## No Calculator

1.

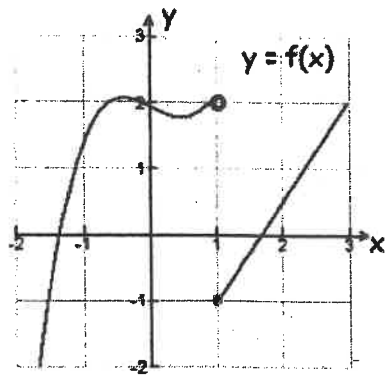
The graph of a function $f$ is given to the right.

(I) $\lim_{x\to 1^-} f(x) = 2$

(II) $\lim_{x\to 1+} f(x) = -1$

(III) $\lim_{x\to 0} f(x) = 2$ debatable unclear graph

(A) I only
(B) II only
(C) I and II
(D) III only
(E) I, II, and III




2. Let $f$ be the function defined by $f(x) = \frac{x^4-4x^2}{x^2-4x}$. Which of the following statements are true?

   a. $f$ has a discontinuity due to a vertical asymptote at $x = 0$ and $x = 4$.
   b. $f$ has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 4$
   c. $f$ has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 4$
   d. $f$ is continuous at $x = 0$, and $f$ has a discontinuity due to a vertical asymptote at $x = 4$.

$$\frac{x^2(x^2-4)}{x(x-4)} = \frac{x^2(x-2)(x+2)}{x(x-4)}$$

$$= \frac{x(x-2)(x+2)}{x-4}$$

$x \neq 0$

3.

Define the function

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases} \rightarrow f(0)=0$$

Then

(I) $f$ is continuous on the set $-\infty < x < \infty$.

(II) $\lim_{x\to\infty} f(x) = 0$.

(III) $\lim_{x\to 0} f(x) = 0$

(A) I only
(B) II only
(C) III only
(D) All are true
(E) None are true.

$\lim_{x\to\infty}\left(\frac{1-\cos x}{x}\right)$

$\lim_{x\to\infty}\frac{1}{x} - \lim_{x\to\infty}\frac{\cos x}{x}$

sandwich thm

$0 - 0$

$0$

$\underbrace{\lim_{x\to 0}\frac{1-\cos x}{x} = 0}$

need to know

$\lim_{x\to\infty}\frac{\cos x}{x} = 0$ Proof $\rightarrow$ $-1 < \cos x < 1$

$-\frac{1}{x} < \frac{\cos x}{x} < \frac{1}{x}$

$\lim_{x\to\infty} -\frac{1}{x} < \lim_{x\to\infty}\frac{\cos x}{x} < \lim_{x\to\infty}\frac{1}{x}$

$0 < \lim_{x\to\infty}\frac{\cos x}{x} < 0$

4.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

The function $f$ is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k$ =?

a. 0
b. 1/2
c. 1
d. 2
e. 3

IVT

5.

| $a$ | $\lim\limits_{x\to a^-} f(x)$ | $\lim\limits_{x\to a^+} f(x)$ | $f(a)$ |
|---|---|---|---|
| −1 | 4 | 6 | 4 |
| 0 | −3 | −3 | 5 |
| 1 | 2 | 2 | 2 |

The function $f$ has the properties indicated in the table above. Which of the following must be true?

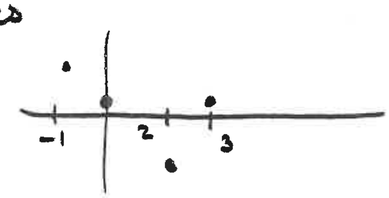
a. $f$ is continuous at $x = -1$
b. $f$ is continuous at $x = 0$
c. $f$ is continuous at $x = 1$
d. $f$ is differentiable at $x = 0$
e. $f$ is differentiable at $x = 1$

cant guarantee not a corner, cusp, etc

6.

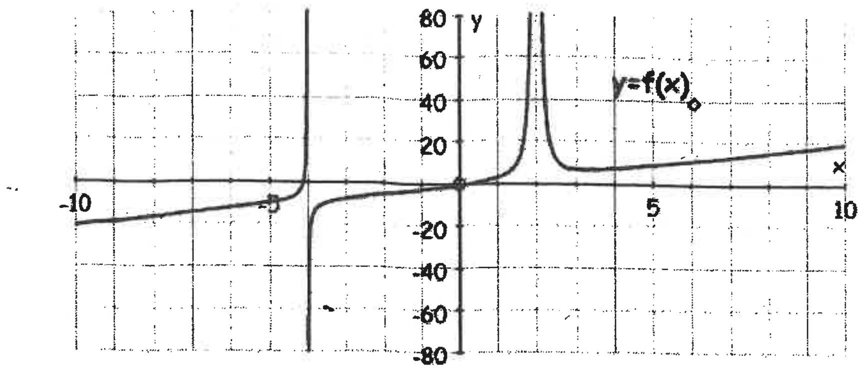
Assume that we have a continuous function $g$ defined on the interval $[-1,3]$ such that $f(-1) = 3$, $f(0) = \frac{1}{2}$, $f(2) = -2$ and $f(3) = 1$. Using the **Intermediate Value Theorem** we may conclude that

(I) $g$ has a zero on the interval ~~$(-1,0)$;~~ $(-1,2)$ yes
(II) $g$ has a zero on the interval $(0,2)$; ✓
(III) $g$ has a zero on the interval $(2,3)$; ✓




(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

7.




The diagram above depicts the graph of a rational function $f$. Judging from the graph,

$-4^-$ yes

(I) $\lim_{x \to -4} f(x) = +\infty = \lim_{x \to 2} f(x)$ ✓

✓(II) $\lim_{x \to +\infty} f(x) = +\infty = \lim_{x \to 2} f(x)$ ✓

✓(III) $\lim_{x \to -4^+} f(x) = -\infty$ and $\lim_{x \to 0} f(x) = 0$ ✓

(A) I only
(B) II only
(C) III only
~~(D) I and II~~
(E) II and III

8. Compute the following limits:

(a) $\lim_{x \to 3} \frac{2x^2 - 7x + 3}{x - 3} = \lim_{x \to 3} \frac{(2x-1)(x-3)}{x-3} = \lim_{x \to 3} 2x - 1 = \boxed{5}$

(b) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

option 1:

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(1) = \boxed{\frac{1}{2}}$

option 2:

$\lim_{x \to 0} \sqrt{x+1} - 1 = 0$

$\lim_{x \to 0} x = 0$

$\frac{0}{0}$ ✓

$\lim_{x \to 0} \frac{\frac{1}{2}(x+1)^{-1/2}}{1} = \boxed{\frac{1}{2}}$

(c) $\lim_{x \to 4} \frac{x + \sqrt{x} - 6}{\sqrt{x} - 2}$.

$\lim_{x \to 4} \frac{(\sqrt{x}+3)(\sqrt{x}-2)}{\sqrt{x} - 2}$

$\lim_{x \to 4} (\sqrt{x} + 3) = \boxed{5}$

or L'hospitals

9. Let:

$$\frac{(x+4)(x-3)}{x+4} = x-3$$

$$f(x) = \begin{cases} \dfrac{x^2+x-12}{x+4} & \text{if } x > -4 \\ -x-11 & \text{if } x \le -4, \end{cases}$$

and compute

(a) $\lim_{x \to -4^-} f(x)$, $= -(-4)-11$

$= -7$

(b) $\lim_{x \to -4^+} f(x)$, $= -4-3$

$= -7$

(c) $\lim_{x \to -4} f(x) = -7$

10. Sketch the graph of a function $f$ that satisfies the following conditions.

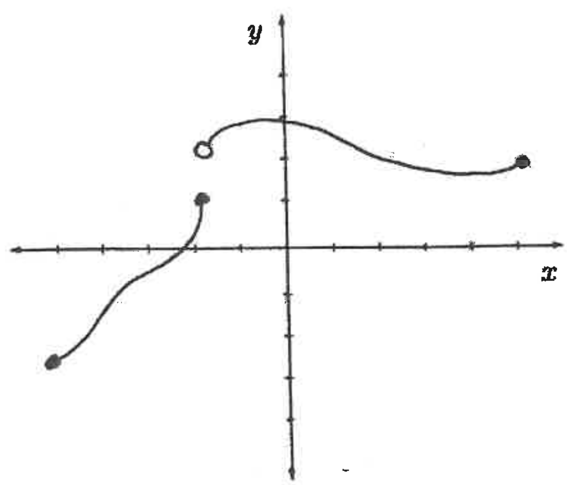
(a) The domain of $f$ is $[-5, 5]$;

(b) $\lim_{x \to -2^-} f(x)$ exists;

(c) $\lim_{x \to -2^-} f(x) = f(-2)$

(d) $\lim_{x \to -2^+} f(x)$ exists;

(e) $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$.



11.

$\lim\limits_{x\to 0}\frac{x^5-16x}{x^3-4x}=$ $\lim\limits_{x\to 0}\frac{x(x^4-16)}{x(x^2-4)}$  02C

(A) −4  (B) −2  (C) 0  (D) 2  (E) 4

$=\lim\limits_{x\to 0}\frac{(x^2-4)(x^2+4)}{x^2-4}$

$=\lim\limits_{x\to 0} x^2+4 = 4$

12.

Let $f(x)=\begin{cases}-2x+6, & \text{if } x<2\\ x^2-1, & \text{if } x\geq 2\end{cases}$. The $\lim\limits_{x\to 2} f(x)$ is  94C-2

(A) 3  (B) 2  (C) 10  (D) 5
(E) non-existent

$\lim\limits_{x\to 2^-} f(x) = -2(2)+6$
$= 2$

$\lim\limits_{x\to 2^+} f(x) = 2^3-1$
$= 7$

13.

The graph of $f(x)=\frac{x^2-5x+6}{x^2-4}$ has vertical asymptotes at  93C-4

(A) $x=0$ only
(B) $x=-2$ only
(C) $x=2$ only
(D) $x=2$ and $x=-2$
(E) $x=3$ only

$f(x)=\frac{(x-2)(x-3)}{(x-2)(x+2)}$  $x\neq 2$ removable

$=\frac{x-3}{x+2}$

$x\neq -2$

14.

95 C -9

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}= \quad \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$$

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 3 (E) $\infty$

$$\lim_{x\to 1}\frac{x^2+x+1}{x+1} = \frac{3}{2}$$

15.

$$\lim_{x\to\infty}\frac{3x^5-4}{x-2x^5}=$$

(A) $-\frac{3}{2}$ (B) $-1$ (C) 0 (D) 1 (E) $\frac{3}{2}$

16.

$$\lim_{x\to\infty}\frac{x^2-1}{1-2x^2}=$$

(A) $-1$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1
(E) non-existent

17.

4. If $f(x)=\dfrac{x^2-4}{x^2-x-2}$, then which of the following is true?

(A) The lines $x=-1$ and $x=2$ are vertical asymptotes

(B) The lines $x=-2$ and $x=2$ are vertical asymptotes

(C) The line $x=1$ is the only vertical asymptote

(D) The line $y=1$ is the only vertical asymptote

(E) The line $x=-1$ is the only vertical asymptote

$f(x)=\dfrac{(x-2)(x+2)}{(x-2)(x+1)}$

$=\dfrac{x+2}{x+1} \quad x\neq 2$

18.

If $f(x)=\begin{cases}3x+1, & \text{if } x<2\\ 9, & \text{if } x=2\\ 6x-4, & \text{if } x>2\end{cases}$, then $\lim\limits_{x\to 2} f(x)$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) undefined

$\lim\limits_{x\to 2^-} = 3(2)+1$
$= 7$

$\lim\limits_{x\to 2^+} = 6(2)-4$
$= 8$

19.

Let $f(x)=\begin{cases}e^x & -\infty<x\le 0\\ |x-2|+k & 0<x<\infty\end{cases}$. Find $k$ so that $f$ is continuous everywhere.

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. $e$

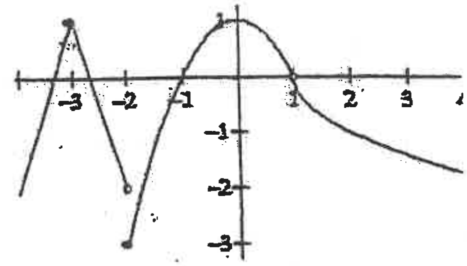
$\lim\limits_{x\to 0^-} f(x) = e^0$
$= 1$

$\lim\limits_{x\to 0^+} f(x) = |0-2| + K$
$= 2 + K$

$1 = 2 + K$
$-1 = K$

20.

The graph of function $f$ is shown below.



Which of the following statements are true?

I. $\lim_{x\to -3} f(x) = 1$

~~II.~~ $\lim_{x\to -2} f(x) = -3$

III. $\lim_{x\to 1} f(x) = 0$

A. I only
B. II only
C. III only
~~D. I and II only~~
E. I and III only

21.

shifted 1/x function

Given: $f(x) = 4 + \dfrac{1}{x-2}$. Which of the following statements is true?

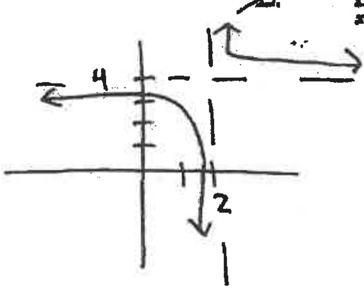
~~A.~~ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

~~B.~~ $\lim_{x\to 2} f(x)$ exists.

C. $\lim_{x\to -\infty} f(x) = \lim_{x\to \infty} f(x)$

~~D.~~ $\lim_{x\to 2} f(x) = -\infty$

~~E.~~ $\lim_{x\to 2} f(x) = \infty$




$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

22.

$$\frac{(x-2)(x+2)}{x-2} = x+2 \quad x \neq 2$$

Let $h(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & \text{if } x \neq 2 \\ 0, & \text{if } x = 2 \end{cases}$. Which of the following statements is true?

$\lim_{x \to 2} h(x) = 4$

~~I.~~ The function is continuous everywhere.

✓II. $\lim_{x \to 2^+} h(x) = \lim_{x \to 2^-} h(x)$.

~~III.~~ The graph of $h(x)$ has a vertical asymptote at $x = 2$.

A. I only
B. II only
C. III only
D. II and III only
E. I and III only

23.

Let $f$ be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

doesn't need to be

(A) $f(0) = 0$

dont know slope

(B) $f'(c) = \frac{4}{9}$ for at least one $c$ between $-3$ and $6$

(C) $-1 \leq f(x) \leq 3$ for all $x$ between $-3$ and $6$. above or below

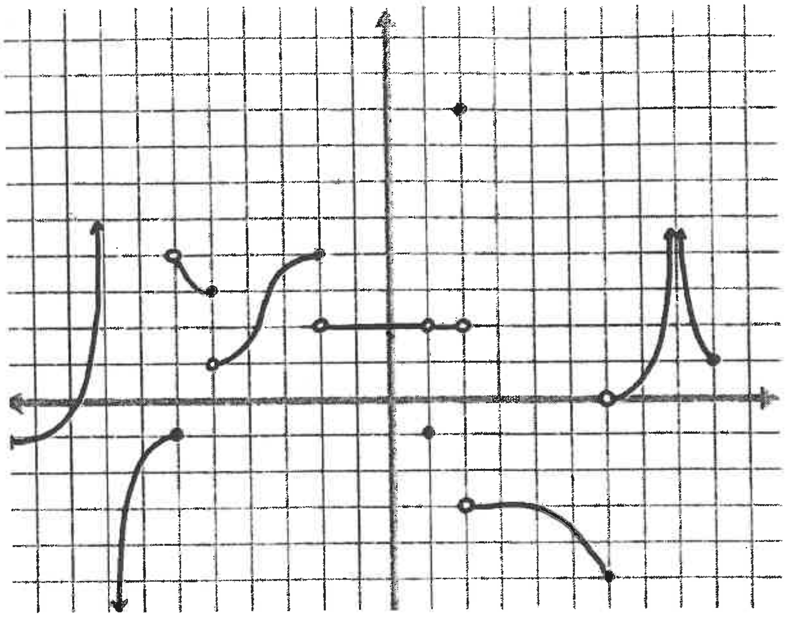
function height could go

(D) $f(c) = 1$ for at least one $c$ between $-3$ and $6$

(E) $f(c) = 0$ for at least one $c$ between $-1$ and $3$

wrong interval

24.



| | | |
|---|---|---|
| 1) $\lim_{x\to1^+} f(x) =$ 2 | 13) $\lim_{x\to6^+} f(x) =$ 0 | 25) $\lim_{x\to-6^+} f(x) =$ 4 |
| 2) $\lim_{x\to1^-} f(x) =$ 2 | 14) $\lim_{x\to6^-} f(x) =$ -5 | 26) $\lim_{x\to-6^-} f(x) =$ -1 |
| 3) $\lim_{x\to1} f(x) =$ 2 | 15) $\lim_{x\to6} f(x) =$ DNE | 27) $\lim_{x\to-6} f(x) =$ DNE |
| 4) $f(1) =$ -1 | 16) $f(6) =$ -5 | 28) $f(-6) =$ -1 |
| 5) $\lim_{x\to2^+} f(x) =$ -3 | 17) $\lim_{x\to8^+} f(x) = \infty$ | 29) $\lim_{x\to-8^+} f(x) = -\infty$ |
| 6) $\lim_{x\to2^-} f(x) =$ 2 | 18) $\lim_{x\to8^-} f(x) = \infty$ | 30) $\lim_{x\to-8^-} f(x) = \infty$ |
| 7) $\lim_{x\to2} f(x) =$ DNE | 19) $\lim_{x\to8} f(x) = \infty$ | 31) $\lim_{x\to-8} f(x) =$ DNE |
| 8) $f(2) =$ 8 | 20) $f(8) =$ DNE | 32) $f(-8) =$ DNE |
| 9) $\lim_{x\to-2^+} f(x) =$ 2 | 21) $\lim_{x\to-5^+} f(x) =$ 1 | |
| 10) $\lim_{x\to-2^-} f(x) =$ 4 | 22) $\lim_{x\to-5^-} f(x) =$ 3 | |
| 11) $\lim_{x\to-2} f(x) =$ DNE | 23) $\lim_{x\to-5} f(x) =$ DNE | |
| 12) $f(-2) =$ 4 | 24) $f(-5) =$ 3 | |



25. Let $g$ and $h$ be the function defined by $g(x) = \sin\left(\frac{\pi}{2}(x+2)\right) + 3$ and $h(x) = -\frac{1}{4}x^3 - \frac{3}{2}x^2 - \frac{9}{4}x + 3$. If $f$ is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-2 < x < 0$, what is $\lim_{x\to-1} f(x)$?

a. 3
b. 3.5
c. 4
d. The limit cannot be determined from the information given.

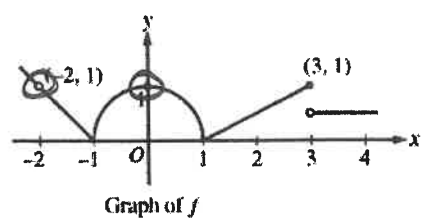
$\lim_{x\to-1} g(x) \le \lim_{x\to-1} f(x) \le \lim_{x\to-1} h(x)$

$\sin \pi/2 + 3 \le \qquad \le \frac{1}{4} - \frac{3}{2} + \frac{9}{4} + 3 \qquad \frac{10}{4} - \frac{6}{4} + 3$

$4 \le \qquad \le 4$

26. The graph of the function $f$ is shown below. For which of the following values of $c$ does $\lim_{x\to c} f(x) = 1$?

a. 0 only
b. 0 and 3 only
c. −2 and 0 only
d. −2 and 3 only
e. −2, 0, and 3 only



Graph of $f$