1973 BC 7

If $y = \ln\left(x^2 + y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

$$\frac{dy}{dx} = \frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{1}{1^2+0^2}\left(2(1) + 2(0)\,dy/dx\right)$$

$$\frac{dy}{dx} = 1(2)$$

$$= 2$$

1985 AB 13

If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

$$2x + \left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x + 3y^2\right) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+3y^2}$$

1973 BC 18

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

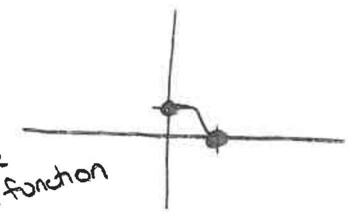
(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$. EVT

(B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$. true otherwise not a function

(C) There exists a number h in $[0,1]$ such that $g(h)=\frac{1}{2}$. ✓

(D) There exists a number h in $[0,1]$ such that $g(h)=\frac{3}{2}$. → not necessarily

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$. → ✓ cont def



1993 AB 25

$\frac{d}{dx}\left(2^x\right)=$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

2^1 2^2 2^3 2^4 2^5 2^6 2^7
2 4 8, 16 32 64 128

1969 AB 6

What is $\lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

$x = 1/2$ $8x^8 = f(x)$ $f'(x) = 64x^7$
$= 64(1/2)^7$
$= 64(1/2^7)$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

or $\frac{2^6}{2^7} = \frac{1}{2}$ $= \frac{64}{128} = \frac{1}{2}$

1988 AB 29

The $\lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

$f(x) = \tan 3x$

$f'(x) = 3\sec^2 3x$

1973 AB 36

If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

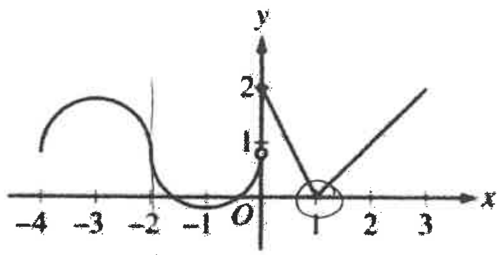
(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) ne^{nx} (D) $n^n e^x$ (E) $n!e^x$

$y' = e^{nx}(n)$

$y'' = n^2 e^{nx}$

$y''' = n^3 e^{nx}$

2008 BC 3



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

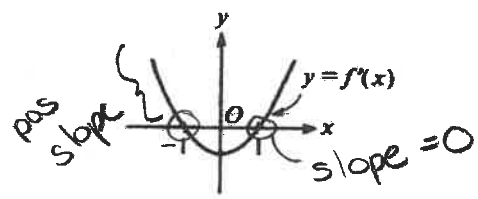
(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

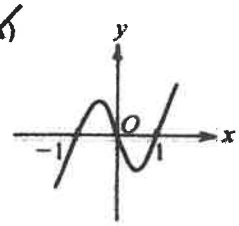
(D) $x = 0$ and $x = 1$

1985 AB 3

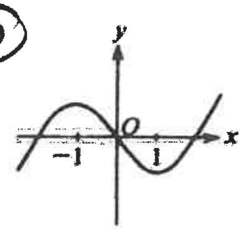


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

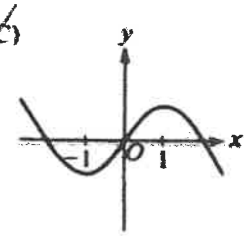
~~(A)~~



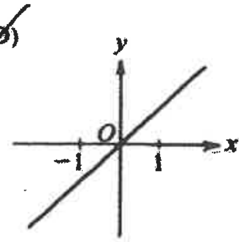
(B)



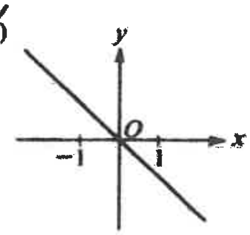
~~(C)~~



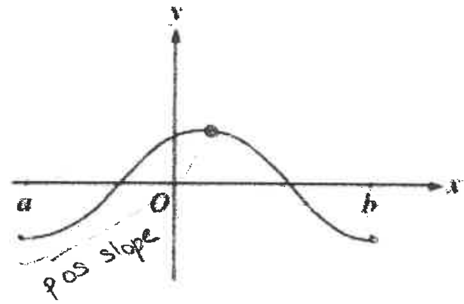
~~(D)~~



~~(E)~~

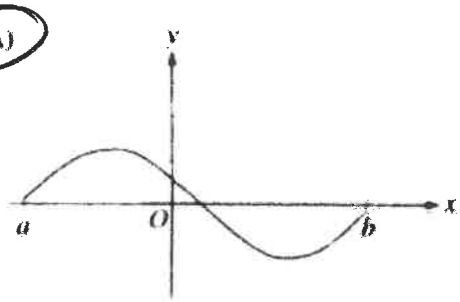


1998 AB 23

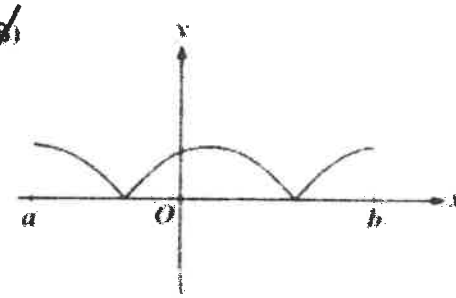


The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

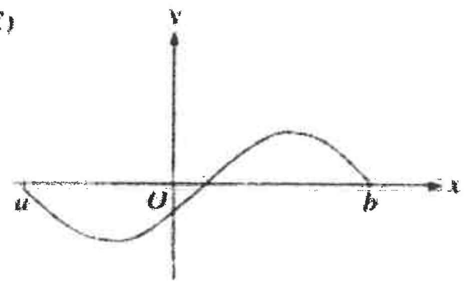
(A)



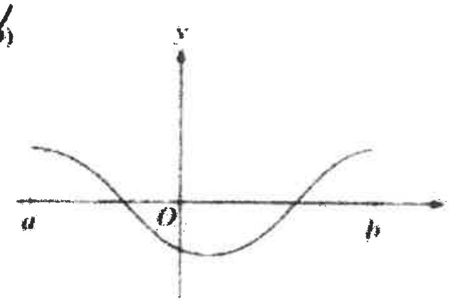
~~(B)~~



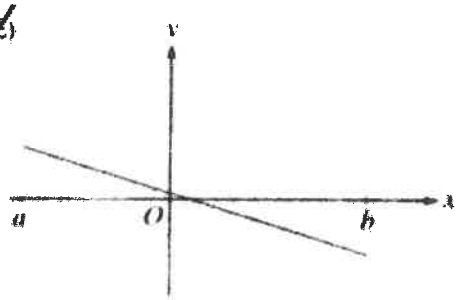
(C)



~~(D)~~



~~(E)~~



2008 AB/BC 6

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

$\sim x+2$

$\lim_{x\to 2^-} = 4 = \lim_{x\to 2^+}$

$f(2)=1 \neq \lim_{x\to 2} f(x)$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

~~II.~~ f is continuous at $x=2$. ↓ part of def

~~III.~~ f is differentiable at $x=2$. ↓ part of def

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

1969 BC 1

$x\to\pm\infty$ $t\to 0$ $y=0$

$y\to\pm\infty$ $t\to -1$ $x=-1$

The asymptotes of the graph of the parametric equations $x=\frac{1}{t}$, $y=\frac{t}{t+1}$ are go to 0

go to 0

(A) $x=0,\ y=0$
(B) $x=0$ only
(C) $x=-1,\ y=0$
(D) $x=-1$ only
(E) $x=0,\ y=1$

$$\frac{dx}{dt} = -1t^{-2} = \frac{-1}{t^2}$$

t^{-1}

$$\frac{dy}{dt} = \frac{(t+1)(1) - t(1)}{(t+1)^2} = \frac{1}{(t+1)^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{(t+1)^2}}{-\frac{1}{t^2}} = -\frac{1}{(t+1)^2}\cdot\frac{t^2}{1} = -\frac{t^2}{(t+1)^2}$$

no VA
HA @ -1

1969 BC 20

An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x-2y=0$ (B) $x-y=0$ (C) $x=0$

(D) $y=0$ (E) $\pi x-2y=0$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x/2)^2}}\left(\frac{1}{2}\right) \quad y'(0) = \frac{1}{\sqrt{1}}(1/2) = \frac{1}{2}$$

$y - 0 = 1/2(x-0)$

$y = 1/2 x$

$y - 1/2 x = 0$

$2y - x = 0$

or

$x - 2y = c$

1993 AB 16

The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

neg reciprocal of m

$\frac{1}{\sqrt{2}}$

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

$$y' = 2\frac{1}{\sec x}(\sec x \tan x)$$

$$y'(\pi/4) = 2\frac{1}{\sec \pi/4}(\sec \pi/4 \tan \pi/4)$$

$$= 2\frac{1}{\sqrt{2}}(\sqrt{2}(1))$$

$$= 2$$

$$m_{nor} = -\frac{1}{2}$$

1993 BC 17

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

$$\frac{1}{xy}\left(x\frac{dy}{dx} + y\right) = 1$$

$$x\frac{dy}{dx} + y = xy$$

$$x\frac{dy}{dx} = xy - y$$

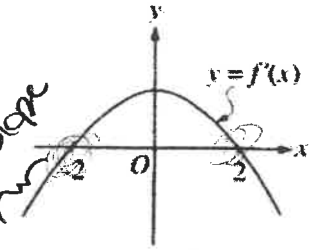
$$\frac{dy}{dx} = \frac{xy - y}{x}$$

$$y'(1) = y - y$$

$$= 0$$

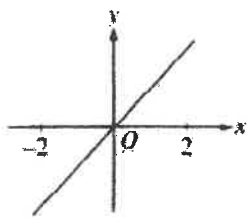
1997 AB 11

neg slope

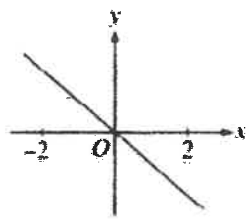


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

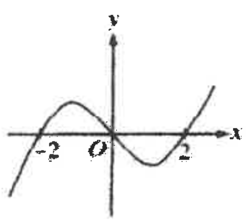
~~(A)~~



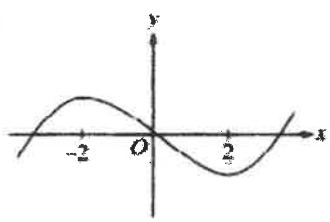
~~(B)~~



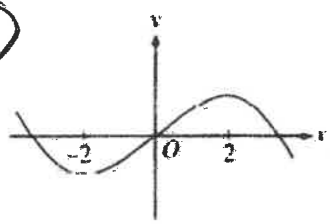
~~(C)~~



(D)



(E)



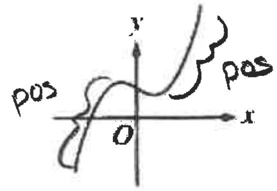
1998 BC 2

In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

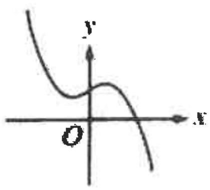
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3}{5}$$

1998 BC 6

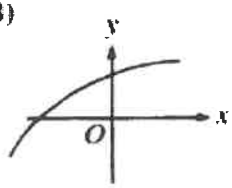


The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

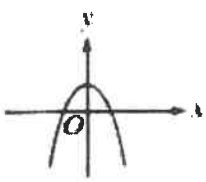
(A)



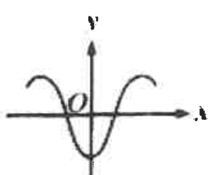
(B)



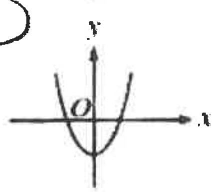
(C)



(D)



(E)



1998 BC 40

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(5)} = \frac{1}{-1/2} = -2$$

$\ln y = x \ln \sin x$

$y' \frac{1}{y} = \ln \sin x + \frac{x}{\sin x} \cos x$

$y' = y(\ln \sin x + x \cot x)$

1985 BC 26

For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx}$ is

$y' = (\sin x)^x (\ln \sin x + x \cot x)$

(A) $x \ln(\sin x)$

(B) $(\sin x)^x \cot x$

(C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x (x \cos x + \sin x)$

(E) $(\sin x)^x (x \cot x + \ln(\sin x))$

1. Find the values of a and b that will make $f(x)$ differentiable at $x = -1$.

$$f(x) = \begin{cases} ax^2 + bx - 3, & x < -1 \\ 2x^3 - 5, & x \geq -1 \end{cases}$$

$f'(x) = \begin{cases} 2ax + b \\ 6x^2 \end{cases}$

$\lim_{x \to -1^-} f(x) = a - b - 3$

$\lim_{x \to -1^+} f(x) = -2 - 5 = -7$

$a - b - 3 = -7$

$a - b = -4$

$\lim_{x \to -1^-} f'(x) = -2a + b$

$\lim_{x \to -1^+} f'(x) = 6$

$-2a + b = 6$

$a - b = -4$
$-2a + b = 6$

$-a = 2$

$\boxed{a = -2 \quad b = 2}$

2. Write an equation for the tangent line to $y = x \cos x$ at $x = \frac{\pi}{2}$.

$y = \frac{\pi}{2} \cos \frac{\pi}{2} = 0$

$y' = \cos x - x \sin x$

$y'|_{\pi/2} = \cos \frac{\pi}{2} - \frac{\pi}{2} \sin \frac{\pi}{2}$

$= -\frac{\pi}{2}$

$y = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right)$

$\boxed{y = -\frac{\pi}{2} x + \frac{\pi^2}{4}}$

3. Write an equation for the **normal** line at $x = 0$ to $y = 2 + e^{-2x}$.

$y' = -2e^{-2x}$

$y'(0) = -2e^0$

$= -2$

$\perp m = \frac{1}{2}$

$y(0) = 2 + e^0$

$= 3$

$y - 3 = \frac{1}{2} x$

$\boxed{y = \frac{1}{2} x + 3}$

4. If the line $y = 4x - 18$ is tangent to the curve $y = ax^2 + bx$ at the point $(3,-6)$, then find a and b.

$\frac{dy}{dx}\Big|_3 = 4$

$y = ax^2 + bx$

$y' = 2ax + b$

$4 = 2a(3) + b$

$4 = 6a + b$

$-6 = a(3)^2 + b(3)$

$-6 = 9a + 3b$

$(4 = 6a + b) \cdot -3$

$-6 = 9a + 3b$

$-12 = -18a - 3b$

$-6 = 9a + 3b$

$-18 = -9a$

$2 = a$

$4 = 6(2) + b$

$-8 = b$

$\boxed{y = 2x^2 - 8x}$

Check

$y' = 4x - 8\Big|_3$

$= 12 - 8$

$= 4$

$y + 6 = 4(x - 3)$

$y = 4x - 18$ ✓

5. Find $y = ax^2 + bx + c$ such that $f(0) = 5$, $f'(0) = 6$, *and* $f''(0) = -3$.

$f(0) = c = 5$

$f' = 2ax + b$

$f'(0) = b = 6$

$f''(x) = 2a$

$f''(0) = 2a = -3$

$a = -3/2$

$\boxed{y = -\frac{3}{2}x^2 + 6x + 5}$

6. The position (in meters) of an object at any time t (in minutes) is given by the function $s(t) = 3t^2 - \cos 2t$.
   a. Find the velocity of the object at time $t = \pi$ using appropriate units.

$s'(t) = 6t - (-\sin 2t)(2)$

$s'(\pi) = 6t + 2\sin 2t \,\Big|_{\pi}$

$= 6\pi + 2\sin 2\pi$ 0

$= 6\pi$ m/min

   b. Find the acceleration of the object at time $t = \pi$ using appropriate units.

$s''(t) = 6 + 4\cos 2t \,\Big|_{\pi}$

$s''(\pi) = 6 + 4\cos 2\pi$

$= 6 + 4$

$= 10$ m/min^2

7. Use the table of values below representing the position of an object at the given times.

t (sec)	1	2	3	4	5
$s(t)$ (cm)	2.3	5.6	6.2	6.4	4.8

   a. Find the average velocity of the object between times $t = 1$ and $t = 4$. Show your computation.

$$\frac{s(4) - s(1)}{4 - 1} = \frac{6.4 - 2.3}{3} = 1.366 \text{ cm/sec}$$

   b. Find an estimate for the velocity of the object at $t = 3$.

$$\frac{s(4) - s(2)}{4 - 2} = \frac{6.4 - 5.6}{2}$$

$= 0.4$ cm/sec

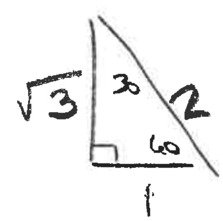
8. Find $\frac{d^2y}{dx^2}$, for the function $y = 2x^4 - 5\sqrt{x}$.

$x^{1/2}$

$$y' = 8x^3 - 5(1/2)x^{-1/2}$$
$$= 8x^3 - 5/2\, x^{-1/2}$$
$$y'' = 24x^2 + 5/4\, x^{-3/2}$$
$$\boxed{= 24x^2 + \frac{5}{4x^{3/2}}}$$

9. Find $\lim_{h \to 0} \frac{\cos(\frac{\pi}{3}+h)-\frac{1}{2}}{h}$

$x = \pi/3$



60°

$$f(x) = \cos x$$
$$f'(x) = -\sin x \Big|_{\pi/3}$$
$$f'(\pi/3) \Big| \boxed{= -\frac{\sqrt{3}}{2}}$$

10. Find $\lim_{h \to 0} \frac{3(2+h)^3-24}{h}$

$$f(x) = 3x^3 \qquad x = 2$$
$$f'(x) = 9x^2 \Big|_2$$
$$f'(2) = 9(4)$$
$$\boxed{= 36}$$

11. Use the table below to find the specified derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	$\frac{1}{3}$	-2	-3
2	3	$\frac{1}{2}$	4	0
3	1	-2	5	-1

a. If $h(x) = f(x) * g(x)$, find $h'(2)$

$$h'(x) = f'(x)g(x) + g'(x)f(x)$$

$$h'(2) = f'(2)g(2) + g'(2)f(2)$$

$$= 1/2\,(4) + 0(3)$$

$$= 2$$

b. If $h(x) = \frac{f(x)}{g(x)}$, find $h'(3)$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

$$h'(3) = \frac{5(-2) - 1(-1)}{5^2} = \frac{-10+1}{25}$$

$$= -9/25$$

c. If $h(x) = x^3 * g(x)$, find $h'(1)$

$$h'(x) = 3x^2 g(x) + x^3 g'(x)$$

$$h'(1) = 3(1)^2 g(1) + 1^3 g'(1)$$

$$= 3(-2) + 1(-3)$$

$$= -6 - 3$$

$$= -9$$

(number 11 continued)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	$\frac{1}{3}$	-2	-3
2	3	$\frac{1}{2}$	4	0
3	1	-2	5	-1

d. If $h(x) = \frac{2f(x)}{x^3}$, find $h'(2)$

$$h'(x) = \frac{x^3(2f'(x)) - 2f(x)3x^2}{x^6}$$

$$h'(2) = \frac{8(2(1/2)) - 2(3)3(4)}{2^6} = \frac{8-72}{64}$$

$$= \frac{-64}{64} = -1$$

e. If $h(x) = g(f(x))$, find $h'(3)$

$$h'(x) = g'(f(x))f'(x)$$

$$h'(3) = g'(f(3))f'(3)$$

$$= g'(1)(-2)$$

$$= (-3)(-2) = 6$$

f. If $h(x) = f(x^2)$, find $h'(1)$

$$h'(x) = f'(x^2)(2x)$$

$$h'(1) = f'(1)(2)$$

$$= \frac{2}{3}$$

g. If $h(x)$ is the inverse of $f(x)$, find $h'(1)$

$x=1$ in der of inverse so y of original function $=1$

$$h(x) = f^{-1}(x)$$

$$f(x) = 1$$

$$\Rightarrow x = 3$$

$$h'(1) = \frac{1}{f'(3)}$$

$$= \frac{1}{-2}$$

$$\boxed{= -1/2}$$

12. Find the 78^{th} derivative of $f(x) = 3^x$

$f'(x) = 3^x \ln 3$

$f''(x) = 3^x(\ln 3)^2 + 3^x(0)$

$= 3^x (\ln 3)^2$

$f'''(x) = 3^x (\ln 3)^3$

$f^{78}(x) = 3^x (\ln 3)^{78}$

13. Find the 95^{th} derivative of $f(x) = \sin(3x)$

$f'(x) = 3\cos 3x$

$f''(x) = -9\sin 3x$

$f^3(x) = -27\cos 3x$

$f^4(x) = 3^4 \sin 3x$

$f^{95} = -3^{95} \cos 3x$

14. Find the derivative of the function $f(x) = \tan^{-1}(3x^2)$

$f'(x) = \dfrac{6x}{1+9x^4}$

15. Find the derivative of $f(x) = \sin^{-1}(\cos(3x))$

$f'(x) = \dfrac{1}{\sqrt{1-(\cos 3x)^2}} (-\sin 3x)3 = \dfrac{-3\sin 3x}{\sqrt{1-\cos^2(3x)}}$

16. Find the derivative of the inverse of the function $f(x) = 3x^5 - 2x^3 - 4$ at $x = -5$.

$-5 = 3x^5 - 2x^3 - 4$

$0 = 3x^5 - 2x^3 + 1$

calc $\to x = -1$

$3+2-4$
-5

$f'(x) = 15x^4 - 6x^2$

$f'(-1) = 15 - 6 = 9$

$(f^{-1})'(5) = \dfrac{1}{f'(-1)} = \dfrac{1}{9}$

17. Find the derivative of the function $y = x^{\cos x}$.

* get $\cos x$ out of exponent

$\ln y = \ln x^{\cos x}$

$\ln y = \cos x \ln x$

$\dfrac{dy}{dx}\dfrac{1}{y} = -\sin x \ln x + \dfrac{\cos x}{x}$

$\dfrac{dy}{dx} = y\left(-\sin x \ln x + \dfrac{\cos x}{x}\right)$

$\dfrac{dy}{dx} = x^{\cos x}\left(-\sin x \ln x + \dfrac{\cos x}{x}\right)$

18. Which of the following are asymptotes of $2y + xy - x + 3 = 0$

I. $x = 3$

II. $x = -2$

III. $y = 1$

a. I only
b. III only
c. I and II only
d. II and III only
e. I, II, and III

$y = \frac{x-3}{2+x}$

$x = -2$ VA

$y = 1$ HA