

Calculus
Chapter 3 Review

1. List the 2 limit-definition of derivatives.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{and} \quad \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

2. What are some other phrases that mean derivative?

instantaneous rate of change
slope
slope of tangent line

3. What is the product rule?

$$uv' + v u'$$

4. What is the quotient rule?

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2}$$

5. Name the three things that must be true for a function to be differentiable.

① $\lim_{x \rightarrow c} f(x)$ exists, ② $\lim_{x \rightarrow c} f(x) = f(c)$, ③ $\lim_{x \rightarrow c^-} f'(x) = \lim_{x \rightarrow c^+} f'(x)$
 $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} f(x)$

6. What are three types of functions that are continuous, but not differentiable at a particular point? Give an example of each type.

corner
 $y = |x|$

cusp
 $y = x^{2/3}$

vertical tangent
 $y = x^{1/3}$

7. Find the derivatives:

a. $y = \sin x$ $y' = \cos x$

b. $y = \cos x$ $y' = -\sin x$

c. $y = \tan x$ $y' = \sec^2 x$

d. $y = \csc x$ $y' = -\csc x \cot x$

e. $y = \sec x$ $y' = \sec x \tan x$

f. $y = \cot x$ $y' = -\csc^2 x$

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8. Find $\frac{dy}{dx}$ for each function:

a. $y = x^5 \tan x$

i. $5x^4 \tan x$ ii. $x^5 \sec^2 x$ iii. $5x^4 \sec^2 x$ iv. $5x^4 + \sec^2 x$

v. $5x^4 \tan x + x^5 \sec^2 x$

$$5x^4 \tan x + x^5 \sec^2 x$$

b. $y = \frac{2-x}{3x+1}$

i. $-\frac{7}{(3x+1)^2}$ ii. $\frac{6x-5}{(3x+1)^2}$ iii. $-\frac{9}{(3x+1)^2}$ iv. $\frac{7}{(3x+1)^2}$

v. $\frac{7-6x}{(3x+1)^2}$

$$y' = \frac{(3x+1)(-1) - (2-x)(3)}{(3x+1)^2}$$

$$= \frac{-3x-1 - 6 + 3x}{(3x+1)^2}$$

$$= \frac{-7}{(3x+1)^2}$$

c. $y = 3x^{\frac{2}{3}} - 4x^{\frac{1}{2}} - 2$

i. $2x^{\frac{1}{3}} - 2x^{-\frac{1}{2}}$ ii. $3x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$ iii. $\frac{9}{5}x^{\frac{5}{3}} - 8x^{\frac{3}{2}}$ iv. $\frac{2}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{2}}} - 2$

v. $2x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$

$$2x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$$

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d. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

i. $x + \frac{1}{x\sqrt{x}}$ ii. $x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$ iii. $\frac{4x-1}{4x\sqrt{x}}$

iv. $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ v. $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

$$2x^{\frac{y_2}{2}} - \frac{1}{2}x^{-\frac{y_2}{2}}$$

$$x^{-\frac{1}{2}} + \frac{1}{4}x^{-\frac{3}{2}}$$

** simplify*

Find $\frac{dy}{dx}$ of the following:

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

$$y = (-8x^3)(-2x^2 + 1) + (-2x^4 - 3)(-4x)$$

$$= 16x^5 - 8x^3 + 8x^5 + 12x$$

$$= \boxed{24x^5 - 8x^3 + 12x}$$

5) $f(x) = (5x^5 + 5)(-2x^3 - 3)$

$$f'(x) = (25x^4)(-2x^3 - 3) + (5x^3 + 5)(-10x^4)$$

$$= -50x^9 - 75x^4 - 50x^9 - 50x^4$$

$$= \boxed{-100x^9 - 125x^4}$$

3) $f(x) = \frac{5}{4x^3 + 4}$

$$f'(x) = \frac{(4x^3 + 4)(0) - 5(12x^2)}{(4x^3 + 4)^2}$$

$$= \frac{-60x^2}{(4x^3 + 4)^2}$$

5) $y = \frac{3x^4 + 2}{3x^3 - 2}$

$$y' = \frac{(3x^3 - 2)(12x^3) - (3x^4 + 2)(9x^2)}{(3x^3 - 2)^2}$$

$$= 36x^4 - 24x^3 - 27x^6 - 18x^2$$

$(3x^3 - 2)^2$

$$= \frac{9x^6 - 24x^3 - 18x^2}{(3x^3 - 2)^2}$$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

$$f'(x) = 8x^3(x^2 + 1) + (2x^4 - 3)(2x)$$

$$= 8x^5 + 8x^3 + 4x^7 - 6x$$

$$= \boxed{12x^5 + 8x^3 - 6x}$$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

$$f'(x) = -3x^{-4}(-4x^3 + 3) + (-3 + x^{-3})(-12x^2)$$

$$= 12x^{-1} - 9x^{-4} + 36x^2 - 12x^{-1}$$

$$= \boxed{-9x^{-4} + 36x^2}$$

4) $y = \frac{4x^3 - 3x^2}{4x^5 - 4}$

$$f'(x) = \frac{(4x^5 - 4)(12x^2 - 6x) - (4x^3 - 3x^2)(20x^4)}{(4x^5 - 4)^2}$$

$$= \frac{-32x^7 + 36x^6 - 48x^2 + 24x}{(4x^5 - 4)^2}$$

6) $y = \frac{4x^5 + 2x^2}{3x^4 + 5}$

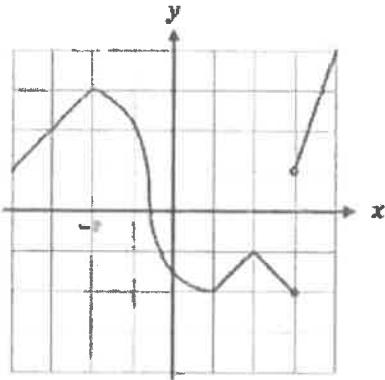
$$y' = \frac{(3x^4 + 5)(20x^4 + 4x) - (4x^3 + 2x^2)(12x^3)}{(3x^4 + 5)^2}$$

$$= \frac{60x^8 + 100x^4 + 12x^5 + 20x - 48x^6 - 24x^5}{(3x^4 + 5)^2}$$

$$= \frac{12x^8 - 12x^5 + 100x^4 + 20x}{(3x^4 + 5)^2}$$

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9. The function for $f(x)$ is graphed below. There is a vertical tangent line when $x = -\frac{1}{2}$. Where is $f(x)$ not differentiable? Why?



$x = -2, 1, 2$ corner

$x = -\frac{1}{2}$ vertical tangent line

$x = 3$ discontinuous

10. If $f(x)$ has a derivative at $x = 2$, tell whether or not each of the following must be true?

- $\lim_{x \rightarrow 2} f(x)$ exists **true**
- $f'(2)$ exists **true**
- $f''(2)$ exists **not necessarily**
- $f(x)$ is continuous at $x = 2$ **true**
- $\lim_{x \rightarrow 2} \frac{f(x)-f(2)}{x-2}$ exists **true**
- $\lim_{h \rightarrow 0} \frac{f(2+h)-f(2)}{h}$ exists **true**

2003 AB 25

11.

A particle moves along the x -axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest.

- $t = 1$ only
- $t = 3$ only
- $t = \frac{7}{2}$ only
- $t = 3$ and $t = \frac{7}{2}$
- $t = 3$ and $t = 4$

$$v(t) = 6t^2 - 42t + 72$$

$$0 = 6t^2 - 42t + 72$$

$$0 = 6(t^2 - 7t + 12)$$

$$= (t - 3)(t - 4)$$

$$t = 3 \text{ and } 4$$

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7. Find $\frac{dy}{dx}$ of the $y = x^5 \tan x$

- a. $5x^4 \tan x$
- b. $x^5 \sec^2 x$
- c. $5x^4 \sec^2 x$
- d. $5x^4 + \sec^2 x$
- e. $5x^4 \tan x + x^5 \sec^2 x$

8. Find $\frac{dy}{dx}$ of $y = \frac{x^2}{\cos x}$

- a. $\frac{2x}{\sin x}$
- b. $-\frac{2x}{\sin x}$
- c. $\frac{2x \cos x - x^2 \sin x}{\cos^2 x}$
- d. $\frac{2x \cos x + x^2 \sin x}{\cos^2 x}$

e. $\frac{2x \cos x + x^2 \sin x}{\sin^2 x}$

$$y' = \frac{\cos x(2x) - x^2(-\sin x)}{\cos^2 x}$$

$$f'(-3) = 2$$

9. If f is a function such that $\lim_{x \rightarrow -3} \frac{f(x) - f(-3)}{x + 3} = 2$, which of the following must be true?

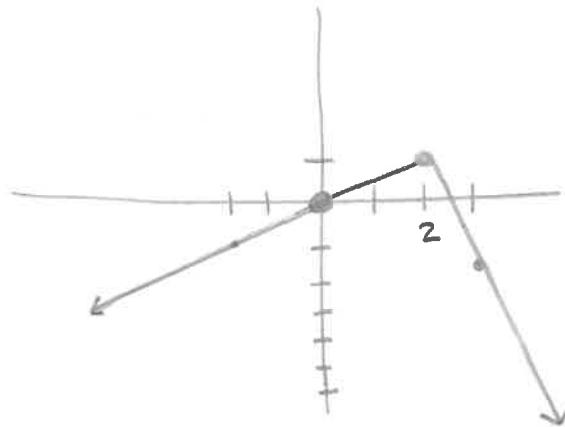
- a. The limit of $f(x)$ as x approaches -3 does not exist
- b. f is not defined at $x = -3$
- c. The derivative of f at $x = -3$ is 2 ✓
- d. f is continuous at $x = 2$
- e. $f(-3) = 2$

$\lim_{x \rightarrow -3} f(x) = f(-3)$ yes

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10. Sketch the graph of a continuous function f with $f(0) = 0$ and

$$f'(x) = \begin{cases} \frac{1}{2}, & x < 2 \\ -3, & x > 2 \end{cases} \quad \text{slope} \quad \star$$



10. If $f(x) = \begin{cases} 2ax^2 + b, & x \geq 1 \\ -3x + 4, & x < 1 \end{cases}$, find a and b so that f is both continuous and differentiable. (Be sure to use definitions to justify your work)

$$\lim_{x \rightarrow 1^-} f(x) = 2a + b$$

$$\lim_{x \rightarrow 1^-} f'(x) = 4a$$

$$2a + b = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

$$\lim_{x \rightarrow 1^+} f'(x) = -3$$

$$-\frac{6}{4} + b = 1$$

$$2a + b = 1$$

$$4a = -3$$

$$b = \frac{5}{2}$$

$$\lim_{h \rightarrow 0} \frac{(1+h)^6 - 1}{h}$$

$$a = -\frac{3}{4}$$

a. 0

b. 1

c. 6

d. ∞

e. Nonexistent

$$a = 1$$

$$f(x) = x^6$$

$$f'(x) = 6x^5$$

$$f'(1) = 6(1)^5$$

$$= 6$$

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$$f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} f = 5 \quad \lim_{x \rightarrow 3^+} f = 5$$

Let f be the function given above. Which of the following statements are true about f ? $f(3) = 5$

- I. $\lim_{x \rightarrow 3} f(x)$ exists. ✓
- II. f is continuous at $x = 3$. ✓
- III. f is differentiable at $x = 3$.

$$f'(x) = \begin{cases} 1 & x \leq 3 \\ 4 & x > 3 \end{cases}$$

- (A) None
- (B) I only
- (C) II only
- (D) I and II only**
- (E) I, II, and III

1998 AB 14

A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

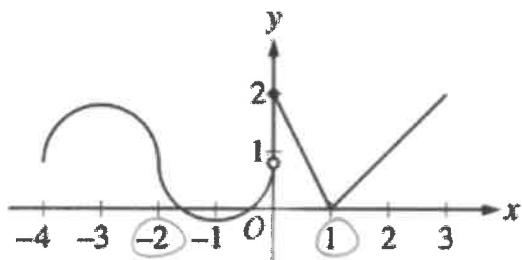
- (A) 1
- (B) 2
- (C) 3**
- (D) 4
- (E) 5

$$v(t) = 2t - 6$$

$$0 = 2t - 6$$

$$3 = t$$

1998 AB 3



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x , $-4 < x < 3$, at which f is continuous but not differentiable?

- (A) $x = 1$
- (B) $x = -2$ and $x = 0$
- (C) $x = -2$ and $x = 1$
- (D) $x = 0$ and $x = 1$

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Calc
1993 AB 41

A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by

$$s(t) = -4 \cos t - \frac{t^2}{2} + 10. \text{ What is the velocity of the particle when its acceleration is zero?}$$

- (A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

$$v(t) = 4 \sin t - t$$

$$a(t) = 4 \cos t - 1$$

$$v(a) = 4 \sin a - a$$

$$0 = 4 \cos t - 1$$

$$= 2.55487$$

$$\cos^{-1}\left(\frac{1}{4}\right) = t$$

$$1.31812 = t$$

store a

1997 AB 12

At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

- (A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

$$y' = x$$

$$2x - 4y = 3$$

$$x = \frac{1}{2}$$

$$-4y = -2x + 3$$

$$y = \frac{1}{2}\left(\frac{1}{2}\right)^2$$

$$y = \frac{1}{2}x + 3$$

$$= \frac{1}{2}\left(\frac{1}{4}\right)$$

$$m = \frac{1}{2}$$

$$= \frac{1}{8}$$

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2003 AB 16

If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

- (A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$$m = \frac{-2 - 7}{-2 - 1} = \frac{-9}{-3} = 3$$

1985 AB 23

$\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

- (A) -6 (B) -4 (C) 0 (D) 2 (E) 6

$$-3x^{-4} + x^{-2} + 2x$$

$$-3(-1)^{-4} + (-1)^{-2} + 2(-1)$$

$$\begin{matrix} -3 + 1 - 2 \\ -4 \end{matrix}$$

1988 AB 41

If $\lim_{x \rightarrow 3} f(x) = 7$, which of the following must be true?

- I. f is continuous at $x = 3$.
- II. f is differentiable at $x = 3$. could be corner or cusp
- III. $f(3) = 7$

- (A) None (B) II only (C) III only
 (D) I and III only (E) I, II, and III

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1997 AB 79 Calc

Let f be a function such that $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

- I. f is continuous at $x = 2$. ✓
 - II. f is differentiable at $x = 2$. ✓
 - III. The derivative of f is continuous at $x = 2$.
- (A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

$$\lim_{x \rightarrow 2^-} f'(x) = \lim_{x \rightarrow 2^+} f'(x)$$

but doesn't mean $f'(x)$
continuous at $x = 2$

1998 AB 10

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$$f'(x) = \frac{(x-1)(2x) - (x^2 - 2)(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2 + 2}{(x-1)^2}$$

$$f'(2) = \frac{2^2 - 2(2) + 2}{1} = 2$$

Calculus
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1998 AB 8

Let f and g be differentiable functions with the following properties:

- (i) $g(x) > 0$ for all x
- (ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

- (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

$f(x)$ must be a constant

$f(0) = 1$ means f can't be 0

1998 AB 87 Calc

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- (A) $y = 8x - 5$
 (B) $y = x + 7$
 (C) $y = x + 0.763$
 (D) $y = x - 0.122$
 (E) $y = x - 2.146$

$$f'(x) = 4x^3 + 4x \quad f(0.236733) =$$

$$f'(x) = 1$$

$$0.115226$$

$$1 = 4x^3 + 4x \quad y - 0.115 = x - 0.237$$

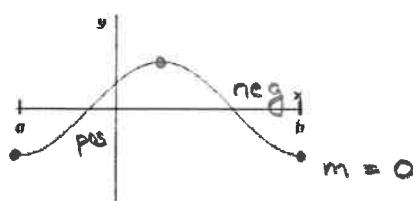
$$0 = 4x^3 + 4x - 1$$

$$y = x - 0.121$$

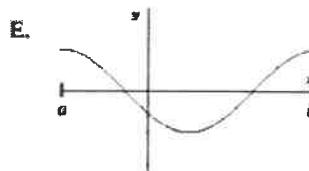
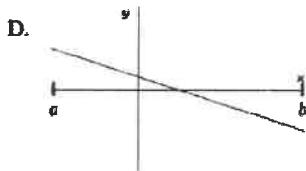
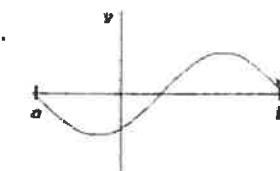
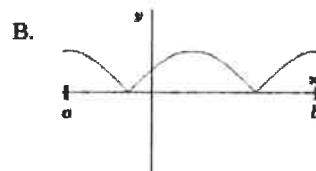
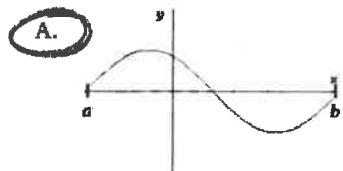
$$x = 0.236733$$

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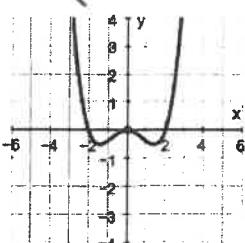
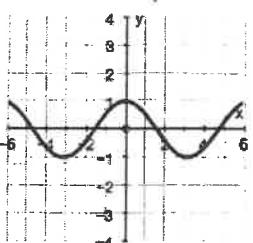
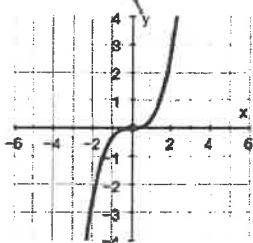
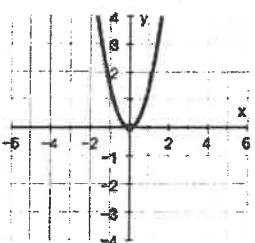
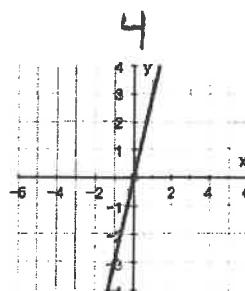
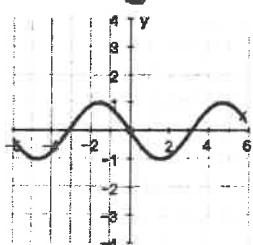
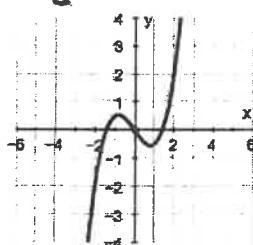
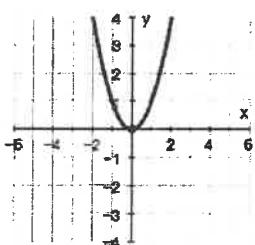
The graph of f is shown below.



Which of the following could be the graph of the derivative of f ?



The graphs in the first row are the derivatives. Match them with the graph of their function shown in the second row.



4

(Graphs of Function)

1

2

3

