

2011 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

6. Let $f(x) = \ln(1 + x^3)$.

- (a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \cdot \frac{x^n}{n} + \dots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f .
- (b) The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.
- (c) Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2) dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.
- (d) The Maclaurin series for g , evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.
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2016 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the n th derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.
- (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
 - (b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.
 - (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.
 - (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.
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END OF EXAM