{x\to\infty}\frac{3x^5-4}{x-2x^5}=$$

(A) $-\frac{3}{2}$ (B) -1 (C) 0 (D) 1 (E) $\frac{3}{2}$

16.

$$\lim_{x\to\infty}\frac{x^2-1}{1-2x^2}=$$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1

(E) non-existent

17.

4. If $f(x)=\dfrac{x^2-4}{x^2-x-2}$, then which of the following is true?

(A) The lines $x=-1$ and $x=2$ are vertical asymptotes

(B) The lines $x=-2$ and $x=2$ are vertical asymptotes

(C) The line $x=1$ is the only vertical asymptote

(D) The line $y=1$ is the only vertical asymptote

(E) The line $x=-1$ is the only vertical asymptote

$$f(x)=\frac{(x-2)(x+2)}{(x-2)(x+1)}$$

$$=\frac{x+2}{x+1} \quad x\neq 2$$

18.

If $f(x)=\begin{cases}3x+1, & \text{if } x<2\\ 9, & \text{if } x=2\\ 6x-4, & \text{if } x>2\end{cases}$, then $\lim\limits_{x\to 2} f(x)$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) undefined

$$\lim_{x\to 2^-} = 3(2)+1$$
$$= 7$$

$$\lim_{x\to 2^+} = 6(2)-4$$
$$= 8$$

19.

Let $f(x)=\begin{cases}e^x & -\infty<x\leq 0\\ |x-2|+k & 0<x<\infty\end{cases}$. Find k so that f is continuous everywhere.

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. e

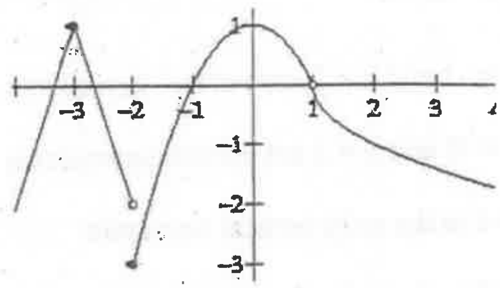
$$\lim_{x\to 0^-} f(x) = e^0$$
$$= 1$$

$$\lim_{x\to 0^+} f(x) = |0-2|+K$$
$$= 2+K$$

$$1 = 2+k$$
$$-1 = k$$

20.

The graph of function f is shown below.



Which of the following statements are true?

~~I. $\lim_{x \to -3} f(x) = 1$~~

~~II.~~ $\lim_{x \to -2} f(x) = -3$

III. $\lim_{x \to 1} f(x) = 0$

A. I only
B. II only
C. III only
~~D. I and II only~~
E. I and III only

21.

shifted 1/x function

Given: $f(x) = 4 + \dfrac{1}{x-2}$; Which of the following statements is true?

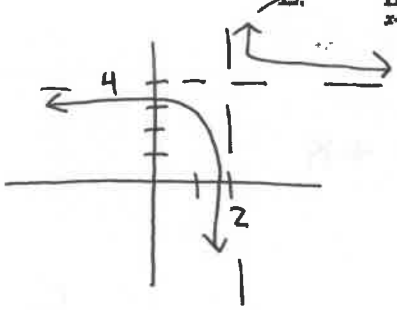
~~A.~~ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

~~B.~~ $\lim_{x \to 2} f(x)$ exists

C. $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x)$

~~D.~~ $\lim_{x \to 2} f(x) = -\infty$

~~E.~~ $\lim_{x \to 2} f(x) = \infty$



$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = \infty$

22.

$$\frac{(x-2)(x+2)}{x-2} = x+2 \quad x \neq 2$$

Let $h(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & \text{if } x\neq 2\\ 0, & \text{if } x=2\end{cases}$. Which of the following statements is true?

$\lim_{x\to 2} h(x) = 4$

~~I.~~ The function is continuous everywhere.

✓II. $\lim_{x\to 2^+} h(x) = \lim_{x\to 2^-} h(x)$.

~~III.~~ The graph of $h(x)$ has a vertical asymptote at $x = 2$.

A. I only
B. II only
C. III only
D. II and III only
E. I and III only

23.

Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

doesn't need to be

(A) $f(0) = 0$

dont know slope

(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6.

function height could go above or below

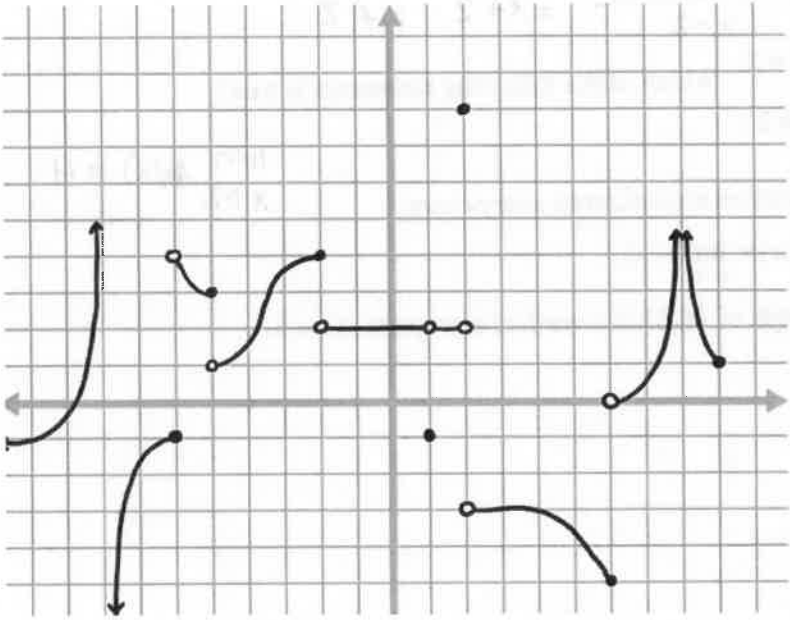
(D) $f(c) = 1$ for at least one c between -3 and 6

(E) $f(c) = 0$ for at least one c between $\underbrace{-1 \text{ and } 3}_{}$

wrong interval

BC Calculus
Chapter 2 Review

24.



1) $\lim_{x\to 1^+} f(x) = 2$
2) $\lim_{x\to 1^-} f(x) = 2$
3) $\lim_{x\to 1} f(x) = 2$
4) $f(1) = -1$
5) $\lim_{x\to 2^+} f(x) = -3$
6) $\lim_{x\to 2^-} f(x) = 2$
7) $\lim_{x\to 2} f(x) =$ DNE
8) $f(2) = 8$
9) $\lim_{x\to -2^+} f(x) = 2$
10) $\lim_{x\to -2^-} f(x) = 4$
11) $\lim_{x\to -2} f(x) =$ DNE
12) $f(-2) = 4$
13) $\lim_{x\to 6^+} f(x) = 0$
14) $\lim_{x\to 6^-} f(x) = -5$
15) $\lim_{x\to 6} f(x) =$ DNE
16) $f(6) = -5$
17) $\lim_{x\to 8^+} f(x) = \infty$
18) $\lim_{x\to 8^-} f(x) = \infty$
19) $\lim_{x\to 8} f(x) = \infty$
20) $f(8) =$ DNE
21) $\lim_{x\to -5^+} f(x) = 1$
22) $\lim_{x\to -5^-} f(x) = 3$
23) $\lim_{x\to -5} f(x) =$ DNE
24) $f(-5) = 3$
25) $\lim_{x\to -6^+} f(x) = 4$
26) $\lim_{x\to -6^-} f(x) = -1$
27) $\lim_{x\to -6} f(x) =$ DNE
28) $f(-6) = -1$
29) $\lim_{x\to -8^+} f(x) = -\infty$
30) $\lim_{x\to -8^-} f(x) = \infty$
31) $\lim_{x\to -8} f(x) =$ DNE
32) $f(-8) =$ DNE