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In Questions 18–24 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

18. $y = x^2$, $x = 2$, and $y = 0$; about the x-axis.

(A) $\frac{64\pi}{3}$ **(B)** 8π **(C)** $\frac{8\pi}{3}$ **(D)** $\frac{128\pi}{5}$ **(E)** $\frac{32\pi}{5}$

19. $y = x^2$, $x = 2$, and $y = 0$; about the y-axis.

(A) $\frac{16\pi}{3}$ **(B)** 4π **(C)** $\frac{32\pi}{5}$ **(D)** 8π **(E)** $\frac{8\pi}{3}$

20. The first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$; about the y-axis.

(A) 8π **(B)** 4π **(C)** $\frac{64\pi}{3}$ **(D)** $\frac{32\pi}{3}$ **(E)** $\frac{16\pi}{3}$

21. $y = x^2$ and $y = 4$; about the x-axis.

(A) $\frac{64\pi}{5}$ **(B)** $\frac{512\pi}{15}$ **(C)** $\frac{256\pi}{5}$

(D) $\frac{128\pi}{5}$ **(E)** $\frac{176\pi}{3}$

22. $y = x^2$ and $y = 4$; about the line $y = 4$.

(A) $\frac{256\pi}{15}$ **(B)** $\frac{256\pi}{5}$ **(C)** $\frac{512\pi}{5}$ **(D)** $\frac{512\pi}{15}$ **(E)** $\frac{64\pi}{3}$

23. An arch of $y = \sin x$ and the x-axis; about the x-axis.

(A) $\frac{\pi}{2}\left(\pi - \frac{1}{2}\right)$ **(B)** $\frac{\pi^2}{2}$ **(C)** $\frac{\pi^2}{4}$ **(D)** π^2 **(E)** $\pi(\pi - 1)$

24. A trapezoid with vertices at (2, 0), (2, 2), (4, 0), and (4, 4); about the x-axis.

(A) $\frac{56\pi}{3}$ **(B)** $\frac{128\pi}{3}$ **(C)** $\frac{92\pi}{3}$

(D) $\frac{112\pi}{3}$ **(E)** 20π

25. The base of a solid is a circle of radius a, and every plane section perpendicular to a diameter is a square. The solid has volume

(A) $\frac{8}{3}a^3$ **(B)** $2\pi a^3$ **(C)** $4\pi a^3$ **(D)** $\frac{16}{3}a^3$ **(E)** $\frac{8\pi}{3}a^3$

26. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

(A) $\frac{64\sqrt{3}}{3}$ **(B)** $64\sqrt{3}$ **(C)** $32\sqrt{3}$

(D) 32 **(E)** $\frac{32\sqrt{3}}{3}$

27. The base of a solid is the region bounded by $y = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$. Each cross section perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{e^2}{2}$ **(B)** $e^2 - 1$ **(C)** $1 - \frac{1}{e^2}$

(D) $\frac{e^2 - 1}{2}$ **(E)** $\frac{1}{2}\left(1 - \frac{1}{e^2}\right)$

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In Questions 48–54 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

48. $y = x^2$ and $y = 4$; about the line $y = -1$.

(A) $4\pi \int_{-1}^{4} (y+1)\sqrt{y}\, dy$ **(B)** $2\pi \int_{0}^{2} (4-x^2)^2\, dx$ **(C)** $\pi \int_{-2}^{2} (16-x^4)\, dx$

(D) $2\pi \int_{0}^{2} (24-2x^2-x^4)\, dx$ **(E)** $2\pi \int_{0}^{2} (x^2+1)^2\, dx$

49. $y = 3x - x^2$ and $y = 0$; about the x-axis.

(A) $\pi \int_{0}^{3} (9x^2+x^4)\, dx$ **(B)** $\pi \int_{0}^{3} (3x-x^2)^2\, dx$ **(C)** $\pi \int_{0}^{\sqrt{3}} (3x-x^2)\, dx$

(D) $2\pi \int_{0}^{3} y\sqrt{9-4y}\, dy$ **(E)** $\pi \int_{0}^{9/4} y^2\, dy$

50. $y = 3x - x^2$ and $y = x$; about the x-axis.

(A) $\pi \int_{0}^{3/2} [(3x-x^2)^2 - x^2]\, dx$ **(B)** $\pi \int_{0}^{2} (9x^2-6x^3)\, dx$

(C) $\pi \int_{0}^{2} [(3x-x^2)^2 - x^2]\, dx$ **(D)** $\pi \int_{0}^{3} [(3x-x^2)^2 - x^4]\, dx$

(E) $\pi \int_{0}^{3} (2x-x^2)^2\, dx$

51. $y = \ln x$, $y = 0$, $x = e$; about the line $x = e$.

(A) $\pi \int_{1}^{e} (e-x)\ln x\, dx$ **(B)** $\pi \int_{0}^{1} (e-e^y)^2\, dy$ **(C)** $2\pi \int_{1}^{e} (e-\ln x)\, dx$

(D) $\pi \int_{0}^{e} (e^2 - 2e^{y+1} + e^{2y})\, dy$ **(E)** $\pi \int_{0}^{1} (e^2 - e^{2y})\, dy$

53. A sphere of radius r is divided into two parts by a plane at distance h $(0 < h < r)$ from the center. The volume of the smaller part equals

CHALLENGE

(A) $\frac{\pi}{3}(2r^3 + h^3 - 3r^2h)$ (B) $\frac{\pi h}{3}(3r^2 - h^2)$ (C) $\frac{4}{3}\pi r^3 + \frac{h^3}{3} - r^2h$

(D) $\frac{\pi}{3}(2r^3 + 3r^2h - h^3)$ (E) $\frac{\pi}{3}(r^3 - h^3)$

54. If the curves of $f(x)$ and $g(x)$ intersect for $x = a$ and $x = b$ and if $f(x) > g(x) > 0$ for all x on (a, b), then the volume obtained when the region bounded by the curves is rotated about the x-axis is equal to

(A) $\pi \int_a^b f^2(x)\,dx - \int_a^b g^2(x)\,dx$

(B) $\pi \int_a^b [f(x) - g(x)]^2\,dx$

(C) $2\pi \int_a^b x[f(x) - g(x)]\,dx$

(D) $\pi \int_a^b [f^2(x) - g^2(x)]\,dx$

(E) $2\pi \int_a^b \left[f^2(x) - g^2(x)\right] dx$

Answer Key

1. C	14. A	27. E	40. D	53. A
2. C	15. C	28. C	41. B	54. D
3. A	16. B	29. A	42. A	55. D
4. D	17. C	30. A	43. C	56. D
5. D	18. E	31. E	44. D	57. B
6. C	19. D	32. B	45. E	58. C
7. E	20. A	33. A	46. B	59. D
8. A	21. C	34. E	47. C	60. E
9. A	22. D	35. C	48. D	
10. D	23. B	36. A	49. B	
11. D	24. A	37. C	50. C	
12. C	25. D	38. E	51. B	
13. D	26. B	39. B	52. C	