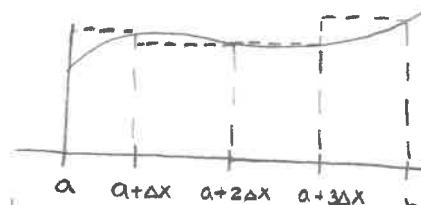


BC Calculus  
Chapter 6 and 7 Review

No calculator unless the problem says "calc".

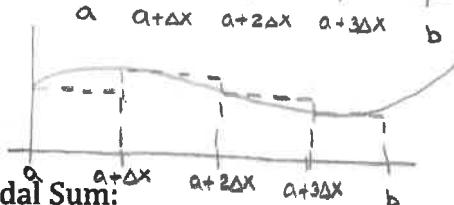
1. List the formulas for:

a. RRAM:



$$\Delta x (y(a+\Delta x) + y(a+2\Delta x) + y(a+3\Delta x) y(b))$$

b. LRAM:



$$\Delta x (y(a) + y(a+\Delta x) + y(a+2\Delta x) + y(a+3\Delta x))$$

c. Trapezoidal Sum:

$$\frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

d. Exponential Growth or Decay:

$$y_0 = y e^{-kt}$$

e. Logistic Growth:

$$\frac{dP}{dt} = kP(M - P) \quad \text{or} \quad \frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

*M = carrying capacity*

2. What is the formula for integration by parts?

$$uv - \int v du \quad \text{LIPET!}$$

3. How do we integrate by Partial Fractions?

- 1) split into parts
- 2) multiply by denominator
- 3) solve A, B, C, ... etc by selecting x values
- 4) plug A, B, C, ... etc in
- 5) integrate

BC Calculus  
Chapter 6 and 7 Review

4. What is the order of integration techniques one should try when approached with an integration problem?

- 1) Basic  $\rightarrow$  reverse power rule
- 2) U-sub (reverse chain)
- 3) Integration by parts (reverse product)
- 5) Partial Fraction

5)  $\frac{d}{dx} \left( \int_0^{x^3} \ln(t^2+1) dt \right) =$

2003 BC 27

(A)  $\frac{2x^3}{x^6+1}$

$3x^2 \ln(x^6 + 1)$

(B)  $\frac{3x^2}{x^6+1}$

(C)  $\ln(x^6+1)$

(D)  $2x^3 \ln(x^6+1)$

(E)  $3x^2 \ln(x^6+1)$

6) Given  $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$   $\int_{-1}^1 f(x) dx =$

1973 BC 41

(A)  $\frac{1}{2} + \frac{1}{\pi}$

(B)  $-\frac{1}{2}$

(C)  $\frac{1}{2} - \frac{1}{\pi}$

(D)  $\frac{1}{2}$

(E)  $-\frac{1}{2} + \pi$

$$\begin{aligned} & \int_{-1}^0 (x+1) dx + \int_0^1 \cos \pi x dx \\ &= \frac{x^2}{2} + x \Big|_{-1}^0 + \frac{\sin \pi x}{\pi} \Big|_0^1 \end{aligned}$$

$$\begin{aligned} &= 0 - \left( \frac{1}{2} - 1 \right) + \frac{\sin \pi}{\pi} - \frac{\sin 0}{\pi} \\ &= \frac{1}{2} + 0 \\ &= \frac{1}{2} \end{aligned}$$

7) Which of the following is equal to  $\int_0^\pi \sin x dx$ ?

1993 BC 33 calc

(A)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$

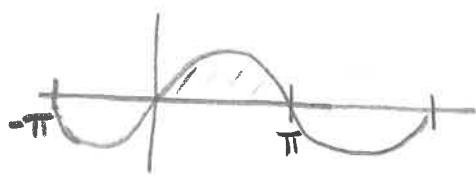
(B)  $\int_0^\pi \cos x dx$

(C)  $\int_{-\pi}^0 \sin x dx$  same area but  
↑ neg of  
 $\int_0^\pi \sin x dx$

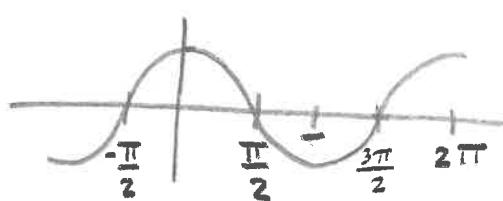
(D)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx$

(E)  $\int_\pi^{2\pi} \sin x dx$

$\sin x$



$\cos x$



\*could find  
integrals of all

8) If  $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \leq x \leq 2, \\ f(x) = x^2 & \text{elsewhere,} \end{cases}$  then  $\int_{-1}^3 f(x) dx$  is a number between

1969 BC 41

- (A) 0 and 8    (B) 8 and 16    (C) 16 and 24    (D) 24 and 32    (E) 32 and 40

$$\begin{aligned} & \int_{-1}^2 8 - x^2 + \int_2^3 x^2 \\ &= 8x - \frac{x^3}{3} \Big|_{-1}^2 + \frac{x^3}{3} \Big|_2^3 \\ &= 16 - \frac{8}{3} - \left(-8 - \frac{1}{3}\right) + \frac{27}{3} - \frac{8}{3} = 27.\bar{3} \end{aligned}$$

9)  $\int_1^e \frac{x^2+1}{x} dx =$

2008 BC 13

- (A)  $\frac{e^2 - 1}{2}$     (B)  $\frac{e^2 + 1}{2}$     (C)  $\frac{e^2 + 2}{2}$     (D)  $\frac{e^2 - 1}{e^2}$     (E)  $\frac{2e^2 - 8e + 6}{3e}$

$$\int_1^e (x + \frac{1}{x}) dx$$

$$= \frac{e^2}{2} + \ln e - \left(\frac{1}{2} - \ln 1\right)$$

$$= \frac{x^2}{2} + \ln x \Big|_1^e$$

$$= \frac{e^2}{2} + 1 - \frac{1}{2}$$

$$= \frac{e^2}{2} + \frac{1}{2}$$

10)  $\int_0^1 \sqrt{x}(x+1) dx =$

1997 BC 1

- (A) 0      (B) 1      (C)  $\frac{16}{15}$       (D)  $\frac{7}{5}$       (E) 2

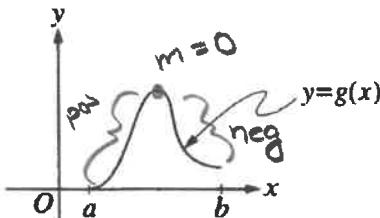
$$= \int_0^1 (x^{3/2} + x^{1/2}) dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} \Big|_0^1$$

$$= \frac{2}{5} + \frac{2}{3} - (0) = \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$$

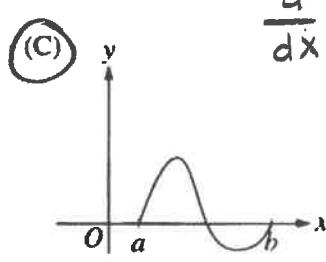
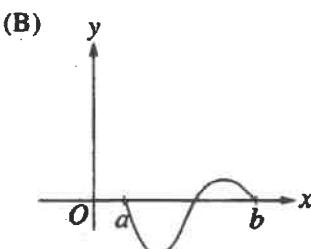
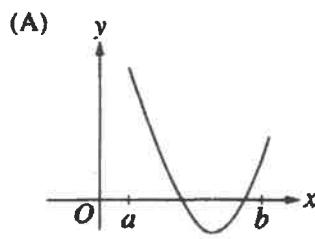

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11)



1998 BC 88 Calc

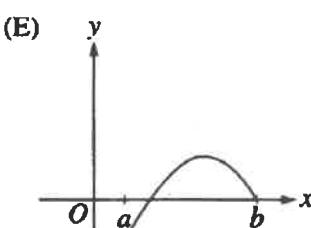
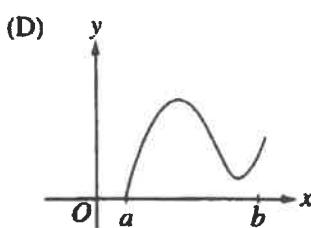
Let  $g(x) = \int_a^x f(t) dt$ , where  $a \leq x \leq b$ . The figure above shows the graph of  $g$  on  $[a, b]$ . Which of the following could be the graph of  $f$  on  $[a, b]$ ?



$$\frac{d}{dx} \left[ \int_0^x f(t) dt \right] = \frac{d}{dx}(g(x))$$

$$= g'(x) = f$$

graph derivative



- (2) What is the average (mean) value of  $3t^3 - t^2$  over the interval  $-1 \leq t \leq 2$ ?      1969 BC 33

- (A)  $\frac{11}{4}$       (B)  $\frac{7}{2}$       (C) 8      (D)  $\frac{33}{4}$       (E) 16

$$\begin{aligned} & \frac{1}{2-(-1)} \int_{-1}^2 (3t^3 - t^2) dt = \frac{1}{3} \left( 12 - \frac{8}{3} - \frac{3}{4} - \frac{1}{3} \right) \\ &= \frac{1}{3} \left( \frac{3}{4}t^4 - \frac{1}{3}t^3 \right) \Big|_{-1}^2 = 4 - \frac{8}{9} - \frac{1}{4} - \frac{1}{9} \\ &= \frac{1}{3} \left[ \left( \frac{3}{4}(2)^4 - \frac{1}{3}(2)^3 \right) - \left( \frac{3}{4}(-1)^4 - \frac{1}{3}(-1)^3 \right) \right] = 4 - 1 - \frac{1}{4} \\ &= 3 - \frac{1}{4} = \frac{12}{4} - \frac{1}{4} = \frac{11}{4} \end{aligned}$$

- (3) The average (mean) value of  $\sqrt{x}$  over the interval  $0 \leq x \leq 2$  is      1973 BC 34

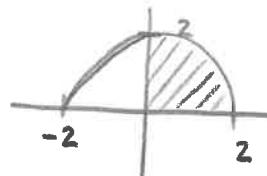
- (A)  $\frac{1}{3}\sqrt{2}$       (B)  $\frac{1}{2}\sqrt{2}$       (C)  $\frac{2}{3}\sqrt{2}$       (D) 1      (E)  $\frac{4}{3}\sqrt{2}$

$$\begin{aligned} & \frac{1}{2-0} \int_0^2 x^{1/2} dx = \frac{1}{2} \left( \frac{2}{3} \sqrt{2} - 0 \right) \\ &= \frac{1}{2} \left( \frac{2}{3} x^{3/2} \right) \Big|_0^2 = \frac{1}{3} (2)\sqrt{2} \\ &= \frac{2}{3} \sqrt{2} \end{aligned}$$

- (4)  $\int_0^2 \sqrt{4-x^2} dx =$       1988 BC 31

- (A)  $\frac{8}{3}$       (B)  $\frac{16}{3}$       (C)  $\pi$       (D)  $2\pi$       (E)  $4\pi$

\* circle



$$\begin{aligned} & \frac{1}{4} \pi r^2 \\ &= \frac{1}{4} \pi (4)^2 \\ &= \pi \end{aligned}$$

---

5)  $\int_2^3 \frac{3}{(x-1)(x+2)} dx =$

1988 BC 17

- (A)  $-\frac{33}{20}$       (B)  $-\frac{9}{20}$       (C)  $\ln\left(\frac{5}{2}\right)$       (D)  $\ln\left(\frac{8}{5}\right)$       (E)  $\ln\left(\frac{2}{5}\right)$

$$\frac{3}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$$

$$3 = A(x+2) + B(x-1)$$

$$x = -2 \quad x = 1$$

$$3 = -3B \quad 3 = 3A$$

$$-1 = B \quad 1 = A$$

$$\int_2^3 \frac{1}{x-1} dx + \int_2^3 \frac{-1}{x+2} dx$$

$$\left( \ln|x-1| - \ln|x+2| \right)_2^3$$

$$\ln \left. \frac{x-1}{x+2} \right|_2^3$$

$$\ln \frac{2}{5} - \ln \frac{1}{4}$$

$$\ln \frac{2/5}{1/4}$$

$$\ln \frac{8}{5}$$

16)  $\int x \sec^2 x dx =$  1993 BC 29

- (A)  $x \tan x + C$       (B)  $\frac{x^2}{2} \tan x + C$       (C)  $\sec^2 x + 2 \sec^2 x \tan x + C$   
 (D)  $x \tan x - \ln |\cos x| + C$       (E)  $x \tan x + \ln |\cos x| + C$

$$u = x \quad v = \tan x$$

$$du = dx \quad dv = \sec^2 x dx$$

$$= x \tan x - \int \tan x dx = x \tan x + \int \frac{1}{u} du$$

$$= x \tan x - \int \frac{\sin x}{\cos x} dx = x \tan x + \ln |\cos x| + C$$

$$u = \cos x$$

$$du = -\sin x dx$$

17)  $\int x \cos x dx =$  1998 BC 15

- (A)  $x \sin x - \cos x + C$   
 (B)  $x \sin x + \cos x + C$   
 (C)  $-x \sin x + \cos x + C$   
 (D)  $x \sin x + C$   
 (E)  $\frac{1}{2} x^2 \sin x + C$

$$u = x \quad v = \sin x$$

$$du = dx \quad dv = \cos x dx$$

$$= x \sin x - \int \sin x dx$$

$$= x \sin x + \cos x + C$$

$$18) \int \frac{7x}{(2x-3)(x+2)} dx =$$

2008 BC 19

(A)  $\frac{3}{2} \ln|2x-3| + 2 \ln|x+2| + C$

(B)  $3 \ln|2x-3| + 2 \ln|x+2| + C$

(C)  $3 \ln|2x-3| - 2 \ln|x+2| + C$

(D)  $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

(E)  $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

$$\frac{7x}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$$

$$7x = A(x+2) + B(2x-3)$$

$$x = -2$$

$$7x = A(x+2) + 2(2x-3)$$

$$x = 0$$

$$-14 = -7B$$

$$0 = 2A + (-6)$$

$$2 = B$$

$$6 = 2A$$

$$3 = A$$

$$= \int \left( \frac{3}{2x-3} + \frac{2}{x+2} \right) dx$$

$$u = 2x-3$$

$$u = x+2$$

$$du = 2 dx$$

$$du = dx$$

$$\frac{3}{2} du = 3 dx$$

$$\frac{3}{2} \ln(2x-3) + 2 \ln(x+2) + C$$

$$19) \int_0^1 (x+1)e^{x^2+2x} dx =$$

1973 BC 21

(A)  $\frac{e^3}{2}$

(B)  $\frac{e^3-1}{2}$

(C)  $\frac{e^4-e}{2}$

(D)  $e^3-1$

(E)  $e^4-e$

$$u = x^2 + 2x$$

$$du = (2x+2)dx$$

$$\frac{1}{2} du = (x+1)dx$$

$$= \frac{1}{2} (e^3 - e^0)$$

$$= \frac{e^3 - 1}{2}$$

$$= \frac{1}{2} \int_0^3 e^u$$

$$= \frac{1}{2} e^u \Big|_0^3$$

20)

$x$	0	1
$f(x)$	2	4
$f'(x)$	6	-3
$g(x)$	-4	3
$g'(x)$	2	-1

2008 BC 22

The table above gives values of  $f$ ,  $f'$ ,  $g$  and  $g'$  for selected values of  $x$ . If

$$\int_0^1 f'(x)g(x)dx = 5, \text{ then } \int_0^1 f(x)g'(x)dx =$$

(A) -14

(B) -13

(C) -2

(D) 7

(E) 15

$$\begin{aligned}
 u &= f(x) & v &= g(x) \\
 du &= f'(x)dx & dv &= g'(x)dx \\
 &= f(x)g(x) \Big|_0^1 - \int_0^1 f'(x)g(x)dx \\
 &= f(1)g(1) - f(0)g(0) - 5
 \end{aligned}$$

21)  $\int_1^2 \frac{x+1}{x^2+2x} dx = \frac{1}{2} \left[ \ln|x^2+2x| \right]_1^2 = \frac{1}{2} (\ln 8 - \ln 3) = \frac{1}{2} \ln \frac{8}{3} = \frac{3 \ln 2 + 2}{2}$  1985 BC 3

(A)  $\ln 8 - \ln 3$ (B)  $\frac{\ln 8 - \ln 3}{2}$ (C)  $\ln 8$ (D)  $\frac{3 \ln 2}{2}$ (E)  $\frac{3 \ln 2 + 2}{2}$ 

$$\begin{aligned}
 u &= x^2 + 2x & &= \frac{1}{2} \ln|u| \Big|_3^8 \\
 du &= (2x + 2)dx & &= \frac{\ln 8 - \ln 3}{2} \\
 \frac{1}{2} du &= (x + 1)dx & & \\
 &= \frac{1}{2} \int_3^8 \frac{1}{u} du
 \end{aligned}$$

22)  $\int_0^{\pi/4} \tan^2 x dx =$

1973 BC 25

- (A)  $\frac{\pi}{4} - 1$       (B)  $1 - \frac{\pi}{4}$       (C)  $\frac{1}{3}$       (D)  $\sqrt{2} - 1$       (E)  $\frac{\pi}{4} + 1$

$$\int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$\tan x - x \Big|_0^{\pi/4}$$

$$\tan \frac{\pi}{4} - \frac{\pi}{4} - (\tan 0 - 0)$$

$$1 - \frac{\pi}{4}$$

23)  $\int (3x+1)^5 dx =$

(A)  $\frac{(3x+1)^6}{18} + C$

2003 BC 3

(B)  $\frac{(3x+1)^6}{6} + C$

$$u = 3x + 1$$

(C)  $\frac{(3x+1)^6}{2} + C$

$$du = 3 dx$$

(D)  $\frac{(\frac{3x^2}{2} + x)^6}{2} + C$

$$\frac{1}{3} du = dx$$

(E)  $\frac{3x^2}{2} + x + C$

$$\frac{1}{3} \int u^5 du$$

$$\frac{1}{3} \cdot \frac{1}{6} u^6 + C$$

$$\frac{1}{18} (3x+1)^6 + C$$

$$24) \int x^2 \cos(x^3) dx =$$

(A)  $-\frac{1}{3} \sin(x^3) + C$

(B)  $\frac{1}{3} \sin(x^3) + C$

(C)  $-\frac{x^3}{3} \sin(x^3) + C$

(D)  $\frac{x^3}{3} \sin(x^3) + C$

(E)  $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int \cos u du$$

$$\frac{1}{3} \sin(x^3) + C$$

2003 BC 8

$$25) \int xe^{x^2} dx =$$

(A)  $\frac{1}{2} e^{x^2} + C$    (B)  $e^{x^2} + C$    (C)  $xe^{x^2} + C$    (D)  $\frac{1}{2} e^{2x} + C$    (E)  $e^{2x} + C$

2008 BC 2

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$\frac{1}{2} \int e^u du$$

$$\frac{1}{2} e^{x^2} + C$$

26)  $\int_0^8 \frac{dx}{\sqrt{1+x}} =$  1969 BC 4

- (A) 1      (B)  $\frac{3}{2}$       (C) 2      (D) 4      (E) 6

$$u = (1+x)$$

$$du = dx$$

$$\int_1^9 u^{-\frac{1}{2}} du$$

$$= 2 u^{\frac{1}{2}} \Big|_1^9$$

$$= 6 - 2$$

$$= 4$$

$x$	2	5	10	14
$f(x)$	12	28	34	30

27)

2003 BC 25

The function  $f$  is continuous on the closed interval  $[2,14]$  and has values as shown in the table above. Using the subintervals  $[2,5]$ ,  $[5,10]$ , and  $[10,14]$ , what is the approximation of  $\int_2^{14} f(x)dx$  found by using a

right Riemann sum?

\* dont need to be  
same width

- (A) 296  
(B) 312  
(C) 343  
(D) 374  
(E) 390

$$3(28) + 5(34) + 4(30)$$

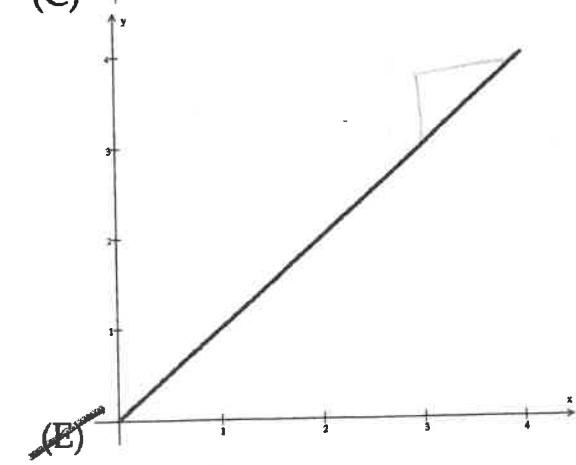
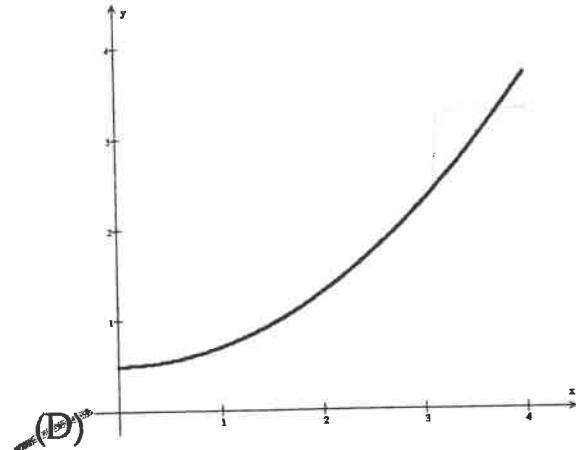
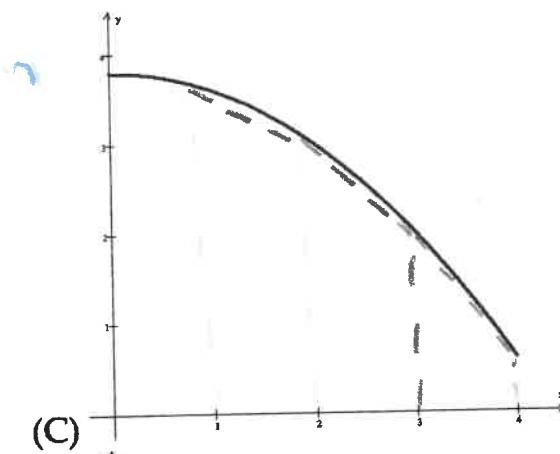
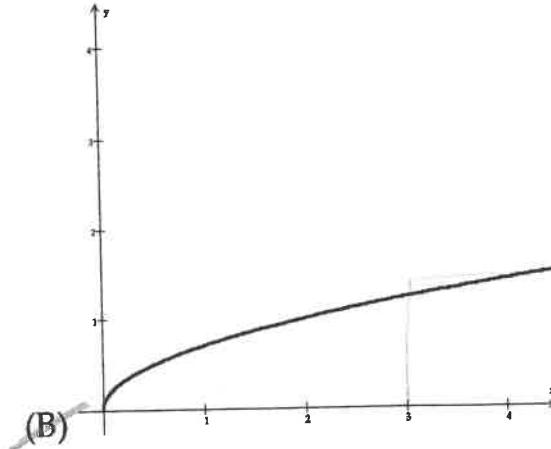
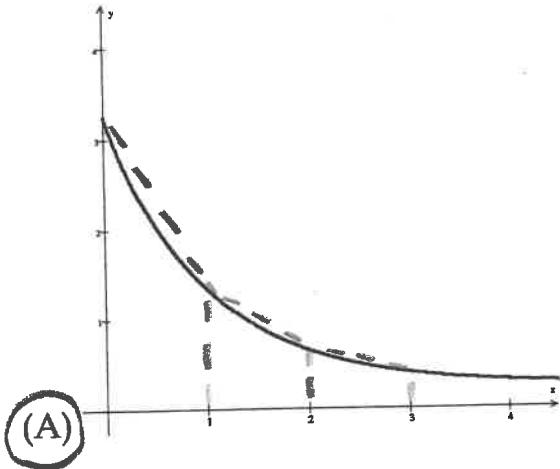
$$= 84 + 170 + 120$$

$$= 84 + 290$$

$$= 374$$

- 28) If a trapezoidal sum overapproximates  $\int_0^4 f(x) dx$ , and a right Riemann sum underapproximates  $\int_0^4 f(x) dx$ , which of the following could be the graph of  $y = f(x)$ ?

2003 BC 85  
616



29) If three equal subdivisions of  $[-4, 2]$  are used, what is the trapezoidal approximation of

$$\int_{-4}^2 \frac{e^{-x}}{2} dx?$$

$$2 - (-4) = 6$$

1988 BC

18

(A)  $e^2 + e^0 + e^{-2}$

(B)  $e^4 + e^2 + e^0$

(C)  $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D)  $\frac{1}{2}(e^4 + e^2 + e^0 + e^{-2})$

(E)  $\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^{-2})$

$$h = \frac{2 - (-4)}{3} = \frac{6}{3} = 2$$

\*watch out

$$\frac{h}{2} (y_0 + 2y_1 + \dots + y_n)$$

function has  $y_2$   
in it \*

$$\frac{2}{2} (e^{-(-4)} + 2e^{-(-2)} + 2e^0 + e^2)$$

$$\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^2)$$

30) Let  $y = f(x)$  be the solution to the differential equation  $\frac{dy}{dx} = x + y$  with 2003 BC

the initial condition  $f(1) = 2$ . What is the approximation for  $f(2)$  if 5

Euler's method is used, starting at  $x=1$  with a step size of 0.5?

(A) 3

(B) 5

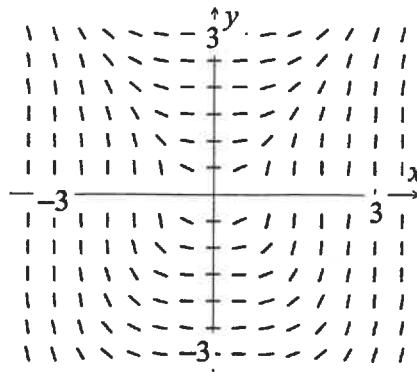
(C) 6

(D) 10

(E) 12

$x$	$y$	$\Delta x$	$\Delta y$	$x + \Delta x$	$y + \Delta y$
1	2	0.5	1.5	1.5	3.5
1.5	3.5	0.5	2.5	2	6
2	6				

31)



when  $x = 0 \quad \frac{dy}{dx} = 0$

when  $y = 0 \quad \frac{dy}{dx} \text{ DNE}$

2003 BC 14

Shown above is a slope field for which of the following differential equations?

- (A)  $\frac{dy}{dx} = \frac{x}{y}$   
 (B)  $\frac{dy}{dx} = \frac{x^2}{y^2}$   
 (C)  $\frac{dy}{dx} = \frac{x^3}{y}$   
 (D)  $\frac{dy}{dx} = \frac{x^2}{y}$   
 (E)  $\frac{dy}{dx} = \frac{x^3}{y^2}$

Quad 1  $\frac{dy}{dx}$  pos

Quad 2  $\frac{dy}{dx}$  neg

Quad 3  $\frac{dy}{dx}$  neg \* both x, y neg

$\Rightarrow$  only option D and E

Quad 4  $\frac{dy}{dx}$  pos \* y neg x pos

$\Rightarrow$  only option E

32)

The general solution for the equation  $\frac{dy}{dx} + y = xe^{-x}$  is

1985 BC 37

(A)  $y = \frac{x^2}{2}e^{-x} + Ce^{-x}$

(B)  $y = \frac{x^2}{2}e^{-x} + e^{-x} + C$

(C)  $y = -e^{-x} + \frac{C}{1+x}$

(D)  $y = xe^{-x} + Ce^{-x}$

(E)  $y = C_1 e^x + C_2 x e^{-x}$

No longer part of AP exam

- 33) If  $\frac{dy}{dx} = x^2 y$ , then  $y$  could be

1993 BC 13 calc

- (A)  $3\ln\left(\frac{x}{3}\right)$       (B)  $e^{\frac{x^3}{3}} + 7$       (C)  $2e^{\frac{x^3}{3}}$       (D)  $3e^{2x}$       (E)  $\frac{x^3}{3} + 1$

$$\int \frac{1}{y} dy = \int x^2 dx$$

$$\ln|y| = \frac{x^3}{3} + C$$

$$y = C e^{\frac{1}{3}x^3}$$

- 34) If  $\frac{dy}{dx} = (1 + \ln x)y$  and if  $y = 1$  when  $x = 1$ , then  $y =$

1997 BC 83 calc

- (A)  $e^{\frac{x^2-1}{x^2}}$   
 (B)  $1 + \ln x$   
 (C)  $\ln x$   
 (D)  $e^{2x+x\ln x-2}$   
 (E)  $e^{x\ln x}$

$$\int \frac{1}{y} dy = \int (1 + \ln x) dx$$

$$\ln|y| = x + \int \ln x dx$$

$$u = \ln x \quad v = x$$

$$du = \frac{1}{x} dx \quad dv = dx$$

$$\ln|y| = x + (x \ln x - \int dx)$$

$$\ln|y| = x + x \ln x - x + C$$

$$\ln|y| = x \ln x + C$$

$$y = C e^{x \ln x}$$

$$1 = C e^{\ln(1)}$$

$$1 = C$$

$$y = e^{x \ln x}$$

35)

The number of bacteria in a culture is growing at a rate of  $3,000e^{2t/5}$  per unit of time  $t$ . At  $t = 0$ , 1973 BC the number of bacteria present was 7,500. Find the number present at  $t = 5$ . 17

- (A)  $1,200e^2$     (B)  $3,000e^2$     (C)  $7,500e^2$     (D)  $7,500e^5$     (E)  $\frac{15,000}{7}e^7$

$$\frac{dN}{dt} = 3000 e^{2t/5}$$

$$\int dN = \int 3000 e^{2t/5} dt$$

$$N = 3000 \left(\frac{5}{2}\right) e^{2t/5} + C$$

$$N = 7500 e^{2t/5} + C \quad N = 7500 e^{2t/5}$$

$$7500 = 7500 e^0 + C \quad N(5) = 7500 e^{10/5} \\ 0 = C \quad = 7500 e^2$$

36)

The rate of change of the volume,  $V$ , of water in a tank with respect to time,  $t$ , is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship? 2003 BC 12

(A)  $V(t) = k\sqrt{t}$

(B)  $V(t) = k\sqrt{V}$

(C)  $\frac{dV}{dt} = k\sqrt{t}$

(D)  $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E)  $\frac{dV}{dt} = k\sqrt{V}$

$$\frac{dV}{dt} = k\sqrt{V}$$

\* recall direct proportion

say  $x$  is directly proportional to  $y$

$$x = ky$$

- 57) The number of moose in a national park is modeled by the function  $M$  that satisfies the logistic differential equation  $\frac{dM}{dt} = 0.6M(1 - \frac{M}{200})$ , where 2003 BC  
21  
 $t$  is the time in years and  $M(0) = 50$ . What is  $\lim_{t \rightarrow \infty} M(t)$ ?

- (A) 50
- (B) 200
- (C) 500
- (D) 1000
- (E) 2000

$$\frac{dM}{dt} = 0.6M \left(1 - \frac{M}{200}\right)$$

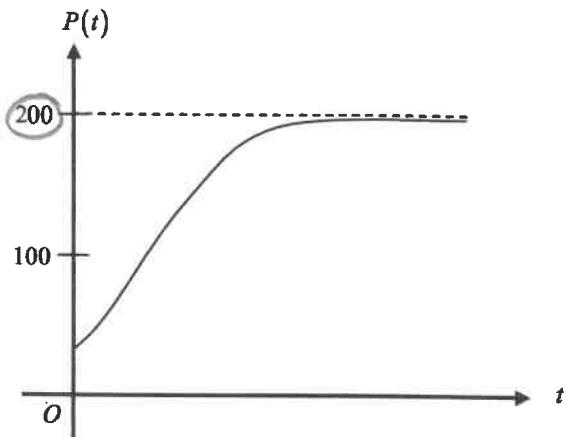
200 is carrying capacity



so when  
 $t \rightarrow \infty$

$\lim_{t \rightarrow \infty} M(t) = \text{carrying capacity}$

38)



Which of the following differential equations for a population  $P$  could model the logistic growth shown in the figure above?

2008 BC 24

(A)  $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B)  $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C)  $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D)  $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E)  $\frac{dP}{dt} = 0.1P^2 + 0.001P$

200 is carrying capacity

$$\frac{dP}{dt} = 0.2P(1 - 0.005P)$$

$$= 0.2P(1 - \frac{P}{200})$$

OR

$$\frac{dP}{dt} = kP(A - P)$$

$$= kP(200 - P)$$

$= k(200P - P^2)$  only A and B work so check both

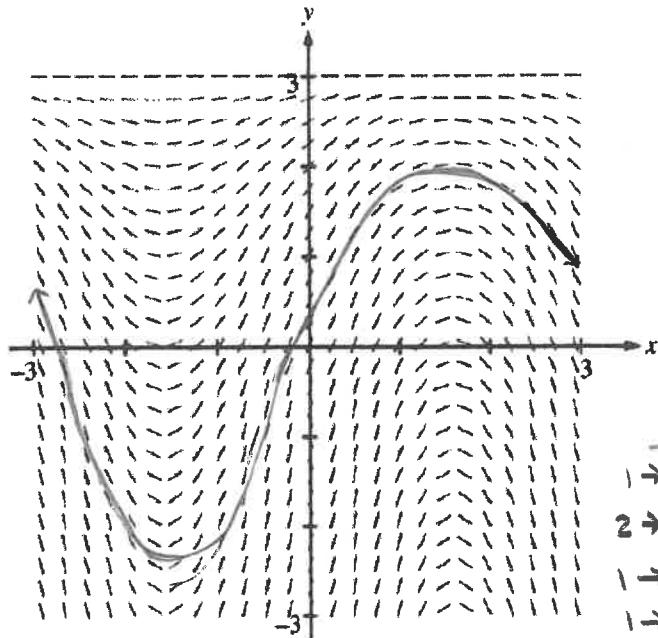
Bonus

2014: AB-6; No Calculator

6. Consider the differential equation  $\frac{dy}{dx} = (3-y)\cos x$ . Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(0) = 1$ . The function  $f$  is defined for all real numbers.

$$\frac{dy}{dx} = (3-y) \cos x \quad \text{(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point } (0, 1).$$

(0, 1)



1 point

part c points

1 → separation of variables

2 → antiderivatives

1 → C

1 → uses initial condition

1 → solves for y

- 2 (b) Write an equation for the line tangent to the solution curve in part (a) at the point  $(0, 1)$ . Use the equation to approximate  $f(0.2)$ .

- 6 (c) Find  $y = f(x)$ , the particular solution to the differential equation with the initial condition  $f(0) = 1$ .

$$(b) \left. \frac{dy}{dx} \right|_{(0,1)} = (3-1)\cos(0) = 2$$

$$y = 1 + 2(x-0)$$

$$y = 2x + 1$$

$$f(0.2) = 2(0.2) + 1 \\ = 1.4$$

tangent line → 1 pt

approximation → 1 pt

$$(c) \frac{dy}{dx} = (3-y)\cos x$$

$$\int \frac{1}{3-y} dy = \int \cos x dx$$

$$-\ln|3-y| = \sin x + C \quad 1 = Ce^{-\sin(0)} + 3$$

$$3-y = e^{-\sin x + C} \quad 1 = C + 3$$

$$-y = Ce^{-\sin x} - 3 \quad -2 = C$$

$$y = Ce^{-\sin x} + 3 \quad y = -2e^{-\sin x} + 3$$

