

2 Exponents and Radicals Dec 2019 (No Calculators)

3 pts 1. Simplify: $\sqrt{\frac{6^3 \cdot 35^4}{75 \cdot 98}}$

Ans. _____

4 pts 2. Find x given that $8^4 \cdot 4^x = 512$.

Ans. _____

5 pts 3. If $3^{x^2-17} \cdot 9^{2x-5} = 27^{x^2-19}$, find the value(s) of $4^{\frac{3x-5}{2x-6}}$.

Ans. _____

Exponents and Radicals

1. $\sqrt{\frac{6^3 \cdot 35^4}{75 \cdot 98}} = \sqrt{\frac{(2^3 \cdot 3^3) \cdot (5^4 \cdot 7^4)}{(3 \cdot 5^2)}} = \sqrt{2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2} = 2 \cdot 3 \cdot 5 \cdot 7 = 210$. Ans. 210

2. $8^4 \cdot 4^x = 512 \rightarrow 2^{12} \cdot 2^{2x} = 2^9$. Thus $12 + 2x = 9$, $2x = -3$, so $x = -3/2$. Ans. -3/2

3. $3^{x^2-17} 9^{2x-5} = 27^{x^2-19} \rightarrow 3^{x^2-17} 3^{4x-10} = 3^{3x^2-57}$, thus $x^2 - 17 + 4x - 10 = 3x^2 - 57 \rightarrow$

$0 = 2x^2 - 4x - 30 \rightarrow x^2 - 2x - 15 = 0 \rightarrow (x - 5)(x + 3) = 0$. So $x = 5$ or -3 .

$4^{\frac{3x-5}{2x-6}} = 4^{\frac{3(5)-5}{2(5)-6}} = 4^{\frac{5}{2}} = 32$ or $4^{\frac{3(-3)-5}{2(-3)-6}} = 4^{\frac{7}{6}} = 4\sqrt[3]{2}$. Ans. 32 or $4\sqrt[3]{2}$

2 Exponents and Radicals Dec 2018 (No Calculators)

3 pts 1. If $35^5 \cdot 15^6 \cdot 21^7 = 3^a \cdot 5^b \cdot 7^c$, find $a + b + c$.

Ans. _____

4 pts 2. 5206 written in scientific notation is 5.206×10^3 . When $5^{24} \cdot 2^{19}$ is written in scientific notation, it takes on the form $a.bc \times 10^d$, when rounded to two decimal places. Find the sum $a + b + c + d$.

Ans. _____

5 pts 3. Express $\frac{\sqrt[5]{25}}{\sqrt[3]{625}}$ in simplest radical form.

Ans. _____

Exponents and Radicals

1. $35^5 \cdot 15^6 \cdot 21^7 = 5^5 \cdot 7^5 \cdot 3^6 \cdot 5^6 \cdot 7^7 \cdot 3^7 = 5^{11} \cdot 7^{12} \cdot 3^{13}$. $11 + 12 + 13 = 36$.

Ans. 36

2. $5^{24} \cdot 2^{19} = 5^5 \cdot 5^{19} \cdot 2^{19} = 5^5 \cdot 10^{19} = 3125 \times 10^{19} = 3.13 \times 10^{22}$. $3+1+3+22 = 29$.

Ans. 29

3. $\frac{\sqrt[5]{25}}{\sqrt[3]{625}} = \frac{5^{2/5}}{5^{4/3}} = 5^{(\frac{2}{5} - \frac{4}{3})} = 5^{(\frac{6}{15} - \frac{20}{15})} = 5^{(-\frac{14}{15})} = \frac{1}{\sqrt[15]{5^{14}}} \cdot \frac{\sqrt[15]{5}}{\sqrt[15]{5}} = \frac{\sqrt[15]{5}}{5}$.

Ans. $\frac{\sqrt[15]{5}}{5}$

2 Exponents and Radicals Dec 2017 (No Calculators)

3 pts 1. For what value of n does $\frac{\sqrt{2} \cdot \sqrt[3]{4}}{8} = 2^n$?

Ans. _____

4 pts 2. Simplify: $1 - \frac{1}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}}$

Ans. _____

5 pts 3. If $\left(r + \frac{1}{r}\right)^2 = 5$, find the value of $r^3 + \frac{1}{r^3}$.

Ans. _____

Exponents and Radicals

1. $\frac{\sqrt{2} \cdot \sqrt[3]{4}}{8} = \frac{2^{1/2} \cdot 2^{2/3}}{2^3} = 2^{(1/2+2/3)-3} = 2^{7/6-3} = 2^{-11/6}$. So $n = -11/6$

Ans. -11/6

2. $1 - \frac{1}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}} \rightarrow 1 - \frac{1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} + \frac{1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} \rightarrow 1 - \frac{1-\sqrt{3}}{1-3} + \frac{1+\sqrt{3}}{1-3} = 1 + \frac{2\sqrt{3}}{-2} =$

$1 - \sqrt{3}$.

Ans. $1 - \sqrt{3}$

3. $\left(r + \frac{1}{r}\right)^2 = 5$, so (1): $r + \frac{1}{r} = \pm\sqrt{5}$ and (2): $r^2 + 2 + \frac{1}{r^2} = 5$ or $r^2 - 1 + \frac{1}{r^2} = 2$

$r^3 + \frac{1}{r^3} = \left(r + \frac{1}{r}\right)\left(r^2 - 1 + \frac{1}{r^2}\right) = \pm\sqrt{5}(2) = \pm 2\sqrt{5}$

Ans. $\pm 2\sqrt{5}$

2 Exponents and Radicals Dec 2016-2017 (No Calculators)

3 pts 1. Find $a + b$, if m is not zero, given that $(m^3)^{2a} = m^{54}$ and $m^b \cdot m^{2b} \cdot m^{4b} = m^{28}$.

Ans. _____

4 pts 2. Solve for x : $\left(\frac{81}{625}\right)^{x-2} = \left(\frac{125}{27}\right)^{5-2x}$.

Ans. _____

5 pts 3. Find the greatest real solution to the equation $3x^8 + 5x^4 = 2$.

Ans. _____

Exponents and Radicals

1. $6a = 54$, so $a = 9$. $7b = 28$, so $b = 4$. $9 + 4 = 13$.

Ans. 13

2. $\left(\frac{81}{625}\right)^{x-2} = \left(\frac{125}{27}\right)^{5-2x} \rightarrow \left(\frac{3}{5}\right)^{4(x-2)} = \left(\frac{3}{5}\right)^{-3(5-2x)}$. $4x - 8 = -15 + 6x \rightarrow 2x = 7$.

Ans. 7/2

3. $3x^8 + 5x^4 - 2 = 0 \rightarrow (3x^4 - 1)(x^4 + 2) = 0$. Only $3x^4 - 1 = 0$, so $x^4 = \frac{1}{3}$ or $x = \frac{\sqrt[4]{27}}{3}$.

Ans. $\frac{\sqrt[4]{27}}{3}$

2 Exponents and Radicals Dec 2015 (No Calculators)

3 pts 1. Find the positive value of N , so that $(\sqrt{2})^{N^2} = 256$.

Ans. _____

4 pts 2. If $729^{\frac{23}{6}} = x^{\frac{13}{10}}$ and $x = 27^A$, find A .

Ans. _____

5 pts 3. Find the value of k , such that $100^{200} = k(200)^{100}$.

Ans. _____

Exponents and Radicals

1. $(\sqrt{2})^{N^2} = 256 \implies 2^{\frac{1}{2}N^2} = 2^8 \rightarrow N^2 = 16 \rightarrow N = 4$.

Ans. 4

2. $729^{\frac{23}{6}} = x^{\frac{13}{10}} \rightarrow (3^6)^{\frac{23}{6}} = (27^A)^{\frac{13}{10}} \rightarrow 3^{23} = (3^{3A})^{\frac{13}{10}} \rightarrow 23 \rightarrow 23 = \frac{39}{10}A \rightarrow A = \frac{230}{39}$. Ans. $\frac{230}{39}$

3. $100^{200} = k(200)^{100} \rightarrow (100^{100})^2 = k(2 \cdot 100)^{100} \rightarrow (100^{100}) = k(2)^{100} \rightarrow \frac{100^{100}}{2^{100}} = K = 50^{100}$. Ans. 50^{100}

2 Exponents and Radicals Dec 2014 (No Calculators)

3 pts 1. Express $\sqrt[4]{16^6}$ in simplest form.

Ans. _____

4 pts 2. $a^{2/3} \cdot b^{1/4} \cdot c^{7/5} = \sqrt[N]{a^X b^Y c^Z}$. Find the number which is the sum of $N + X + Y + Z$.

Ans. _____

5 pts 3. Find all real values of x such that $\sqrt{5x+4} - 2 = \sqrt{3x-2}$.

Ans. _____

Exponents and Radicals

1. $\sqrt[4]{16^6} = (2^4)^{6/4} = 2^6 = 64.$

Ans. 64

2. $a^{2/3} \cdot b^{1/4} \cdot c^{7/5} = a^{40/60} \cdot b^{15/60} \cdot c^{84/60} = \sqrt[60]{a^{40} b^{15} c^{84}}. 60+40+15+84 = 199.$

Ans. 199

3. $\sqrt{5x+4} - \sqrt{3x-2} = 2$. Squaring: $5x+4 - 2\sqrt{15x^2+2x-8} + 3x-2 = 4.$

$\sqrt{15x^2+2x-8} = 4x-1$. Squaring: $15x^2+2x-8 = 16x^2-8x+1 \rightarrow 0 = x^2-10x+9.$

Factoring: $(x-9)(x-1) = 0$. $x = 9$ or 1 .

Ans. 9 or 1

2 Exponents and Radicals Dec 2013 (No Calculators)

3 pts 1. If $9^6 \cdot 27^5 \cdot 81^4 = 3^x$, find x .

Ans. _____

4 pts 2. Solve the following equation for x :

$$\left(\frac{1}{4}\right)^{-1} \cdot 8^{x-1} = \left(\frac{1}{16}\right)^{\frac{1}{2}x-1}$$

Ans. _____

5 pts 3. What does $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$ equal?

Ans. _____

Exponents and Radicals

1. $9^6 \cdot 27^5 \cdot 81^4 = (3^2)^6 \cdot (3^3)^5 \cdot (3^4)^4 = 3^{12} \cdot 3^{15} \cdot 3^{16} = 3^{43} = 3^x$. So $x = 43$.

Ans. 43

2. $\left(\frac{1}{4}\right)^{-1} \cdot 8^{x-1} = \left(\frac{1}{16}\right)^{\frac{1}{2}x-1} \rightarrow 2^2(2)^{3x-3} = 2^{-2x+4} \rightarrow 3x-1 = -2x+4 \rightarrow 5x=5$. Ans. 1

3. Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$, then $x = \sqrt{6+x} \rightarrow x^2 = 6+x \rightarrow x^2 - x - 6 = 0$.

Factoring: $(x-3)(x+2) = 0$. x cannot = -2. So $x = 3$.

Ans. 3

2 Exponents and Radicals Dec 2012 (No calculators)

3 pts 1. Simplify:

$$\sqrt[3]{\frac{8x^{15}y^9}{z^{12}}}$$

Ans. _____

4 pts 2. Express in simplest form:

$$\sqrt{\frac{60}{7} \cdot \frac{3}{68} \cdot \frac{17}{4} \cdot \frac{51}{19} \cdot \frac{5}{84}}$$

Ans. _____

5 pts 3. Find all values of x such that:

$$\sqrt{6x+5} - \sqrt{9x-2} = 1$$

Ans. _____

Exponents and Radicals

1. $\sqrt[3]{\frac{8x^{15}y^9}{z^{12}}} = \frac{2x^5y^3}{z^4}$

Ans. $\frac{2x^5y^3}{z^4}$

2. $\sqrt{\frac{60}{7} \cdot \frac{3}{68} \cdot \frac{17}{4} \cdot \frac{51}{19} \cdot \frac{5}{84}} = \sqrt{\left(\frac{3}{4} \cdot \frac{51}{68}\right) \left(\frac{5}{7} \cdot \frac{60}{84}\right) \left(\frac{17}{19}\right)} = \frac{3}{4} \cdot \frac{5}{7} \sqrt{\frac{17}{19} \cdot \frac{19}{19}} = \frac{15\sqrt{323}}{532}$

Ans. $\frac{15\sqrt{323}}{532}$

3. Squaring both sides of $\sqrt{6x+5} - \sqrt{9x-2} = 1$: $15x + 3 - 2\sqrt{54x^2 + 33x - 10} = 1 \rightarrow$
 $2\sqrt{54x^2 + 33x - 10} = 15x + 2$. Squaring again: $4(54x^2 + 33x - 10) = 225x^2 + 60x + 4 \rightarrow$
 $216x^2 + 132x - 40 = 225x^2 + 60x + 4 \rightarrow 9x^2 - 72x + 44 = 0$ or $(3x - 2)(3x - 22) = 0$.
 So $x = 2/3$ or $22/3$. Plugging $22/3$ into the original equation produces $7 - 8 = 1$, which is
 not correct. Plugging in $2/3$ produces $3 - 2 = 1$, which is correct. **Ans. 2/3**

2. Exponents and Radicals Dec 2011 (No Calculators)

3 pts 1. If $a^p b^q = \frac{a^5 b^{-3}}{b^{12} a^{-10}} \cdot \frac{a^{-3} b^8}{b^{-2} a^6}$, find the value of $p + q$.

Ans. _____

4 pts 2. Find the positive number x , such that $x^6 = \frac{12^3 \cdot 15^3 \cdot 20^6}{125}$.

Ans. _____

5 pts 3. Solve for x among the real numbers: $4^{4x} - 4^{2x} - 2 = 0$.

Ans. _____

Exponents and Radicals

$$1. a^p b^q = \frac{a^5 b^{-3}}{b^{12} a^{-10}} \cdot \frac{a^{-3} b^8}{b^{-2} a^6} = \frac{a^5 a^{10}}{b^{12} b^3} \cdot \frac{b^8 b^2}{a^6 a^3} = \frac{a^{15} b^{10}}{b^{15} a^9} = \frac{a^6}{b^5} = a^6 b^{-5}.$$

Ans. 1

$$2. x^6 = \frac{12^3 \cdot 15^3 \cdot 20^6}{125} = \frac{(2^3 \cdot 2^3 \cdot 3^3)(3^3 \cdot 5^3)(20^6)}{5^3} = 2^6 \cdot 3^6 \cdot 20^6. \text{ So } x = 120.$$

Ans. 120

3. Let $a = 4^{2x}$. Then $a^2 = 4^{4x}$ and thus $a^2 - a - 2 = 0$ or $(a - 2)(a + 1) = 0$. Thus (1) $4^{2x} = 2$ or (2) $4^{2x} = -1$. In (1) $2^{4x} = 2^1$, so $4x = 1$ or $x = \frac{1}{4}$. (2) will produce an unreal number.

Ans. $\frac{1}{4}$

2 Exponents and Radicals (No Calculators) Dec 2010

3 pts 1. Simplify: $\sqrt[3]{3672}$

Ans. _____

4 pts 2. Find all values of x such that $\frac{\sqrt{x+12}}{\sqrt{x+4}} = \frac{\sqrt{33-x}}{\sqrt{x+7}}$.

Ans. _____

5 pts 3. If $4^x - 4^{x-1} = 24$, find the exact value of $(2x)^x$.

Ans. _____

Exponents and Radicals

1. $\sqrt[3]{3672} = \sqrt[3]{36(102)} = \sqrt[3]{36 \cdot 6 \cdot 17} = 6\sqrt[3]{17}$ Ans. $6\sqrt[3]{17}$

2. $\frac{\sqrt{x+12}}{\sqrt{x+4}} = \frac{\sqrt{33-x}}{\sqrt{x+7}} \rightarrow x^2 + 19x + 84 = -x^2 + 29 + 132 \rightarrow 2x^2 - 10x - 48 = x^2 - 5x - 24.$

$(x-8)(x+3) = 0$. So $x = 8$ or -3 .

Ans. 8 or -3

3. $4^x - 4^{x-1} = 24 \rightarrow 2^{2x} - 2^{2(x-1)} = 24 \rightarrow 2^{2x} - 2^{2x-2} = 24 \rightarrow 2^{2x} - \frac{1}{4}(2^{2x}) = 24 \rightarrow \frac{3}{4}(2^{2x}) = 24$

$2^{2x} = 32 = 2^5$. $x = 5/2$. $(2x)^x = (5)^{5/2} = 25\sqrt{5}$.

Ans. $25\sqrt{5}$

2 Exponents and Radicals Dec 09 (No Calculators)

3 pts 1. Solve the following for x: $\sqrt[3]{432} = x\sqrt[3]{2}$.

Ans. _____

4 pts 2. Simplify: $\left(\frac{3^0 \cdot 2^{-2} \cdot 5^3}{3 \cdot 5 \cdot 12}\right)^{1/2}$

Ans. _____

5 pts 3. Express the following in simplest form: $\frac{1+\sqrt[4]{2}}{1-\sqrt[4]{2}}$

Ans. _____

Exponents and Radicals

1. $432/2 = 216 = 6^3$. So $x = 6$.

Ans. 6

2. $\left(\frac{3^0 \cdot 2^{-2} \cdot 5^3}{3 \cdot 5 \cdot 12}\right)^{1/2} = \left(\frac{5^3}{3 \cdot 5 \cdot 4 \cdot 12}\right)^{1/2} = \left(\frac{5^2}{3^2 \cdot 4^2}\right)^{1/2} = \frac{5}{12}$

Ans. 5/12

3. $\frac{1+\sqrt[4]{2}}{1-\sqrt[4]{2}} \cdot \frac{1+\sqrt[4]{2}}{1+\sqrt[4]{2}} = \frac{1+2\sqrt[4]{2}+\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+2\sqrt[4]{2}+\sqrt{2}+\sqrt{2}+2\sqrt[4]{2} \cdot \sqrt{2}+2}{-1} =$
 $\frac{3+2\sqrt[4]{2}+2\sqrt{2}+2\sqrt[4]{8}}{-1}$

Ans. $-3-2\sqrt[4]{2}-2\sqrt{2}-2\sqrt[4]{8}$