BC Calculus
Chapter 9
Review

No Calculator!

1) Write an explicit formula for the sequence $\frac{2}{4}, \frac{3}{7}, \frac{4}{12}, \frac{5}{19}, \frac{6}{42}, \ldots$. Then find a_{14}.

~~A~~ $a_n = a_{n-1} - \frac{n-1}{7n}; \frac{15}{199}$ recursive

C $a_n = \frac{n+1}{n^2+3}; \frac{15}{199}$

~~B~~ $a_n = \frac{a_{n+1}}{n^2+3}; \frac{15}{199}$ not explicit

D $a_n = \frac{n}{n^3-1}; \frac{14}{2743}$

2) Write the explicit formula for the sequence. Then find the fifth term in the sequence: $a_1 = 3, r = -3$.

A $a_n = 3 \cdot (-3)^{n-1}; 243$

B $a_n = -3 \cdot (3)^{n-1}; -243$

C $a_n = 3 \cdot (3)^n; 243$

D $a_n = 3 \cdot (-3)^n; -729$

$a_n = 3(-3)^{n-1}$

3) . Determine whether the sequence defined by

$$a_n = x^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$$

converges or diverges. If it converges, find its limit.

a. -2

b. -1

c. 0

d. π

e. Diverges

$\lim_{n\to\infty} n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$

$\infty \cdot 0$

$\lim_{n\to\infty} \frac{\cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)}{1/n^2}$ $\frac{0}{0}$

L'hosp.

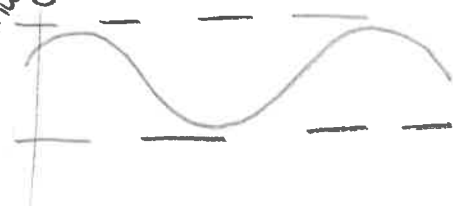
$\lim_{n\to\infty} \frac{-\sin\left(2/n^2 + \pi/2\right)\left(-4n^{-3}\right)}{-2n^{-3}}$

$\lim_{n\to\infty} -2\sin\left(2/n^2 + \pi/2\right)$

$= -2$

4) The sequence $a_n = \sin \frac{n\pi}{6}$

bounded but never converges



(A) is unbounded ~~(B) is monotonic~~
(C) converges to a number less than 1 (D) is bounded
(E) diverges to infinity

5) If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge? 1993 BC 31 Calc

(A) $\frac{1}{5}$ (B) 1 (C) $\frac{5}{4}$ (D) $\left(\frac{5}{4}\right)^{100}$ (E) The sequence does not converge.

$$s_n = \left(\frac{(5+n)^{100}}{5^n 5}\right)\left(\frac{5^n}{(4+n)^{100}}\right) = \left(\frac{(5+n)^{100}}{5(4+n)^{100}}\right)$$

$$\approx \lim_{n\to\infty} \frac{n^{100}+\ldots}{5(n^{100}+\ldots)} = \frac{1}{5}$$

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6) If k is a positive integer, then $\lim\limits_{x\to+\infty} \frac{x^k}{e^x}$ is 1988 BC 35

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

e^x will take over any x^k

★ will be one on the test and will NOT be multiple choice

7) Show which function, $\ln x$ or $\log_2 x$ grows faster.

$$\lim_{n\to\infty} \frac{\ln x}{\log_2 x} \quad \frac{\infty}{\infty}$$

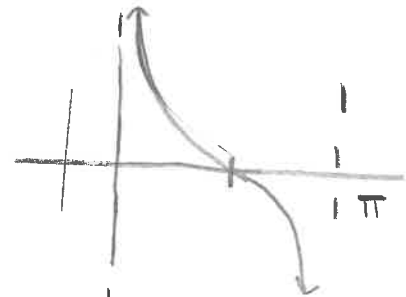
$$= \lim_{n\to\infty} \frac{1/x}{1/x\ln 2}$$

$$= \ln 2$$

grow at same rate

8) $\lim\limits_{x\to 0^+} \dfrac{\cot x}{\ln x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞



$$\frac{\infty}{-\infty}$$

$$\lim_{x\to 0^+} \frac{-x}{\sin^2 x}$$

$$\lim_{x\to 0^+} \frac{-1}{2\sin x\cos x} = \frac{-1}{0^+} = -\infty$$

$$\lim_{x\to 0^+} \frac{-\csc^2 x}{1/x}$$

$$= \lim_{x\to 0^+} -x\csc^2 x = -\infty$$

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9) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(2x)}{x^2} = \dfrac{0}{0}$ 1973 BC 37

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

$$\lim_{x\to 0} \frac{-2\cos(2x)(-\sin(2x))(\cancel{2})}{\cancel{2}x}$$ ★ nested chain

$$\lim_{x\to 0} \frac{2\cos(2x)\sin(2x)}{x} \quad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{2(-\sin(2x)(2)\sin 2x) + \cos(2x)\cos(2x)(2))}{1}$$

$$\lim_{x\to 0} -4\sin^2(2x) + 4\cos^2(2x)$$

$$0 + 4$$

10) $\lim_{x \to \frac{\pi}{4}} \dfrac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is $\dfrac{0}{0}$ 1985 BC 29

(A) 0 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\pi}{4}$ (D) 1 (E) nonexistent

$$\lim_{x \to \pi/4} \frac{\cos(x-\pi/4)}{1}$$

$$= \cos 0$$

$$= 1$$

11) $\lim_{x \to 0} \dfrac{\tan 3x}{2x} =$ $\dfrac{0}{0}$

(A) 0 (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{3}{2}$ (E) ∞

$$\lim_{x \to 0} \frac{3\sec^2 3x}{2}$$

$$\frac{3}{2}(\sec 0)^2$$

$$\frac{3}{2}$$

12) $\lim_{x \to 1} \dfrac{\int_1^x e^{t^2}\,dt}{x^2-1}$ is $\dfrac{0}{0}$ 1998 BC 28

(A) 0 (B) 1 (C) $\dfrac{e}{2}$ (D) e (E) nonexistent

$$\lim_{x \to 1} \frac{(e^x)^2}{2x}$$

$$\lim_{x \to 1} \frac{e^{x^2}}{2x}$$

$$\frac{e}{2}$$

13) $\int_0^{\infty} xe^{x^2}\,dx$

a. 0

b. $-\frac{1}{2}$

c. $\frac{1}{2}$

d. 1

b. Divergent

$$\lim_{b\to\infty} \int_0^b xe^{x^2}\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\lim_{b\to\infty} \int_0^{b^2} e^u \, \tfrac{1}{2}du$$

$$\lim_{b\to\infty} \frac{1}{2}e^u \Big|_0^{b^2}$$

$$\lim_{b\to\infty} \left(\frac{1}{2}e^{b^2} - \frac{1}{2}e^0\right)$$

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14) $\int_2^{+\infty} \frac{dx}{x^2}$ is

1988 BC 7

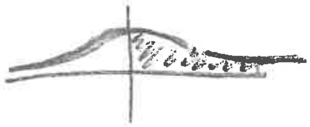
(A) $\frac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) nonexistent

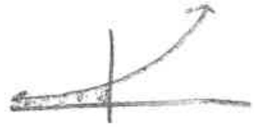
$$\lim_{b\to\infty} \int_2^b x^{-2}\,dx$$

$$\lim_{b\to\infty} -x^{-1}\Big|_2^b$$

$$\lim_{b\to\infty} -\frac{1}{b} + \frac{1}{2}$$

$$\frac{1}{2}$$

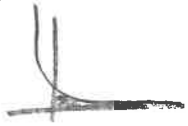


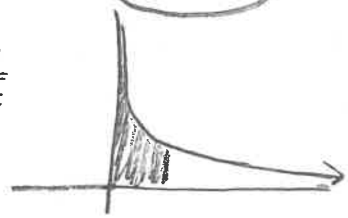


15) Which of the following improper integrals diverges?

(A) $\int_0^{\infty} e^{-x^2}\,dx$ ~~(B)~~ $\int_{-\infty}^{0} e^{x}\,dx$ (C) $\int_0^1 \frac{dx}{x}$

~~(D)~~ $\int_0^{\infty} e^{-x}\,dx$ (E) $\int_0^1 \frac{dx}{\sqrt{x}}$





$$\lim_{a\to 0^+} \int_a^1 \frac{1}{x}\,dx$$

$$\lim_{a\to 0^+} \ln|x|\,\Big|_a^1$$

$$\lim_{a\to 0^+} (\ln 1 - \ln a)$$

∞

diverge

$$\lim_{b\to\infty} \int_0^b e^{-x^2}$$

$$\lim_{b\to\infty} -$$

$$= \lim_{a\to -\infty} \int_a^0 e^x\,dx$$

$$= \lim_{a\to -\infty} (e^0 - e^a)$$

$$= 1 - 0$$

$$= 1$$

$$\lim_{b\to\infty} \int_0^b e^{-x}\,dx$$

$$\lim_{b\to\infty} -e^{-b} + e^0$$

$$= 1$$

$$\lim_{a\to 0^+} \int_a^1 x^{-1/2}\,dx$$

$$= \lim_{a\to 0^+} 2x^{1/2}\,\Big|_a^1$$

$$= \lim_{a\to 0^+} (2 - 2a^{1/2})$$

$$= 2$$

16) $\int_{-\infty}^{\infty} \frac{2}{1+x^2}\,dx$

a. 0

c. $\frac{\pi}{2}$

c. Divergent

b. 2π

b. π

$$\int_{-\infty}^{0} \frac{2}{1+x^2}\,dx + \int_0^{\infty} \frac{2}{1+x^2}\,dx$$

$$\lim_{a\to -\infty} \int_a^0 \frac{2}{1+x^2}\,dx + \lim_{b\to\infty} \int_0^b \frac{2}{1+x^2}\,dx$$

$$\lim_{a\to -\infty} 2\tan^{-1}x\,\Big|_a^0 + \lim_{b\to\infty} 2\tan^{-1}x\,\Big|_0^b$$

$$\lim_{a\to -\infty} 2\tan^{-1}0 - 2\tan^{-1}a + \lim_{b\to\infty} 2\tan^{-1}b - 2\tan^{-1}0$$

$$0 - 2(-\pi/2) + 2(\pi/2) - 0$$

$$\pi + \pi$$