1. List the 2 limit-definition of derivatives.

2. What are some other phrases that mean derivative?

3. What is the product rule?

4. What is the quotient rule?

5. Name the three things that must be true for a function to be differentiable.

6. What are three types of functions that are continuous, but not differentiable at a particular point? Give an example of each type.

7. Find the derivatives:
   a. $y = \sin x$
   b. $y = \cos x$
   c. $y = \tan x$
   d. $y = \csc x$
   e. $y = \sec x$
   f. $y = \cot x$

8. Find $\frac{dy}{dx}$ for each function:

a. $y = x^5 \tan x$

i. $5x^4 \tan x$ ii. $x^5 \sec^2 x$ iii. $5x^4 \sec^2 x$ iv. $5x^4 + \sec^2 x$

v. $5x^4 \tan x + x^5 \sec^2 x$

b. $y = \frac{2-x}{3x+1}$

i. $-\frac{7}{(3x+1)^2}$ ii. $\frac{6x-5}{(3x+1)^2}$ iii. $-\frac{9}{(3x+1)^2}$ iv. $\frac{7}{(3x+1)^2}$

v. $\frac{7-6x}{(3x+1)^2}$

c. $y = 3x^{\frac{2}{3}} - 4x^{\frac{1}{2}} - 2$

i. $2x^{\frac{1}{3}} - 2x^{-\frac{1}{2}}$ ii. $3x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$ iii. $\frac{9}{5}x^{\frac{5}{3}} - 8x^{\frac{3}{2}}$ iv. $\frac{2}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{2}}} - 2$

v. $2x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$

d. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

i. $x+\frac{1}{x\sqrt{x}}$ ii. $x^{-\frac{1}{2}}+x^{-\frac{3}{2}}$ iii. $\frac{4x-1}{4x\sqrt{x}}$ iv. $\frac{1}{\sqrt{x}}+\frac{1}{4x\sqrt{x}}$ v. $\frac{4}{\sqrt{x}}+\frac{1}{x\sqrt{x}}$

Find $\frac{dy}{dx}$ of the following:

3) $y=(-2x^4-3)(-2x^2+1)$

4) $f(x)=(2x^4-3)(x^2+1)$

5) $f(x)=(5x^5+5)(-2x^5-3)$

6) $f(x)=(-3+x^{-3})(-4x^3+3)$

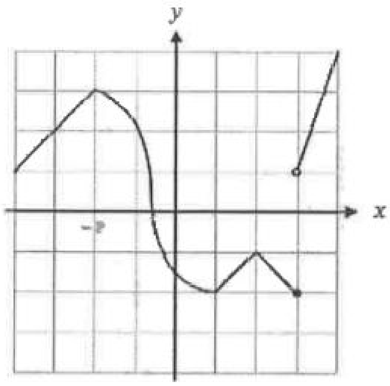
3) $f(x)=\dfrac{5}{4x^3+4}$

4) $y=\dfrac{4x^3-3x^2}{4x^5-4}$

5) $y=\dfrac{3x^4+2}{3x^3-2}$

6) $y=\dfrac{4x^5+2x^2}{3x^4+5}$

9. The function for $f(x)$ is graphed below. There is a vertical tangent line when $x=-\frac{1}{2}$. Where is $f(x)$ not differentiable? Why?



10. If $f(x)$ has a derivative at $x = 2,$ tell whether or not each of the following must be true?
    a. $\lim_{x \to 2} f(x)$ exists
    b. $f'(2)$ exists
    c. $f''(2)$ exists
    d. $f(x)$ is continuous at $x = 2$
    e. $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$ exists
    f. $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$ exists

2003 AB 25

11.
A particle moves along the $x-$axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest.

a. $t = 1$ only
b. $t = 3$ only
c. $t = \frac{7}{2}$ only
d. $t = 3$ and $t = \frac{7}{2}$
e. $t = 3$ and $t = 4$

8. Find $\frac{dy}{dx}$ of $y = \frac{x^2}{\cos x}$
   a. $\frac{2x}{\sin x}$ b. $-\frac{2x}{\sin x}$ c. $\frac{2x\cos x - x^2 \sin x}{\cos^2 x}$ d. $\frac{2x\cos x + x^2 \sin x}{\cos^2 x}$

   e. $\frac{2x\cos x + x^2 \sin x}{\sin^2 x}$

9. If f is a function such that $\lim_{x \to -3} \frac{f(x)-f(-3)}{x+3} = 2$, which of the following must be true?
   a. The limit of $f(x)$ as x approaches -3 does not exist
   b. f is not defined at $x = -3$
   c. The derivative of f at $x = -3$ is 2
   d. f is continuous at $x = 2$
   e. $f(-3) = 2$

10. Sketch the graph of a continuous function f with $f(0) = 0$ and

$$f'(x) = \begin{cases} \frac{1}{2}, & x < 2 \\ -3, & x > 2 \end{cases}$$

10. If $f(x) = \begin{cases} 2ax^2 + b, & x \geq 1 \\ -3x + 4, & x < 1 \end{cases}$, find a and b so that f is both continuous and

differentiable. (Be sure to use definitions to justify your work)

$\lim_{h \to 0} \frac{(1+h)^6 - 1}{h}$ is

a. 0 b. 1 c. 6 d. ∞ e. Nonexistent

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) None

(B) I only

(C) II only

(D) I and II only

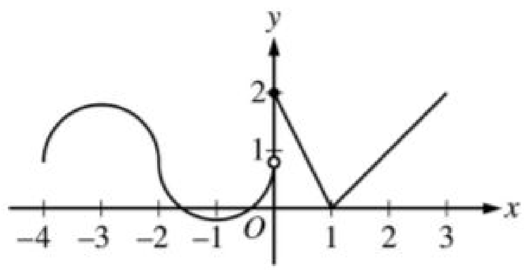
(E) I, II, and III

1998 AB 14

A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

1998 AB 3



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$. at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

2003 AB 16

If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

1985 AB 23

$\dfrac{d}{dx}\left(\dfrac{1}{x^3}-\dfrac{1}{x}+x^2\right)$ at $x=-1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

1988 AB 41

If $\lim\limits_{x\to 3} f(x)=7$, which of the following must be true?

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

1993 AB 41 Calc

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t)=-4\cos t-\dfrac{t^2}{2}+10$. What is the velocity of the particle when its acceleration is zero?

(A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

1997 AB 12

At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

(A) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (B) $\left(\frac{1}{2},\frac{1}{8}\right)$ (C) $\left(1,-\frac{1}{4}\right)$ (D) $\left(1,\frac{1}{2}\right)$ (E) $(2,2)$

1997 AB 79 Calc

Let f be a function such that $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=5$. Which of the following must be true?

I. f is continuous at $x=2$.

II. f is differentiable at $x=2$.

III. The derivative of f is continuous at $x=2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

1998 AB 10

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

1998 AB 8

Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

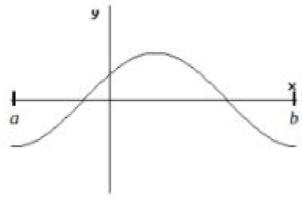
(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

1998 AB 87 Calc

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

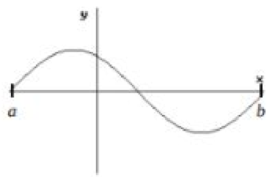
(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

The graph of f is shown below.

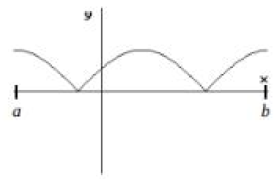


Which of the following could be the graph of the derivative of f?

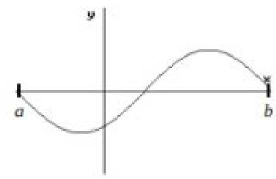
A.



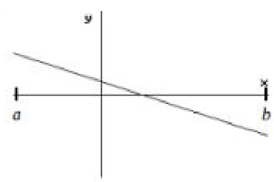
B.



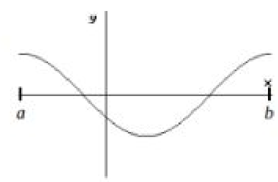
C.



D.

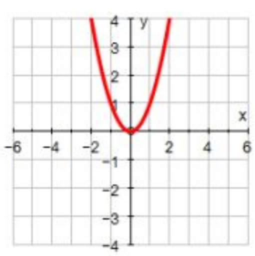


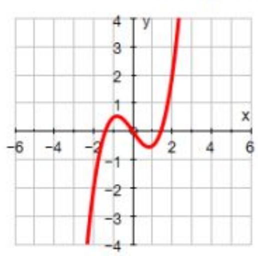
E.

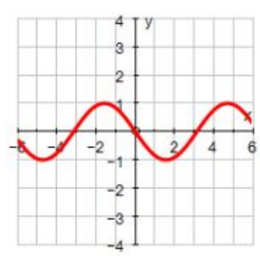


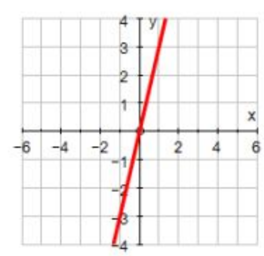
The graphs in the first row are the derivatives. Match them with the graph of their function shown in the second row.

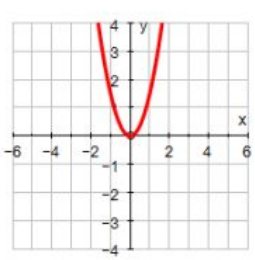
(Graphs of Derivative)

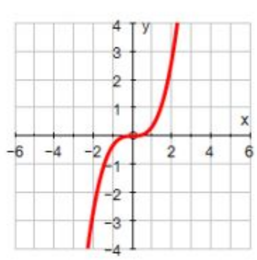


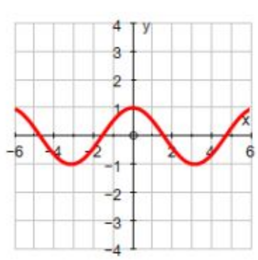


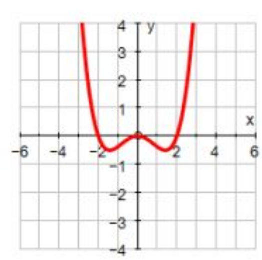












(Graphs of Function)