AB Calculus
Review Chapter Review

1. Solve the inequality and write the answer in interval notation.

a. $2x - 1 \geq 0$

$x \geq 1/2$

$(1/2, \infty)$

b. $-4 \leq 2x - 3 < 4$

$-1 \leq 2x < 7$

$-1/2 \leq x < 7/2$

$[-1/2, 7/2)$

c. $\left(\frac{x}{2} - \frac{x}{3} > 5\right)6$

$3x - 2x > 30$

$x > 30$

$(30, \infty)$

d. $5(x - 3) \leq 8(x + 5)$

$5x - 15 \leq 8x + 40$

$-55 \leq 3x + 40$

$-55 \leq 3x$

$-55/3 \leq x$

$(-\infty, -55/3]$

e. $4 - \frac{5x}{3} > -(2x + \frac{1}{2})$

$\left(4 - \frac{5x}{3} > -2x - 1/2\right)6$

$24 - 10x > -12x - 3$

$2x > -27$

$x > -27/2$

$(-27/2, \infty)$

f. $\frac{3}{4} > x + 1 > \frac{1}{2}$

$-1/4 > x > -1/2$

$(-1/4, -1/2)$

g. $x + 7 \geq |5 - 3x|$

$|5 - 3x| \leq x + 7$

$5 - 3x \leq x + 7$ and $5 - 3x \geq -(x + 7)$

$-2 \leq 4x$

$-1/2 \leq x$

$5 \geq 2x - 7$

$12 \geq 2x$

$6 \geq x$

$[-1/2, 6]$

h. $(x + 2)^2 < 25$

$(x+2) < |5|$

$x + 2 < 5$ $\quad$ $x + 2 > -5$

$x < 3$ $\quad$ $x > -7$

$(-7, 3)$

i. $\frac{x-7}{x-1} < 0$

$x \neq 1$ $\quad$ $x = 7$

$+ \quad - \quad +$

$1 \quad 7$

$(1, 7)$

j. $\frac{x+6}{x^2-5x-24} \geq 0$

$\frac{x+6}{(x-8)(x+3)} \geq 0$

$x \neq -3, 8$ $\quad$ $x = -6$

$- \quad + \quad - \quad +$

$-6 \quad -3 \quad 8$

$[-6, -3) \cup (8, \infty)$

2. Find the domain of the following functions.

a. $y = \frac{x-4}{x^2-16} = \frac{x-4}{(x+4)(x-4)}$

$= \frac{1}{x+4}, \quad x \neq 4$

$\boxed{(-\infty, -4) \cup (-4, 4) \cup (4, \infty)}$ $\quad x \neq -4$

b. $y = \sqrt{2x-9}$

$2x - 9 \geq 0$

$x \geq 9/2$

$\boxed{[9/2, \infty)}$

c. $y = \log(x - 10)$

$x - 10 > 0$

$x > 10$

$\boxed{(10, \infty)}$

d. $y = \frac{\sqrt{2x+14}}{x^2-49}$

$2x + 14 \geq 0$ $\quad x \neq \pm 7$

$x \geq -7$

$\boxed{(-7, 7) \cup (7, \infty)}$

3. Simplify each of the following:

a. $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$

$= \log_2 \frac{5(x^2-1)}{x-1}$

$= \log_2 \frac{5(x+1)(x-1)}{x-1}$

$\boxed{= \log_2 5(x+1)}$

b. $3^{2\log_3 5}$

$= 3^{\log_3 5^2}$

$= 5^2$

$\boxed{= 25}$

4. Solve the following equations for x:

a. $5^{(x+1)} = 25$

$\ln 5^{x+1} = \ln 25$

$(x+1)\ln 5 = \ln 25$

$x + 1 = \frac{\ln 25}{\ln 5}$

$\boxed{x = \frac{\ln 25}{\ln 5} - 1}$

b. $\frac{1}{3} = 3^{2x+2}$

$3^{-1} = 3^{2x+2}$ $\quad$ * relating bases

$-1 = 2x + 2$

$-3 = 2x$

$\boxed{-3/2 = x}$

c. $\log_2 x^2 = 3$

$2^3 = x^2$

$\pm\sqrt{8} = x$

$\boxed{\pm 2\sqrt{2} = x}$

d. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 16 - \log_3 5^4$

$\log_3 x^2 = \log_3 16/625$

$x^2 = 16/625$ $\boxed{x = \pm 4/25}$

e. $\log_2(x-1) + \log_2(x+3) = 5$

$\log_2 (x-1)(x+3) = 5$

$2^5 = x^2 + 2x - 3$ $0 = (x+7)(x-5)$

$32 = x^2 + 2x - 3$ $x = \cancel{-7}, 5$

$0 = x^2 + 2x - 35$ $\boxed{x = 5}$

f. $\log_5(x+3) - \log_5 x = 2$

$\log_5 \frac{x+3}{x} = 2$

$5^2 = \frac{x+3}{x}$

$25x = x + 3$

$24x = 3$

$\boxed{x = 3/24}$

g. $\ln x^3 - \ln x^2 = \frac{1}{2}$

$\ln x^3/x^2 = 1/2$

$\ln x = 1/2$

$e^{1/2} = x$ $\boxed{\sqrt{e} = x}$

h. $3^{x-2} = 18$

$(x-2)\ln 3 = \ln 18$

$x - 2 = \frac{\ln 18}{\ln 3}$

$\boxed{x = 2 + \frac{\ln 18}{\ln 3}}$

i. $e^{3x+1} = 10$

$3x + 1 = \ln 10$

$\boxed{x = \frac{\ln 10 - 1}{3}}$

j. $8^x = 5^{2x-1}$

$x\ln 8 = (2x-1)\ln 5$

$x\ln 8 = 2x\ln 5 - \ln 5$

$x\ln 8 - 2x\ln 5 = -\ln 5$

$x(\ln 8 - 2\ln 5) = -\ln 5$

$\boxed{x = \frac{-\ln 5}{\ln 8 - 2\ln 5}}$

5. For the given functions find the compositions, $f \circ g$ and $g \circ f$.

$f(x) = 9 - x$

$g(x) = x^2 + 2$

$f \circ g = f(g(x)) = 9 - (x^2 + 2)$

$= -x^2 + 7$

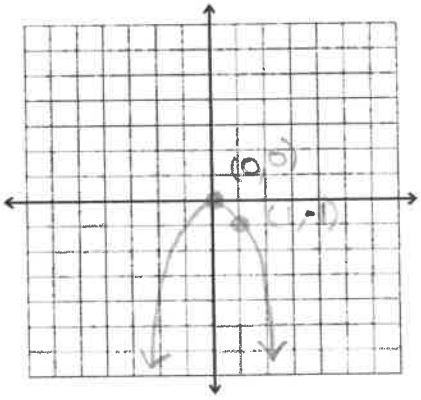
$g \circ f = g(f(x)) = (9 - x)^2 + 2$

$= x^2 - 18x + 81 + 2$

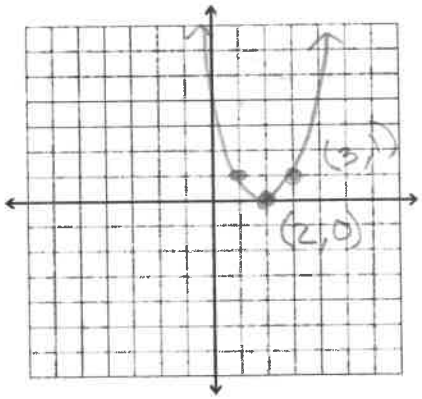
$= x^2 - 18x + 83$

6. Graph the following functions. The graphs must have at least two points labeled with coordinate.

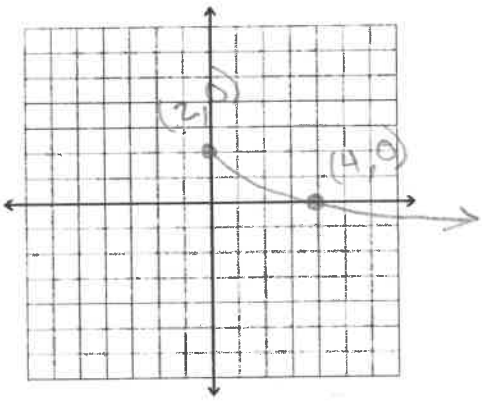
a. $y = -x^2$



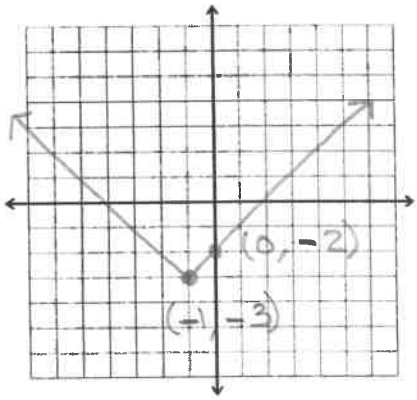
b. $y = (x-2)^2$



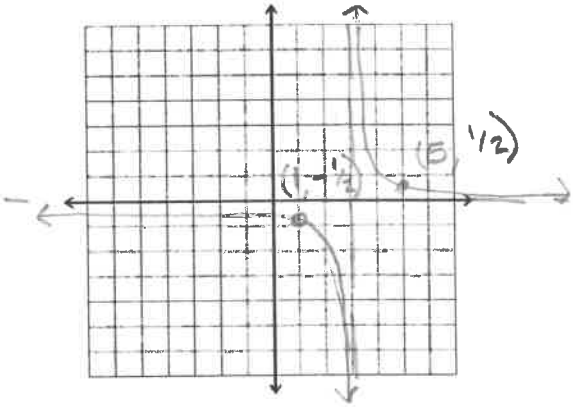
c. $y = 2 - \sqrt{x}$



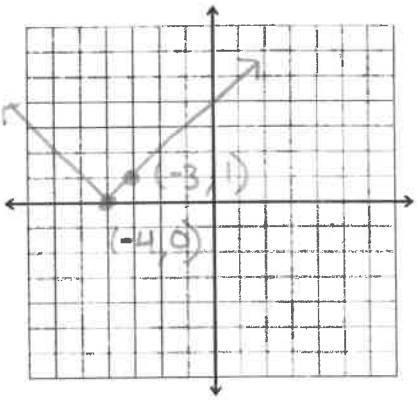
d. $y = |x+1| - 3$



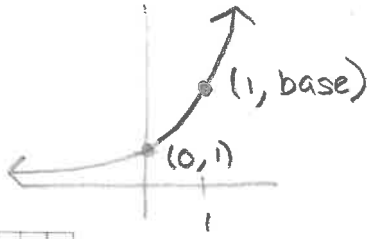
e. $y = \frac{1}{x-3}$

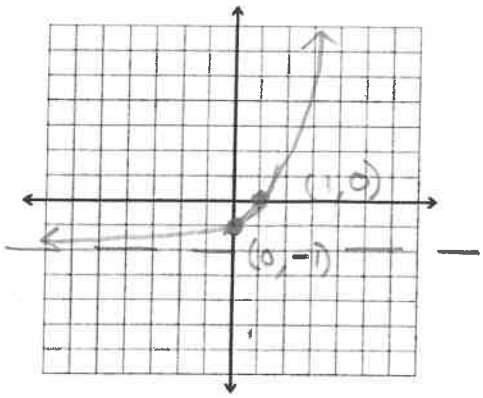


f. $y = |x+4|$

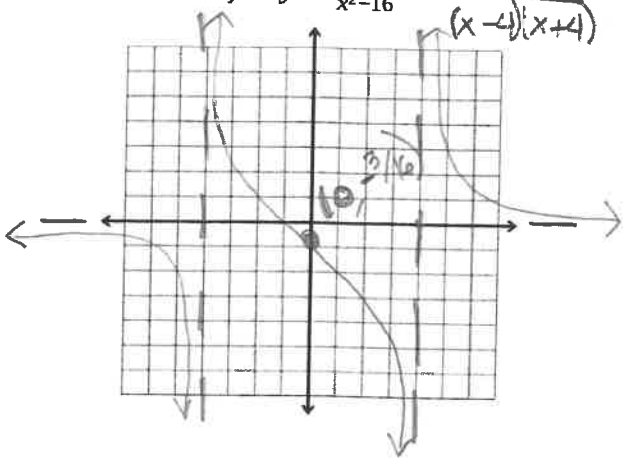


g. $y = 2^x - 2$

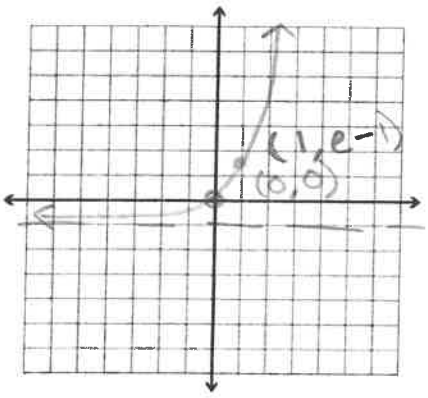




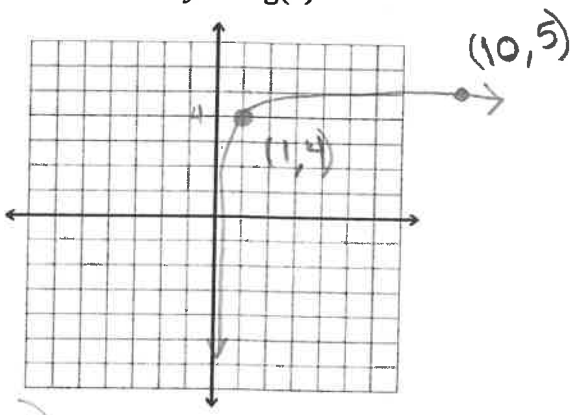
j. $y = \frac{x+3}{x^2-16} = \frac{x+3}{(x-4)(x+4)}$



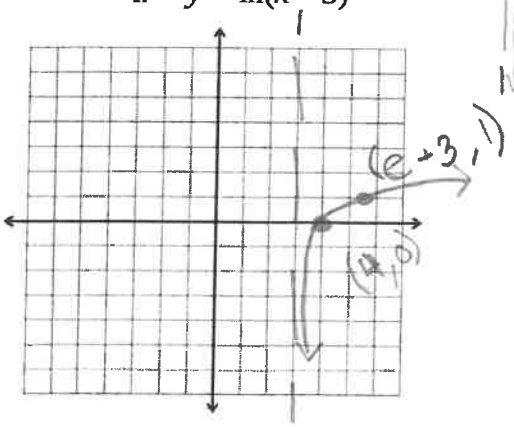
h. $y = e^x - 1$

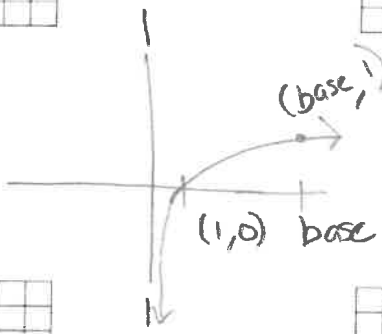


k. $y = \log(x) + 4$

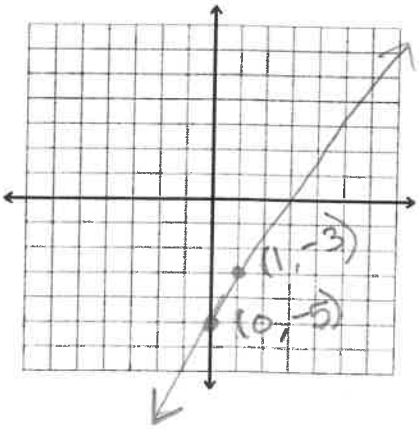


i. $y = \ln(x - 3)$



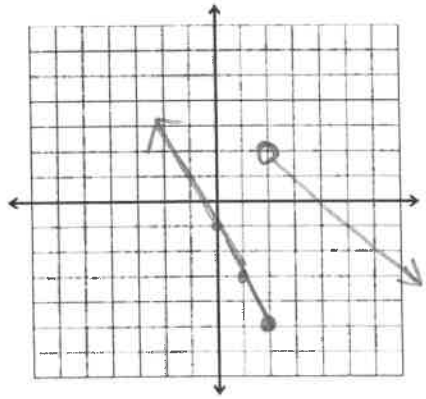


l. $y = 2x - 5$



7. Graph the following piecewise function

$$f(x) = \begin{cases} -2x - 1, & x \le 2 \\ -x + 4, & x > 2 \end{cases}$$



8. Sketch the following rational function. Identify point(s) of discontinuity, hole(s), vertical asymptote(s), and horizontal asymptote(s).

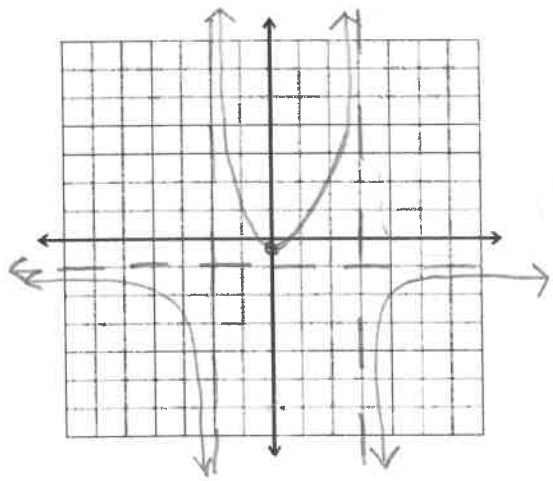
$f(x) = \frac{-x^2}{x^2-x-6} = \frac{-x^2}{(x-3)(x+2)}$

Point(s) of Discontinuity: $x = -2, 3$

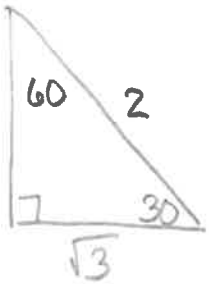
Hole(s): none

Vertical Asymptote(s): $x = -2, 3$

Horizontal Asymptote(s): $y = -1$

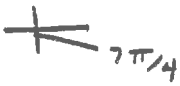


9. Find the exact value of the following:



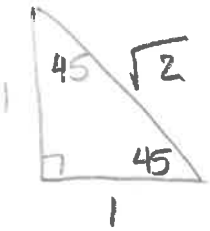
a. $\cos 270° = 0$

b. $\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$

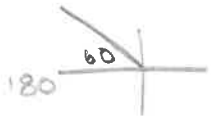


c. $\tan 90° =$ und

d. $\sin \frac{\pi}{4} = 1/\sqrt{2}$

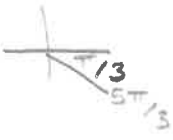


e. $\tan 120° = -\sqrt{3}$



f. $\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

g. $\tan \frac{5\pi}{3} = -\sqrt{3}$



h. $\sin 210° = -\frac{1}{2}$

i. $\cos \frac{4\pi}{3} = -\frac{1}{2}$

j. $\tan 0 = 0$

10. Find the exact value of the following:

a. $\sin^{-1} -\frac{1}{2} = -\frac{\pi}{6}$

b. $\cos^{-1}\frac{1}{2} = \pi/3$

c. $\tan^{-1}\frac{\sqrt{3}}{3} = \pi/6$

d. $\sin^{-1}\frac{\sqrt{2}}{2} = \pi/4$

e. $\cos^{-1}\frac{\sqrt{3}}{2} = \pi/6$

f. $\sin^{-1}0 = 0$

g. $\cos^{-1}0 = \pi/2$

h. $\tan^{-1} -\sqrt{3} = -\pi/3$