AB Calculus

Chapter 8 Review

No calculator unless it says "calc" in top right corner of question.

$v(0)=0$

At $t=0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

1969 AB 35

[✓ (A) 32] (B) 48 (C) 64 (D) 96 (E) 192

$v(t) = \int 24t^2\,dt$

$= 8t^3 + C$

$v(0) = 0 = 8(0)^3 + C$

$0 = C$

$v(t) = 8t^3$

$s(t) = \int 8t^3\,dt$

$s(2) = s(0) + \int_0^2 8t^3\,dt$

$s(2) = 0 + 2t^4 \Big|_0^2$

$= 2(2)^4 - 2(0)^4$

$\boxed{= 32}$

The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

1993 AB 11

Calc

$v(3) = 25$ $x(1) = 10$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

[✓ (C) $t^3 - t^2 + 4t + 6$]

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

$v(t) = \int (6t-2)\,dt$

$= 3t^2 - 2t + C$

$v(3) = 25 = 3(3)^2 - 2(3) + C$

$25 = 27 - 6 + C$

$25 = 21 + C$

$4 = C$

$v(t) = 3t^2 - 2t + 4$

$x(t) = \int (3t^2 - 2t + 4)\,dt$

$= t^3 - t^2 + 4t + C$

$x(1) = 10 = 1^3 - 1^2 + 4(1) + C$

$6 = C$

2003 AB 82 Calc

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

[✓ (A) $\int_{1.572}^{3.514} r(t)\,dt$]

(B) $\int_{0}^{8} r(t)\,dt$

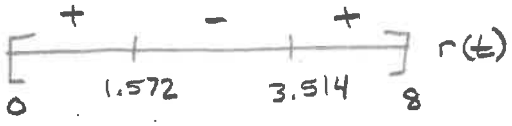
(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

$0 = t^3 - 4t^2 + 6$

$t = -1.086,\ 1.572,\ 3.514$



2008 AB 7

A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \ge 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

$x(0) = 2$ $x(1) = ?$

(A) 4 [✓ (B) 6] (C) 9 (D) 11 (E) 12

$$x(1) = x(0) + \int_0^1 (3t^2 + 6t)\,dt$$

$$x(1) = 2 + \left(t^3 + 3t^2 \Big|_0^1\right)$$

$$= 2 + \left[1 + 3(1) - (0)\right]$$

$$= 2 + 4$$

$$= 6$$

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

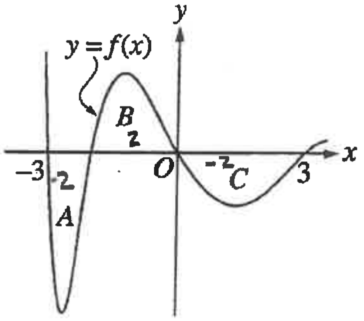
1998 AB 88
Calc

(A) 0.048 (B) 0.144 [✓ (C) 5.827] (D) 23.308 (E) 1,640.250

$$F(9) = F(1) + \int_{1}^{9} \frac{(\ln x)^3}{x}\, dx$$

$$= 0 + 5.827$$

$$= 5.827$$



2003 AB 77
Calc

The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x)+1)\,dx$?

(A) −2 (B) −1 [✓ (C) 4] (D) 7 (E) 12

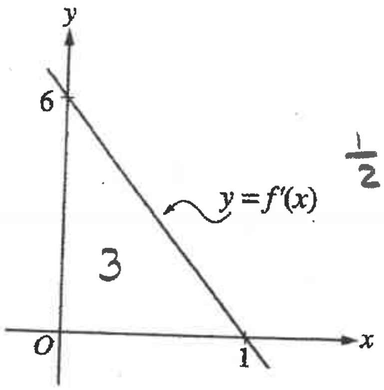
$$\int_{-3}^{3} (f(x)+1)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$$

$$= -2 + x \Big|_{-3}^{3}$$

$$= -2 + (3-(-3))$$

$$= -2 + 6$$

$$= 4$$



$\frac{1}{2}(1)(6) = 3$

2003 AB 22

The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 [✓ (D) 8] (E) 11

$$f(1) = f(0) + \int_0^1 f'(x)dx$$

$$= 5 + 3 \quad = \boxed{8}$$

At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

1997 AB
87
Calc

(A) 1.02 [✓ (B) 1.48] (C) 1.85 (D) 2.81 (E) 3.14

$v(0) = -2$

$v(t) = 0$?

$$v(t) = \int(t + \sin t)dt$$

$$= \frac{1}{2}t^2 - \cos t + C$$

$$v(0) = -2 = \frac{1}{2}(0)^2 - \cos(0) + C$$

$$-2 = -1 + C \qquad -1 = C$$

$$v(t) = \frac{t^2}{2} - \cos t - 1$$

$$0 = \frac{1}{2}t^2 - \cos t - 1$$

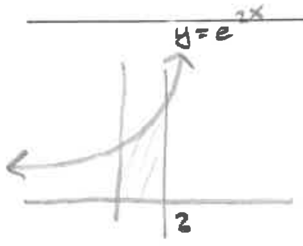
$$t = 1.478$$

The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

1969 AB
23

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ [✓ (C) $\frac{e^4}{2} - \frac{1}{2}$]

(D) $2e^4 - e$ (E) $2e^4 - 2$



$$\int_0^2 (e^{2x})dx$$

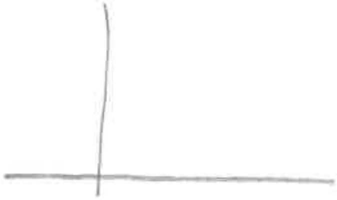
$$= \frac{e^{2x}}{2} \Big|_0^2$$

$$= \frac{e^4}{2} - \frac{1}{2}$$

The area of the region bounded by the lines $x=0$, $x=2$, and $y=0$ and the curve $y=e^{\frac{x}{2}}$ is

1973 AB
15

(A) $\frac{e-1}{2}$ (B) $e-1$ [✓ (C) $2(e-1)$] (D) $2e-1$ (E) $2e$



$$\int_0^2 e^{x/2}\,dx$$

$$= 2e^{x/2}\Big|_0^2$$

$$= 2e - 2$$

The area of the region in the first quadrant that is enclosed by the graphs of $y=x^3+8$ and $y=x+8$ is

1985 AB
34

[✓ (A) $\frac{1}{4}$] (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

$$\int_0^1 [(x+8)-(x^3+8)]\,dx$$

$$= -x^4/4 + x^2/2 \Big|_0^1$$

$$= -1/4 + 1/2 - (0)$$

$$= 1/4$$

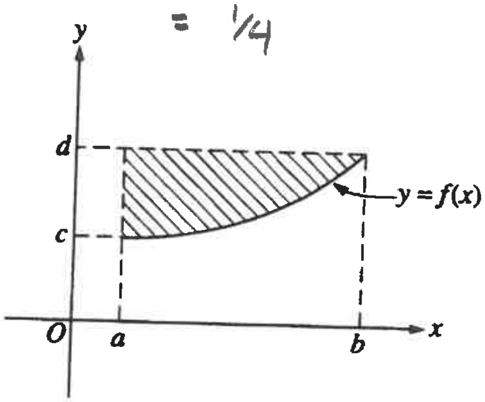
$x+8 = x^3+8$

$0 = x^3 - x$

$0 = x(x-1)$

$x = 0, 1$

1993 AB 2 Calc



Which of the following represents the area of the shaded region in the figure above?

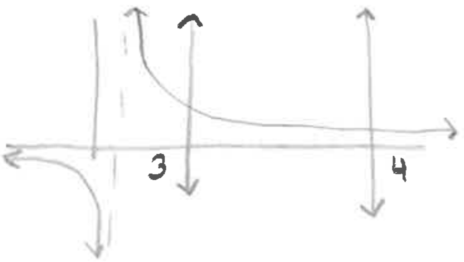
(A) $\int_c^d f(y)dy$ [✓ (B) $\int_a^b (d-f(x))dx$] (C) $f'(b)-f'(a)$

(D) $(b-a)[f(b)-f(a)]$ (E) $(d-c)[f(b)-f(a)]$

The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

1993 AB
6 Calc

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ [✓ (D) $\ln\frac{3}{2}$] (E) $\ln 6$



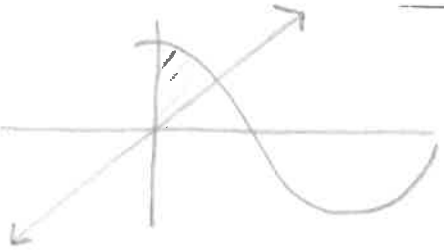
$$\int_3^4 \left(\frac{1}{x-1}\right) dx$$

$$= \ln(3/2)$$

What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

1997 AB 83
Calc

(A) 0.127 (B) 0.385 [✓ (C) 0.400] (D) 0.600 (E) 0.947



$$\int_0^{0.739085} (\cos x - x)dx$$

$$= 0.4004$$

If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

1998 AB 92
Calc

(A) 1.471 (B) 1.414 (C) 1.277 [✓ (D) 1.120] (E) 0.436

$$0.1 = \int_K^{\pi/2} \cos x \, dx$$

$$0.1 = \sin x \Big|_K^{\pi/2}$$

$$0.1 = \sin \pi/2 - \sin K$$

$$0.1 = 1 - \sin K$$

$$0.9 = \sin K$$

$$\sin^{-1}(0.9) = K$$

$$1.119 = K$$

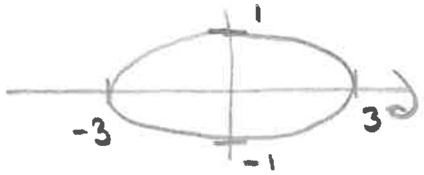
shrinks by 3

$x^2+(3y)^2=9$ r=3

The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2+9y^2=9$ about the x-axis is

1988 AB 43

(A) 2π [✓ (B) 4π] (C) 6π (D) 9π (E) 12π



$x^2+9y^2=9$

$y=\sqrt{1-\frac{1}{9}x^2}$

$$\int_{-3}^{3}\pi\left(\sqrt{1-\tfrac{1}{9}x^2}\right)^2dx$$

$$\pi\int_{-3}^{3}\left(1-\tfrac{1}{9}x^2\right)dx$$

$$\pi\left(x-\tfrac{1}{27}x^3\right)\Big|_{-3}^{3}$$

$$\pi\left[3-\tfrac{1}{27}(3)^3-\left(-3-\tfrac{1}{27}(-3)^3\right)\right]$$

$$\pi\left[3-1+3-1\right]$$

4π

A region in the first quadrant is enclosed by the graphs of $y=e^{2x}$, $x=1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

1988 AB 30

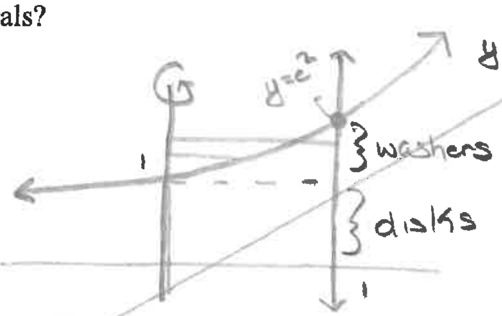
[✓ (A) $2\pi\int_0^1 xe^{2x}\,dx$]

(B) $2\pi\int_0^1 e^{2x}\,dx$

(C) $\pi\int_0^1 e^{4x}\,dx$

(D) $\pi\int_0^e y\ln y\,dy$

(E) $\frac{\pi}{4}\int_0^e \ln^2 y\,dy$



$y=e^{2x}$

$\frac{\ln y}{2}=x$

$$\int_0^1\pi(1)^2\,dy+\left[\int_1^{e^2}\pi(1)^2-\pi\left(\ln y/2\right)^2dy\right]$$

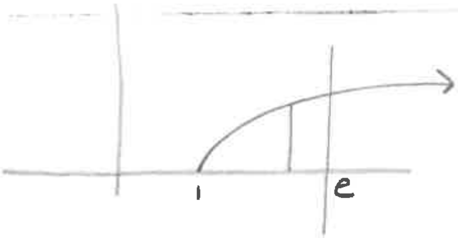
=

answers are for shell method which is no longer ashed on AB test

1997 AB 84
Calc

The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ [✓ (C) 1] (D) 2 (E) $\frac{1}{3}(e^3 - 1)$



$$\int_1^e (\sqrt{\ln x})^2 \, dx$$

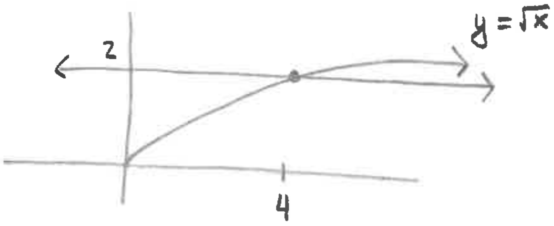
$$\ln x \Big|_1^e$$

$$\ln e - \ln 1$$

$$1$$

Find the **volume** of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=2$ and $x=0$ about:

- a) the x-axis
- b) the y-axis
- c) the line $y=2$
- d) the line $x=4$



a) $$\int_0^4 \left[\pi(2)^2 - \pi(\sqrt{x})^2\right] dx$$

$$= \pi \left(4x - \tfrac{1}{2}x^2 \Big|_0^4\right)$$

$$= \pi \left(16 - 8 - (0)\right)$$

$$= 8\pi$$

b) $$\int_0^2 \pi (y^2)^2 \, dy$$

$$= \frac{\pi}{5} y^5 \Big|_0^2$$

$$= \frac{32\pi}{5} - (0)$$

$$= \frac{32\pi}{5}$$

c) $$\int_0^4 \pi(2-\sqrt{x})^2 \, dx$$

$$= \frac{8\pi}{3}$$

d) $$\int_0^2 \left[\pi(4)^2 - \pi(4-y^2)^2\right] dy$$

$$= \frac{224\pi}{15}$$

calculator

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

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a) $|v(t)| = 2$ *speed $= |v(t)|$

$v(t) = 2$

$t = 3.128$ and 3.473

b) $s(t) = \int_0^t v(x)\,dx$

$s(5) = s(0) + \int_0^5 v(t)dt$

$= 10 + \int_0^5 v(t)dt$

$= -9.207$

d) $v(4) = -11.475758$

$a(4) = v'(4) = -22.295714$

The speed is increasing at time $t=4$ b/c velocity and acceleration have the same sign

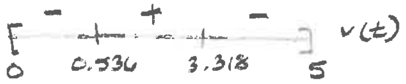
* want same sign for inc
different sign for dec

think about ball thrown in air:
v(t) pos b/c going in pos direction
a(t) neg b/c gravity slowing it

END OF PART A OF SECTION II

c) $v(t) = 0$

$t = 0.536033$ and 3.317756



v(t) changes from negative to positive at time $t = 0.536033$

v(t) changes from positive to negative at time $t = 3.317756$

Therefore the particle changes direction at time $t = 0.536$ and $t = 3.318$

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

calculator

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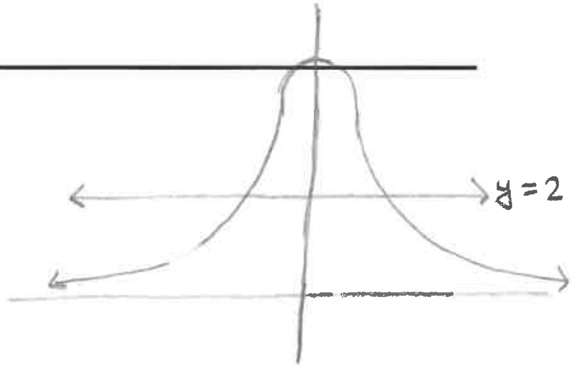
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.



$$\frac{20}{1+x^2} = 2$$

$$x = \pm 3$$

a) $\text{Area} = \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right) dx$

$= 37.962$

b) ★ washers ★

$R = \dfrac{20}{1+x^2} \qquad r = 2$

olume $= \int_{-3}^{3} \left(\pi \left(20/1+x^2\right)^2 - \pi(2)^2\right) dx$

$= 187.190$

c) ★ Diameter $= \dfrac{20}{1+x^2} - 2$

Volome $= \dfrac{\pi}{2} \int_{-3}^{3} \left[\frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)\right]^2 dx$

$= \dfrac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right)^2 dx$

$= 174.268$

2013 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

calculator

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer. compare arriving to processing

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

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a) $G'(5) = -24.588$

The rate at which gravel is arriving is decreasing by 24.588 tons per hour per hour at time $t=5$ hours

* tons per hour per hour b/c before derivative units were tons per hour and derivative = rate * respect to time*

b) $\int_0^8 G(t)\,dt = 825.551$ tons

c) $G(5) = 98.140764 < 100$

At time $t=5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t=5$

d) Amount of gravel at the plant at time t is

$A(t) = A(0) + \int_0^t (G(x) - 100)\,dx$

$= 500 + \int_0^t (G(x) - 100)\,dx$

* max when derivative = 0

$A'(t) = G(t) - 100$

$0 = G(t) - 100$

$t = 4.923480$

t	A(t)
0	500
4.923480	635.376123
8	525.551089

* check endpoints

The max amount of gravel at the plant during the workday is 635.376 tons