BC Calculus

Chapter 8 Review

At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

1969 BC 35

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

$a(t) = 24t^2$

$v(t) = 8t^3 + C$

$v(0) = 0 = 8(0)^3 + C$

$0 = C$

$\int_0^2 (8t^3)\,dt$

$= 2t^4 \Big|_0^2$

$= 2(2)^4$

$= 32$

2008 BC 77

CALC

Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

$\int_0^5 R(t)\,dt$

(A) 9.439 cubic meters

(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

2003 BC 8 2
Calc

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

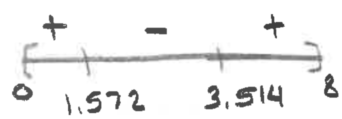
(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

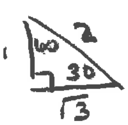
$0 = r(t)$

$t = 1.572, \ 3.514$



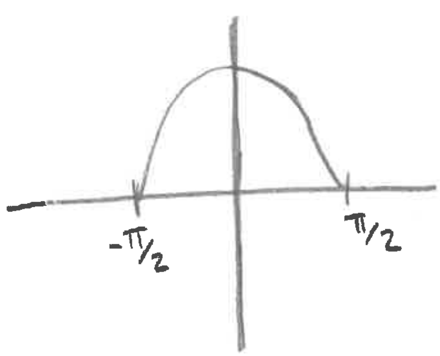
The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

1969 BC
13



(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

$30° \to \frac{\pi}{6}$



$\int_{-\pi/2}^{k} \cos x\,dx = 3\int_{k}^{\pi/2} \cos x\,dx$

$\sin x \Big|_{-\pi/2}^{k} = 3\sin x \Big|_{k}^{\pi/2}$

$\sin k - \sin(-\pi/2) = 3\sin \pi/2 - 3\sin k$

$4\sin k + 1 = 3$

$\sin k = 1/2$

$k = \pi/6$

Let R be the region enclosed by the graph of $y = 1 + \ln\left(\cos^4 x\right)$, the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

1998 BC 30
calc

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$$\int_{-2/3}^{2/3} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$$

A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

1969 BC 25

(A) is independent of m.

(B) increases as m increases.

(C) decreases as m increases.

(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.

(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

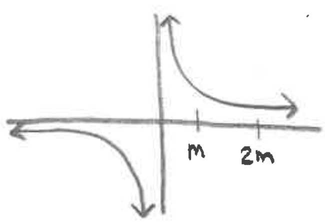
$$\boxed{\int_m^{2m} \frac{1}{x}\, dx = \ln x \Big|_m^{2m} = \ln 2m - \ln m = \ln \frac{2m}{m} = \ln 2}$$

ind. of m!

when $m > \frac{1}{2}$ interval inc.

$m = \frac{3}{4}$ $\quad$ $m = 4$

$[\frac{3}{4}, \frac{3}{2}]$ $\quad$ $[4, 8]$



$m > 0$

when $m < \frac{1}{2}$ interval decreases

$m = \frac{1}{4}$ $\quad$ $m = \frac{1}{8}$

$[\frac{1}{4}, \frac{1}{2}]$ $\quad$ $[\frac{1}{8}, \frac{1}{4}]$

The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

1973 BC
15

(A) $\frac{e-1}{2}$ (B) $e - 1$ (C) $2(e-1)$ (D) $2e - 1$ (E) $2e$

$$\int_0^2 e^{x/2}\, dx$$

$$= 2e^{x/2} \Big|_0^2$$

$$= 2e^1 - 2e^0$$

$$= 2e - 2$$

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

1987 BC 87 Calc

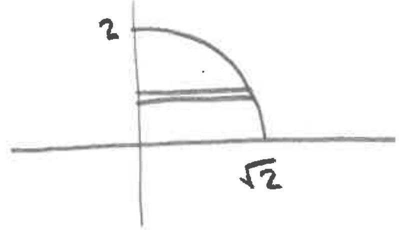
(A) $\pi\int_0^2 (2-y)^2\,dy$

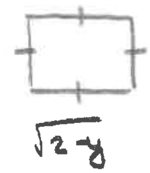
(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$





$y = 2 - x^2$

$y - 2 = -x^2$

$2 - y = x^2$

$\pm\sqrt{2-y} = x$

$\sqrt{2-y} = x$

$A = s^2 = \left(\sqrt{2-y}\right)^2$

$\int_0^2 \left(\sqrt{2-y}\right)^2 dy$

$\int_0^2 (2-y)\,dy$

The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

1993 BC 1 Calc

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

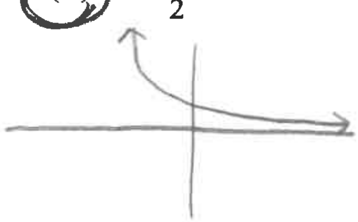
$\int_0^1 (x - x^2)\,dx$

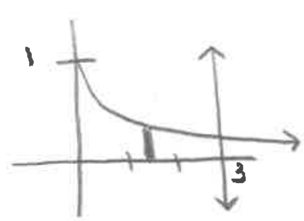
$= \frac{1}{6}$

The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

1985 BC 38

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$





$s = e^{-x}$

$A = \left(e^{-x}\right)^2$

$\int_0^3 e^{-2x}\,dx$

$-\frac{1}{2}e^{-2x}\Big|_0^3$

$-\frac{1}{2}e^{-6} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2}e^{-6} + \frac{1}{2}$

What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

1993 BC 30 Calc

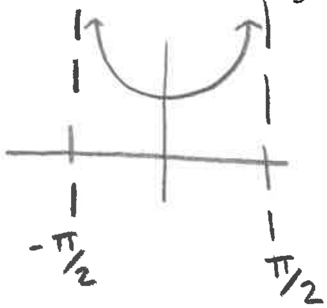
(A) $\frac{\pi}{\sqrt{3}}$

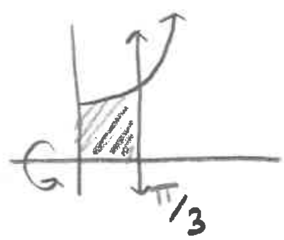
(B) π

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$





$$V = \int_0^{\pi/3} \pi (\sec x)^2 \, dx$$

The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

$x(6-x)$

$x = 0, 6$

1985 BC 35

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

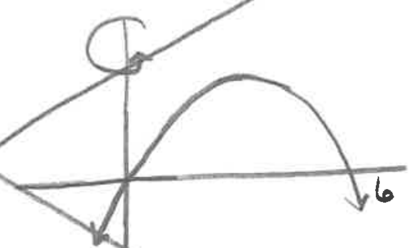
(B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$

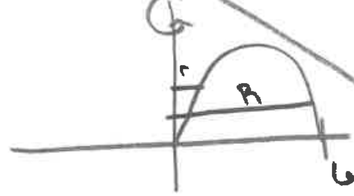
(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

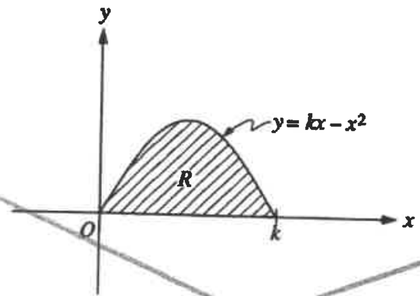
(E) $\int_0^9 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

$y = 6x - x^2$





Shell Method



1993 BC 19 Calc

~~The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k?~~

~~(A) 1.51 (B) 2.09 (C) 2.49 (D) 4.18 (E) 4.77~~

Shell Method

Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

1969 BC 43

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

$\frac{dy}{dx} = \sec^2 x$

The length of a curve from $x=1$ to $x=4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

2003 BC 15

(A) $y=3+3x^2$

(B) $y=5+x^3$

(C) $y=6+x^3$

(D) $y=6-x^3$

(E) $y=\frac{16}{5}+x+\frac{9}{5}x^5$

$$9x^4 = \left(\frac{dy}{dx}\right)^2$$

$$3x^2 = \frac{dy}{dx}$$

$$y = \int 3x^2\,dx = x^3 + C$$

$$6 = 1^3 + C$$

$$5 = C$$

$$y = x^3 + 5$$