

AB Calculus
Chapter 7 Review

$$\int x^2 \cos(x^3) dx =$$

(A) $-\frac{1}{3} \sin(x^3) + C$

(B) $\frac{1}{3} \sin(x^3) + C$

(C) $-\frac{x^3}{3} \sin(x^3) + C$

(D) $\frac{x^3}{3} \sin(x^3) + C$

(E) $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

2003 AB 8

$$\frac{1}{3} \int \cos u du$$

$$\frac{1}{3} \sin x^3 + C$$

$$\int_0^1 e^{-4x} dx =$$

2003 AB 2

(A) $-\frac{e^{-4}}{4}$

(B) $-4e^{-4}$

(C) $e^{-4} - 1$

(D) $\frac{1}{4} - \frac{e^{-4}}{4}$

(E) $4 - 4e^{-4}$

$$u = -4x$$

$$du = -4 dx$$

$$-\frac{1}{4} \int_0^{-4} e^u du$$

$$\rightarrow \frac{1}{4} - \frac{1}{4e^4}$$

$$-\frac{1}{4} du = dx$$

$$= -\frac{1}{4} \int_{-4}^0 e^u du = \frac{1}{4} e^0 - \frac{1}{4} e^{-4}$$

What is the average value of $y = x^2 \sqrt{x^3 + 1}$ on the interval $[0, 2]$?

1998 AB 27

1985 AB 44

(different
answer
options)

(A) $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

(E) 24

$$\frac{1}{2-0} \int_0^2 x^2 \sqrt{x^3 + 1} dx$$

$$u = x^3 + 1$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{6} \int_1^9 \sqrt{u} du$$

$$\frac{1}{6} \cdot \frac{2}{3} u^{3/2} \Big|_1^9$$

$$\left. \frac{1}{9} (27) - \frac{1}{9} \right. \frac{26}{9}$$

$$\int \tan(2x) dx = \int \frac{\sin(2x)}{\cos(2x)} dx$$

1985 AB 30

(A) $-2 \ln |\cos(2x)| + C$

(B) $-\frac{1}{2} \ln |\cos(2x)| + C$

(C) $\frac{1}{2} \ln |\cos(2x)| + C$

(D) $2 \ln |\cos(2x)| + C$

(E) $\frac{1}{2} \sec(2x) \tan(2x) + C$

$$u = \cos 2x$$

$$du = -\sin 2x \cdot (2) dx$$

$$-\frac{1}{2} du = \sin 2x dx$$

$$-\frac{1}{2} \int \frac{1}{u} du$$

$$-\frac{1}{2} \ln |\cos 2x| + C$$

$$u = 3x$$

$$du = 3dx$$

$$\int_0^{\frac{\pi}{3}} \sin(3x) dx =$$

$$\frac{1}{3} du = dx$$

$$\frac{1}{3} \int_0^{\pi} \sin u du$$

$$-\frac{1}{3} \cos \pi + \frac{1}{3} \cos 0$$

AB 32

(A) -2

(B) $-\frac{2}{3}$

(C) 0

(D) $\frac{2}{3}$

(E) 2

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

U-sub doesn't work

1973 AB 30

$$\int_1^2 \frac{x-4}{x^2} dx =$$

(A) $-\frac{1}{2}$

(B) $\ln 2 - 2$

(C) $\ln 2$

(D) 2

(E) $\ln 2 + 2$

$$\int_1^2 \left(\frac{x}{x^2} - \frac{4}{x^2} \right) dx$$

$$= \int_1^2 (x^{-1} - 4x^{-2}) dx$$

$$= \ln 2 + 2 - 0 - 4$$

$$= \ln 2 + 4x^{-1} \Big|_1^2$$

$$= \ln 2 - 2$$

$$= \ln 2 + 4 \cdot \frac{1}{2} - (\ln 1 + 4)$$

$$\int \frac{x^2}{e^{x^3}} dx =$$

$u = x^3$
 $du = 3x^2 dx$
 $\frac{1}{3} du = x^2 dx$

(A) $-\frac{1}{3} \ln e^{x^3} + C$

(B) $-\frac{e^{x^3}}{3} + C$

(C) $-\frac{1}{3e^{x^3}} + C$

$\frac{1}{3} \int e^{-u} du$

1969 AB 38

(D) $\frac{1}{3} \ln e^{x^3} + C$

(E) $\frac{x^3}{3e^{x^3}} + C$

$-\frac{1}{3} e^{-x^3} + C$

$\int (x^2 + 1)^2 dx =$

1993 AB 17 Calc

(A) $\frac{(x^2 + 1)^3}{3} + C$

$\int (x^4 + 2x^2 + 1) dx$

(B) $\frac{(x^2 + 1)^3}{6x} + C$

$\frac{1}{5} x^5 + \frac{2}{3} x^3 + x + C$

(C) $\left(\frac{x^3}{3} + x \right)^2 + C$

(D) $\frac{2x(x^2 + 1)^3}{3} + C$

(E) $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

1969 AB 29

(A) $\ln \sqrt{2}$

(B) $\ln \frac{\pi}{4}$

(C) $\ln \sqrt{3}$

(D) $\ln \frac{\sqrt{3}}{2}$

(E) $\ln e$

$u = \sin x$

$du = \cos x dx$

$\int_{\pi/2}^1 \frac{1}{u} du$

$\ln|1| - \ln|\frac{\sqrt{2}}{2}|$

$\ln \frac{2}{\sqrt{2}}$

$\ln \frac{2\sqrt{2}}{2} = \ln \sqrt{2}$

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$$

1993 AB 32
Calc

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{1}{2} \ln 2$

(E) $-\ln 2$

$$\int_0^{\sqrt{3}} \frac{1}{2\sqrt{1-(\frac{1}{2}x)^2}} dx$$

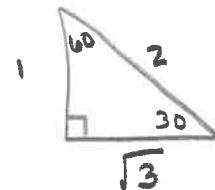
$$u = \frac{1}{2}x$$

$$du = \frac{1}{2}dx$$

$$2du = dx$$

$$2 \cdot \frac{1}{2} \int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}} du$$

$$\sin^{-1} u \Big|_0^{\sqrt{3}/2}$$



$$\sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} 0$$

$$\frac{\pi}{3} - (0)$$

$$\frac{\pi}{3}$$

1988 AB 7

$$\int \frac{x dx}{\sqrt{3x^2+5}} =$$

(A) $\frac{1}{9}(3x^2+5)^{\frac{3}{2}} + C$

(B) $\frac{1}{4}(3x^2+5)^{\frac{3}{2}} + C$

(C) $\frac{1}{12}(3x^2+5)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}(3x^2+5)^{\frac{1}{2}} + C$

(E) $\frac{3}{2}(3x^2+5)^{\frac{1}{2}} + C$

$$u = 3x^2 + 5$$

$$du = 6x dx$$

$$\frac{1}{6} du = x dx$$

$$\frac{1}{6} \int \frac{1}{\sqrt{u}} du$$

$$\frac{1}{6} \cdot 2 u^{\frac{1}{2}} + C$$

$$\frac{1}{3} \sqrt{3x^2+5} + C$$

$$\int_0^8 \frac{dx}{\sqrt{1+x}} =$$

1969 AB 4

(A) 1 (B) $\frac{3}{2}$

(C) 2

(D) 4

(E) 6

$$u = 1 + x$$

$$du = dx$$

$$\int_1^9 u^{-\frac{1}{2}} du$$

$$2u^{\frac{1}{2}} \Big|_1^9$$

$$2\sqrt{9} - 2\sqrt{1}$$

$$\frac{6 - 2}{4}$$

If $\frac{dy}{dx} = \cos(2x)$, then $y =$

1985 AB 4

(A) $-\frac{1}{2}\cos(2x) + C$

(B) $-\frac{1}{2}\cos^2(2x) + C$

(C) $\frac{1}{2}\sin(2x) + C$

(D) $\frac{1}{2}\sin^2(2x) + C$

(E) $-\frac{1}{2}\sin(2x) + C$

$$\int dy = \int \cos 2x \, dx$$

$$u = 2x$$
$$du = 2dx$$

$$y = \frac{1}{2} \int \cos u \, du$$

$$\frac{1}{2} du = dx$$

$$y = \frac{1}{2} \sin 2x + C$$

Which of the following are antiderivatives of $f(x) = \sin x \cos x$? 1997 AB 90 calc

I. $F(x) = \frac{\sin^2 x}{2}$ ✓

$$\int \sin x \cos x \, dx$$

II. $F(x) = \frac{\cos^2 x}{2}$

$$u = \cos x$$

$$u = \sin x$$

III. $F(x) = \frac{-\cos(2x)}{4}$

$$du = -\sin x \, dx$$

$$du = \cos x \, dx$$

(A) I only

$$-\int u \, du$$

$$\int u \, du$$

(B) II only

$$-\frac{u^2}{2} + C$$

$$\frac{u^2}{2} + C$$

(C) III only

$$-\frac{\cos^2 x}{2}$$

$$\frac{\sin^2 x}{2} + C$$

(D) I and III only

(E) II and III only

$$\frac{d}{dx} \left(F(x) = \frac{-\cos(2x)}{4} \right)$$

$$F'(x) = +\frac{2 \sin(2x)}{4} = \frac{1}{2} \sin(2x) = \sin x \cos x$$

1988 AB 19

$$\int_2^3 \frac{x}{x^2 + 1} \, dx =$$

(A) $\frac{1}{2} \ln \frac{3}{2}$

(B) $\frac{1}{2} \ln 2$

(C) $\ln 2$

(D) $2 \ln 2$

(E) $\frac{1}{2} \ln 5$

$$u = x^2 + 1$$

$$\frac{1}{2} \int_5^{\infty} \frac{1}{u} \, du$$

$$du = 2x \, dx$$

$$\frac{1}{2} \, du = x \, dx$$

$$\frac{1}{2} \ln 10 - \frac{1}{2} \ln 5$$

$$\frac{1}{2} \ln \frac{10}{5}$$

$$\frac{1}{2} \ln 2$$

A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

2003 AB 19

- (A) $y = 5x - 3$
- (B) $y = x^2 + 1$
- (C) $y = x^2 + 3x$
- (D) $y = x^2 + 3x - 2$
- (E) $y = 2x^2 + 3x - 3$

$$\int (2x+3)dx$$

$$y = x^2 + 3x + C$$

$$y = x^2 + 3x - 2$$

$$2 = 1 + 3 + C$$

$$-2 = C$$

If $\frac{dy}{dx} = \tan x$, then $y =$

$$\int \frac{\sin x}{\cos x} dx$$

1969 AB 27

- (A) $\frac{1}{2} \tan^2 x + C$
- (B) $\sec^2 x + C$
- (C) $\ln |\sec x| + C$
- (D) $\ln |\cos x| + C$
- (E) $\sec x \tan x + C$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-\int \frac{1}{u} du$$

$$-\ln |\cos x| + C$$

$$\ln |\sec x| + C$$

If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

1973 AB 37

- (A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

$$\int \frac{1}{y} dy = 4 dx$$

$$|\ln y| = 4x + C$$

$$\ln 4 = 4(0) + C$$

$$\ln 4 = C$$

$$|\ln y| = 4x + \ln 4$$

$$y = e^{4x + \ln 4} = 4e^{4x}$$

If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

1998 AB 21

- (A) $2e^{kt}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kt + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

2008 AB 22

A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t , where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

$$\frac{dp}{dt} = k p (N - p)$$

(B) $\frac{dp}{dt} = kp(N - p)$

(C) $\frac{dp}{dt} = kp(p - N)$

(D) $\frac{dp}{dt} = kt(N - t)$

(E) $\frac{dp}{dt} = kt(t - N)$

$N = \text{total pop}$

as rumor spreads gets smaller
b/c people hear rumor

$\Rightarrow N - p$

Separable Differential Equations Practice

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Find the general solution of each differential equation.

1) $\frac{dy}{dx} = \frac{x^3}{y^2}$ $\int y^2 dy = \int x^3 dx$

$$\frac{1}{3}y^3 = \frac{1}{4}x^4 + C$$

$$y = \sqrt[3]{\frac{3}{4}x^4 + C}$$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$$\int \sec^2 y dy = \int dx$$

$$\tan y = x + C$$

$$y = \tan^{-1}(x + C)$$

3) $\frac{dy}{dx} = 3e^{x-y}$

4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$ $\int e^{2y} dy = \int 2x dx$

$$\int e^y dy = \int 3e^x dx$$

$$\frac{1}{2}e^{2y} = x^2 + C$$

$$e^y = 3e^x + C \quad y = \ln(3e^x + C)$$

$$y = \frac{1}{2}\ln(2x^2 + C)$$

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

5) $\frac{dy}{dx} = \frac{2x}{y^2}, y(2) = \sqrt[3]{13}$

$$\frac{1}{3}y^3 = 2^2 + C$$

$$\int y^2 dy = \int 2x dx$$

6) $\frac{dy}{dx} = 2e^{x-y}, y(-3) = \ln \frac{3e^3 + 2}{e^3}$

$$\frac{1}{3}y^3 = x^2 + C$$

$$\frac{13}{3} - 4 = C$$

$$\frac{1}{3} = C$$

7) $\frac{dy}{dx} = \frac{1}{\sec^2 y}, y(3) = 0$

$$\frac{1}{3}y^3 = x^2 + \frac{1}{3}$$

$$y = \sqrt[3]{3x^2 + 1}$$

8) $\frac{dy}{dx} = \frac{e^x}{y^2}, y(-1) = \frac{\sqrt[3]{e^3 + 3e^2}}{e}$

9) $\frac{dy}{dx} = -\frac{1}{\sin y}, y(3) = \frac{\pi}{2}$

10) $\frac{dy}{dx} = \frac{2x}{e^{2y}}, y(2) = \frac{\ln 5}{2}$

$$\frac{1}{2}e^{2y} = x^2 + C$$

$$\frac{1}{2}e^{2(\ln 5/2)} = 2^2 + C$$

$$\frac{1}{2}(5) = 4 + C$$

$$\frac{5}{2} - 4 = C$$

11) $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}, y(1) = 0$

12) $\frac{dy}{dx} = \frac{1+x^2}{y^2}, y(-1) = -\sqrt[3]{4}$ $-\frac{3}{2} = C$

$$\frac{1}{2}e^{2y} = x^2 - 3/2$$

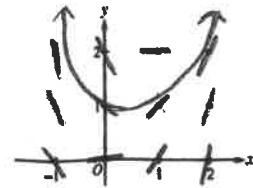
$$y = \frac{1}{2}\ln(2x^2 - 3)$$

On BC Test mean score was 4.68 pts

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2005 SCORING GUIDELINES**

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.



- 3 (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$. (Note: Use the axes provided in the pink test booklet.)
- 2 (b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y -coordinate of this local minimum?
- 2 (c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.
- 2 (d) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

b) local min $\Rightarrow \frac{dy}{dx} = 0$

- a) 1: zero slopes
1: nonzero slopes
1: curve through $(0, 1)$

$$0 = 2x - y$$

$$2x = y$$

$$2 \ln\left(\frac{3}{2}\right) = y$$

The y -coordinate is
 $2 \ln\left(\frac{3}{2}\right)$

- b) 1: sets $\frac{dy}{dx} = 0$
1: answer

(x, y)	$\frac{dy}{dx}$	Δx	Δy	$(x + \Delta x, y + \Delta y)$
$(0, 1)$	-1	-0.2	0.2	$(-0.2, 1.2)$
$(-0.2, 1.2)$	$\frac{-0.4 - 1.2}{-0.2} = -1.6$	-0.2	0.32	$(-0.4, 1.52)$

- c) 1: Euler's method w/
2 steps

1: Euler's approximation
to $f(-0.4)$

d) 1: $\frac{d^2y}{dx^2}$

1: answer w/ reason

$$\begin{aligned} \frac{d^2y}{dx^2} &= 2 - \frac{dy}{dx} \\ &= 2 - (2x - y) \\ &= 2 - 2x + y \end{aligned}$$

*only care about quad II
in quad II b/c $(-0.4, f(-0.4))$ is in quad II

$$\frac{d^2y}{dx^2} \text{ is positive b/c}$$

$$x < 0 \text{ and } y > 0$$

since f is concave up $1.52 < f(-0.4)$

