3 Lines, Angles and Polygons Dec 2019 (No Calculators)

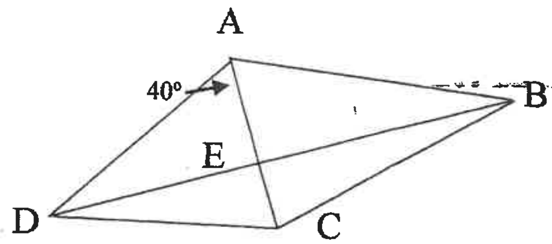
3 pts 1. Find the sum of the number of sides, number of vertices and number of diagonals of a convex heptagon.

Ans. ______________

4 pts 2. Each exterior angle of a regular polygon measures $3\frac{3}{4}$ degrees. How many diagonals can be drawn in the polygon that do not pass through the center?

Ans. ______________

5 pts 3. Given $m\angle CAD = 40^o$, $\overline{AC}$ bisects $\angle DCB$, ~~AD = AB, and the ratio~~ of the measure of $\angle ADE$ to the measure of $\angle ACB$ is 1:2. If $m\angle DBC > 0^o$, find the minimum integer measure of $\angle BDC$.



Ans. ______________

Lines, Angles and Polygons

1. 7 sides, 7 vertices and $\frac{7\cdot 4}{2} = 14$ diagonals. $7 + 7 + 14 = 28$. **Ans. 28**

2. The polygon has $\frac{360}{3\frac{3}{4}} = \frac{360\cdot 4}{15} = 24(4) = 96$ sides and 96 vertices. The total number of diagonals is $\frac{96\cdot 93}{2} = 48(93) = 4464$. Since there are an even number of vertices, one diagonal from each vertex crosses through the center and connects to another vertex, 48 in all. Thus required answer id $4464 - 48 = 4416$. **Ans. 4416**

3. Let $m\angle ADE = x$. Then $m\angle ABC = x$ and $m\angle ACB = m\angle ACD = 2x$. $m\angle DEC = 40 + x$, because of the Exterior Angle Theorem [redacted]
$m\angle CEB = 180 - (40+x) = 140 - x$. $m\angle DBC = 180 - (140 - x + 2x) = 40 - x$
Since $m\angle$[redacted] DBC > 0, then $40 - x > 0$ or $x < 40$. Thus $m\angle DCE < 80$ and $m\angle DEC < 80$. So in triangle DEC, the $m\angle EDC$ must be greater than 20. Minimum integer is 21. **Ans. 21**

3 Lines, Angles and Polygons Dec 2018 (No Calculators)

3 pts 1. A regular quadrilateral and a regular hexagon have the same perimeter. If a side of the hexagon is 8, what is a side of the quadrilateral?

Ans. ________________

4 pts 2. A regular polygon with whole number sides, each side greater than 1, has a perimeter of 161. If it has at least 8 sides, how many diagonals does it have?

Ans. ________________

5 pts 3. The midpoints of the sides of a square, which is 20 units on a side, are connected to form a second square. The midpoints of the sides of this new square are joined to form a third square, and so on until 10 new squares are formed. What is the area of the tenth square formed?

Ans. ________________

Lines, Angles and Polygons

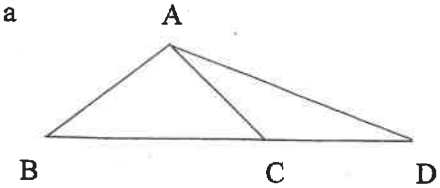
1. Perimeter of hexagon $= 6(8) = 48$. $4s = 48$, $s = 12$ for side of quadrilateral **Ans. 12**

2. $161 = 7(23)$, so it has 23 sides. It has $\frac{23(20)}{2} = 23(10) = 230$ diagonals. **Ans. 230**

3. The side of the first new rectangle made if $10\sqrt{2}$ long. The next is 10 units long. The 10^{th} is $20\left(\frac{1}{\sqrt{2}}\right)^{10}$ long or $\frac{20}{\left(\sqrt{2}\right)^{10}} = \frac{20}{2^5} = \frac{20}{32} = \frac{5}{8}$. So the area of the square is $\frac{25}{64}$. **Ans. 25/64**

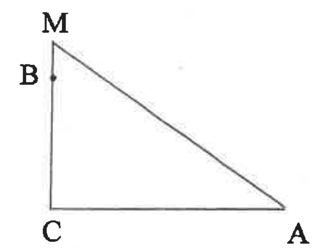
3 Lines, Angles and Polygons Dec 2017 (No Calculators)

3 pts 1. In the figure AB = AC = CD, angle BAC is a right angle and C lies on side BD. Find $m\angle DAB$.



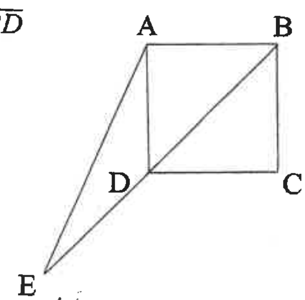
Ans. ____________

4 pts 3. In right triangle ACM shown, MA + MB = BC + AC. If BC = 8 and AC = 10, find the length of segment MB.



Ans. ____________

5 pts 2. In rectangle ABCD, AB = 4 and BC = 5. Extend $\overline{BD}$ to E so that DE = BD. How long is segment AE?



Ans. ____________

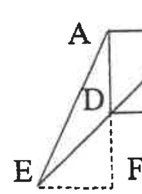
Lines, Angles and Polygons

1. $m\angle ACB = 45°$, so $m\angle 22.5°$. Thus $m\angle DAB = 112.5°$. **Ans. 112.5°**

2. Let BM = x and AM = y. Then (1): $x + y = 18$, and (2): $(x + 8)^2 + 10^2 = y^2$. In (2):

$x^2 + 16x + 64 + 100 = (18 - x)^2 = 324 - 36x + x^2 \Rightarrow 52x = 160,\ x = 3\frac{1}{13}$. **Ans.** $\mathbf{3\frac{1}{13}}$

3. Extending $\overline{AD}$ through point D to F the length of $\overline{AD}$ and then connecting E to F will make right triangle DEF. Since DF = 5, then EF = 4. $(AE)^2 = 10^2 + 4^2 = 116$. So $AE = \sqrt{116} = 2\sqrt{29}$.



Ans. $\mathbf{2\sqrt{29}}$

3 Lines, Angles and Polygons Dec 2016-17 (No Calculators)

3 pts 1. Consider the parallelogram ABCD, where $m\angle A = 54 + x$ and the $m\angle B = 84 + 2x$, both measured in degrees. Find the measure of angle D in degrees.

Ans. ________________

4 pts 2. The complement of the supplement of angle A is equal to twice the complement of half of angle A. Give the degree measure of angle A.

Ans. ________________

5 pts 3. Suppose that a sheet of paper with dimensions 8 inches by 8 inches has the four corners folded toward the center in such a way that it forms a regular octagon. What is the length of each side of the octagon?

Ans. ________________

Lines, Angles and Polygons

1. $54 + x + 84 + 2x = 180 \Rightarrow 3x + 138 = 180 \Rightarrow 3x = 42$, so $x = 14$ and $D = 84 + 28$. **Ans. 112**
2. $90 - (180 - A) = 2(90 - \frac{1}{2}A) \Rightarrow A - 90 = 180 - A \Rightarrow 2A = 270$, so $A = 135$. **Ans. 135**
3. Let the side of the octagon be s. Then $8 = s + 2\left(\frac{s}{\sqrt{2}}\right) = s + s\sqrt{2} = s\left(1+\sqrt{2}\right)$. Threfore

$$s = \frac{8\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} = \frac{8-8\sqrt{2}}{1-2} = 8\sqrt{2} - 8.$$

Ans. $8\sqrt{2} - 8$

3 Lines, Angles and Polygons Dec 2015 (No Calculators)

3 pts 1. Seven of the interior angles in an octagon measure $100^o, 110^o, 120^o, 130^o, 140^o, 150^o$ and 160^o. Find the degree measure of one of the two exterior angles at the vertex of the missing angle.

Ans. ________________

4 pts 2. Transversals h and m cross the two parallel lines j and k. One angle formed at the h-j intersection measures 100 degrees and one angle formed at the m-k intersection measures 60 degrees. Find the greatest possible measure of the acute angle formed at the h-m intersection.

Ans. ________________

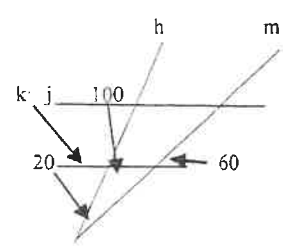
5 pts 3. Regular hexagon ABCDEF, where the vertices are labeled clockwise in alphabetical order, is mounted off-center with the hour and minute hands of a clock so that the hands turn on an axis through vertex A. At 12:00, the hour hand and minute hands point exactly from A to B. If the minute hand is observed when the hour hand points from A to C, from A to D, from A to E, and from A to F, at how many of these four will the minute hand point from A to B?

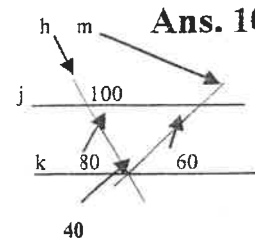
Ans. ________________

Lines, Angles and Polygons

1. Sum of all angles of an octagon: 6(180) = 1080. Sum of the 7: 910. 1080 − 910 = 170. The exterior angle measure is 10 degrees. **Ans. 10**

2. The figures at right show the two possibilities. In one the angle is 20°. In the other it is 40°.

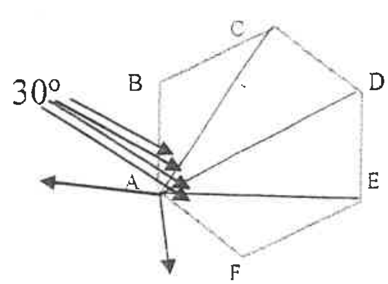




Ans. 40°

3. There are 120 degrees in each angle of a regular hexagon. From the diagram, $\angle BAC = \angle CAD = \angle DAE = \angle EAF = 30^o$.

Since $\frac{360}{12} = 30$, the hour hand points to C at 1:00, to D at 2:00, to E at 3:00, and to F at 4:00, so all 4. **Ans. 4**



3 Lines, Angles and Polygons Dec 2014 (No Calculators)

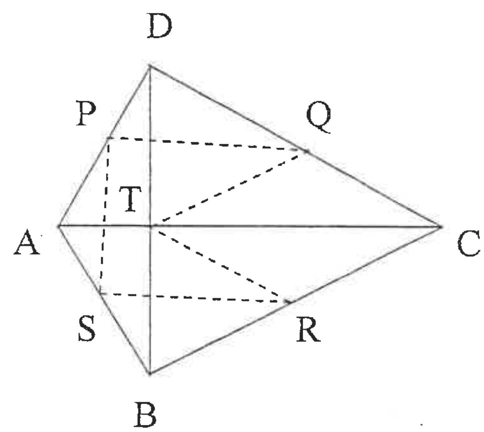
3 pts 1. Find the measure of an angle if the supplement of the angle is 40° larger than twice its complement.

Ans. ________________

4 pts 2. The measure of each angle of a regular polygon is $162\frac{6}{7}$ degrees. How many diagonals does the polygon have?

Ans. ________________

5 pts 3. In the figure at right $\overline{AC} \perp \overline{BD}$ at T. P, Q, R, and S are midpoints. AT = 5, BT = 10, CT = 16, and DT = 12. Find the perimeter of pentagon TRSPQ.



Ans. ________________

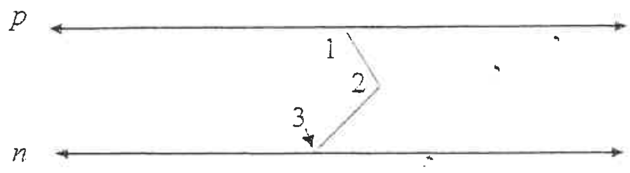
Lines, Angles and Polygons

1. $180 - n = 40 + 2(90 - n) \rightarrow -n = 40 - 2n.\ \ n = 40.$ **Ans. 40**

2. $180 - 162\frac{6}{7} = 17\frac{1}{7}.\ \ \frac{360}{17\frac{1}{7}} = \frac{360}{\frac{120}{7}} = 360 \cdot \frac{7}{120} = 21$ (# of sides). $\frac{21(18)}{2} = 189.$ **Ans. 189**

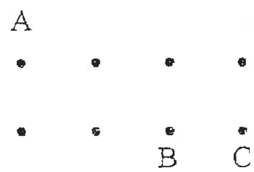
3. Using the Pythagorean Theorem: DC = 20, BC = $\sqrt{356} = 2\sqrt{89}$, and. TQ = ½ DC = 10, TR = ½ BC = $\sqrt{89}$, because the median to the hypotenuse is half as long as the hypotenuse. SR = ½ (21) = 10.5, PS = ½ BD = 11, and PQ = ½ (21) = 10.5, because the segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long. $10 + \sqrt{89} + 10.5 + 11 + 10.5 = \sqrt{89} + 42.$ **Ans. $42 + \sqrt{89}$**

3 pts 1. In the figure, line p is parallel to line n, $m\angle 1 = 100°$, and $m\angle 2 = 120°$. Find the measure of angle 3.



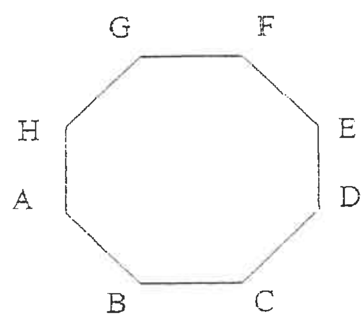
Ans. ______________

4 pts 2. On the square grid shown each of the points, vertical and horizontal, is one unit apart and $AB = 3\sqrt{5}$. Find the length AC.



Ans. ______________________

5 pts 3. Regular octagon ABCDEFGH has a perimeter of 48. Find the length AE.



Ans. __________________

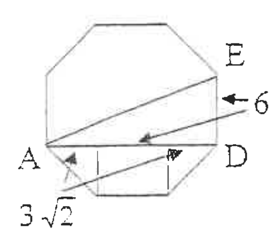
Lines, Angles and Polygons

1. If you run line m parallel to the other two lines but passing through the vertex at angle 2, then angle two is split into an $80°$, the supplement of $\angle 1$, and a $40°$ angle. The supplement of $40°$ is $140°$, the measure of $\angle 3$. **Ans. 140°**

2. Let the unit between dots be x. Then $x^2 + (2x)^2 = (3\sqrt{5})^2 \Rightarrow 5x^2 = 45$, so $x = 3$. So $AC^2 = 3^2 + 9^2 = 90$, thus $AC = 3\sqrt{10}$. **Ans. $3\sqrt{10}$**

3. At right $AD = 6 + 6\sqrt{2}$. $AE^2 = (6 + 6\sqrt{2})^2 + 6^2 =$

$36 + 72\sqrt{2} + 72 + 36 = 144 + 72\sqrt{2}$. $AE = \sqrt{144 + 72\sqrt{2}} =$
$\sqrt{36(4 + 2\sqrt{2})} = 6\sqrt{4 + 2\sqrt{2}}$.



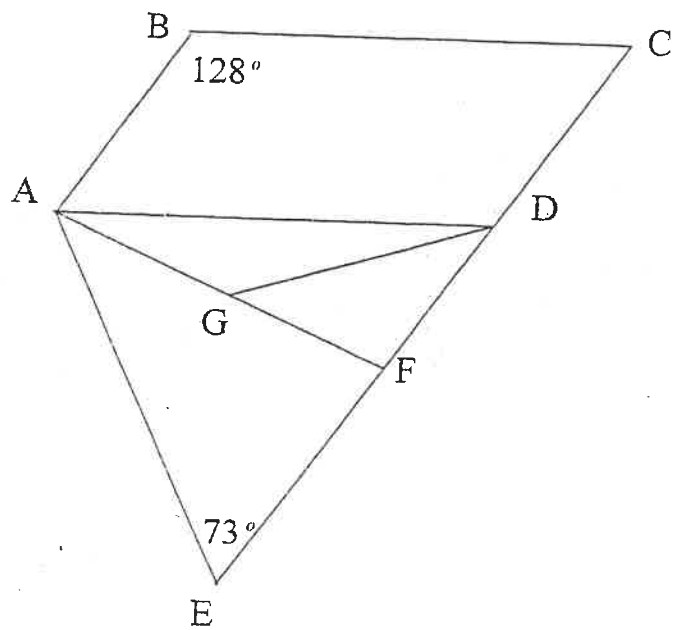
Ans. $6\sqrt{4 + 2\sqrt{2}}$

3 Lines, Angles and Polygons Dec 2012 (No calculators)

3 pts 1. One of the angles of a regular pentagon is given as $5x - 2$. Find x.

Ans. ____________________

4 pts 2. Quadrilateral ABCD is a parallelogram. $\overline{CD}$ is extended through D to point E. A and E are then connected. $\overline{AF}$ bisects $\angle DAE$ and $\overline{DG}$ bisects $\angle ADF$. If $m\angle B = 128°$, and $m\angle E = 73°$. Find $m\angle DGF$.



Ans. ____________________

5 pts 3. A circle is circumscribed about a regular dodecagon whose perimeter is 108. Find the radius of the circle.

Ans. ____________________

Lines, angles and Polygons 2012

1. The number of degrees in each angle of a regular pentagon is $180 - \frac{360}{5} = 108$.

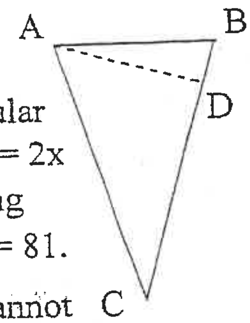
Ans. 22

$5x - 2 = 108 \Rightarrow 5x = 110$, so $x = 22$.

2. $\angle ADB$ is a supplement of $128 = 52$. $\angle DAE = 180 - (73 + 52) = 55$.
So $m\angle DAG = 27\frac{1}{2}$ and $m\angle ADG = 26$. Since $\angle DGF$ is an exterior angle of $\triangle ADG$, then $m\angle DGF = 26 + 27\frac{1}{2} = 53\frac{1}{2}$.

Ans. 53½°

3. The measure of each vertex angle of the isosceles triangle made by the center of the circle and each chord of the dodecagon is $360/12 = 30$. Using the triangle ABC at right, dropping a perpendicular From A to meet $\overline{BC}$ at D forms a 30-60-90$\triangle$. Let $AD = x$, then $AC = 2x$ and $CD = x\sqrt{3}$. $\triangle ABD$ is a right triangle then $BD = 2x - x\sqrt{3}$. Using the Pyth. Thm. $x^2 + (2x - x\sqrt{3})^2 = 9^2 \Rightarrow x^2 + 4x^2 - 4x^2\sqrt{3} + 3x^2 = 81$.
$8x^2 - 4x^2\sqrt{3} = 81 \Rightarrow x^2(8 - 4\sqrt{3}) = 81 \Rightarrow x = \frac{\pm 9}{\sqrt{8 - 4\sqrt{3}}}$. Since x cannot be negative: $x = \frac{9\sqrt{2+\sqrt{3}}}{2\sqrt{2-\sqrt{3}}\sqrt{2+\sqrt{3}}} = \frac{9\sqrt{2+\sqrt{3}}}{2}$. Therefore $AC = 9\sqrt{2+\sqrt{3}}$. **Ans. $9\sqrt{2+\sqrt{3}}$**



3 Lines, Angles and Polygons Dec 2011 (No Calculators)

3 pts 1. If all the diagonals are drawn inside a regular hexagon, at how many distinct points do the diagonals intersect inside of, but not on the boundary, of the polygon?

Ans. ____________________

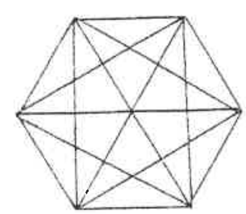
4 pts 2. In a plane, a triangle is constructed as follows: Line segment AB is drawn and its midpoint, X, marked. $30°$ of arc is drawn from point A with center at X. The end of the arc is marked as C. Then line segments AC and BC are drawn, completing the triangle. Find the measure of angle ACB.

Ans. ____________________

5 pts 3. Each side of a regular octagon is 4 units long. How long is its longest diagonal?

Ans. ____________________

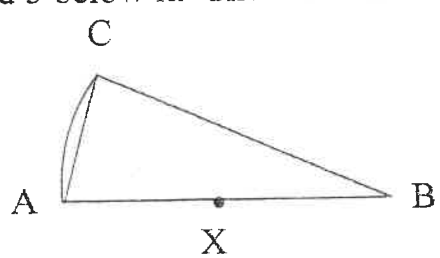
Lines, angles and Polygons



1. At right there are 5 above the horizontal center diagonal and 5 below it. There are three on it. That is a total of 13.

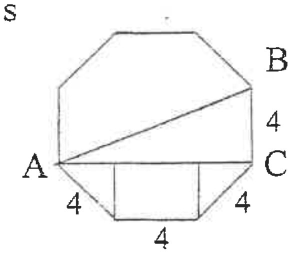
Ans. 13

2.

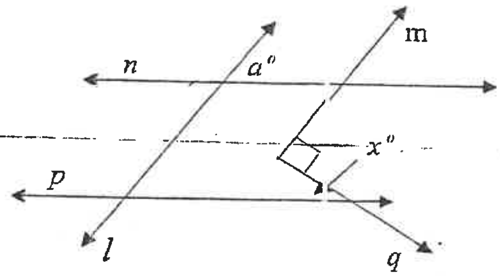


Since A, B and C are all points on a circle centered a X, then $\angle ACB$ is inscribed in a semicircle and the measure of angle ACB is $90°$. **Ans. 90**

3. At right $\overline{AB}$ is one of the longest diagonals. BC in triangle ABC is 4 and AC = $4 + 2(2\sqrt{2})$, since each of the legs of the 45-45-90 Δ's formed is $2\sqrt{2}$. So AB = $\sqrt{4^2 + (4+4\sqrt{2})^2} = \sqrt{16+16+32\sqrt{2}+32} = \sqrt{64+32\sqrt{2}} = 4\sqrt{4+2\sqrt{2}}$. **Ans.** $4\sqrt{4+2\sqrt{2}}$



3 pts 1. In the figure at right, line l is parallel to ray m, line n is parallel to line p, and ray m is perpendicular to ray q. Find x in terms of a.

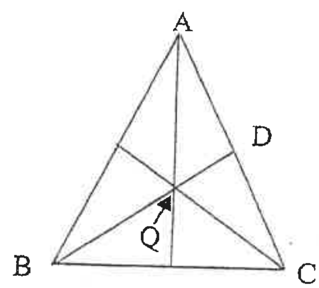


Ans. ____________

4 pts 2. A polygon has 135 diagonals. Find the sum of its interior angles.

Ans. ____________

5 pts 3. Triangle ABC is isosceles. Q is the centroid of the triangle. AC = AB = 2BC. BC = 8. Find the unit length of $\overline{BD}$ in simplest form.



Ans. ____________

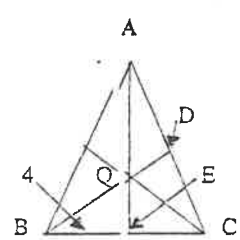
Lines, Angles and Polygons

1. If you make the ray opposite to m intersect p, you make a right triangle. Where m intersects n, the alternate interior angle is a. Where the ray opposite to m intersect p, the alternate interior angle is a. So $x = 90 - a$. **Ans. 90 - a**

2. $\frac{n(n-3)}{2} = 135 \rightarrow$ n(n-3) = 270. 18(15) = 270. So n = 18. (18-2)180 = **Ans. 2880**

3. AE = $\sqrt{16^2 - 4^2} = 4\sqrt{15}$. Thus EQ = $\frac{4}{3}\sqrt{15}$, since the centroid is 2/3 of the distance from a vertex to the opposite side.

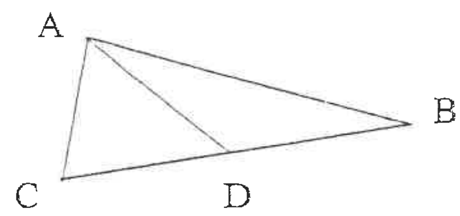
BQ = $\sqrt{\left(\frac{4}{3}\sqrt{15}\right)^2 + 4^2} = \sqrt{\frac{16\cdot15}{9} + \frac{16\cdot9}{9}} = \frac{4}{3}\sqrt{24}$ or $\frac{8}{3}\sqrt{6}$.



BD = $\frac{3}{2}\cdot\frac{8}{3}\sqrt{6} = 4\sqrt{6}$. **Ans. $4\sqrt{6}$**

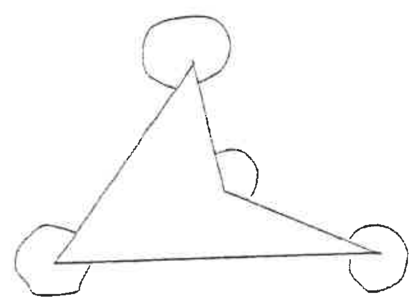
3 Lines, Angles and Polygons Dec 09 (No Calculators)

3 pts 1. In the triangle below, AB = CB and $\overline{AC} \cong \overline{AD} \cong \overline{BD}$. Determine the measure of angle ABC.



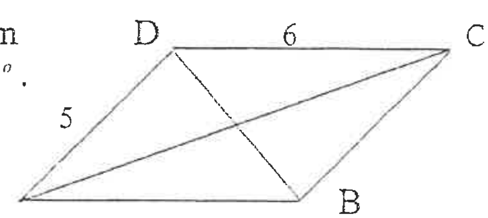
Ans. ____________________

4 pts 2. Let's define a new kind of exterior angle and call it the "outer angle". The quickest way to define the "outer" angle is visually. The "outer angles" are drawn freehand in the quadrilateral below. What is the sum of the "outer angles" of a convex or concave polygon of n sides?



Ans. ____________________

5 pts 3. The lengths of the sides in the parallelogram ABCD at right are 5 and 6, and $0^{\circ} < m\angle DAB < 90^{\circ}$. If diagonal AC = w and diagonal BD = z, find the value of $w^2 + z^2$.



Ans. ____________________

2009

1. m∠ADC = m∠ADC, let each be y. m∠B = m∠BAD, let each be x.
$y = 2x$, because the measure of the exterior ∠ADC = sum of remote interior angles.
Since AB = BC, then m∠BAC = y. Thus in △ABC, (2) $x + 2y = 180$. Substituting:
$x + 2(2x) = 180 \rightarrow 5x = 180$ or $x = 36$. **Ans. 36 or 36°**

2. The sum of the interior angles of any polygon is $180(n-2)$. Each "outer angle" + each of its corresponding interior angle = 360. Since there are n such angles in an n-gon, then the sum of all of these is 360n. $360n - 180(n-2) = 180n + 360$. **Ans. 180n + 360**

3. Let w = length of the other diagonal. In the figure:
$25 = x^2 + y^2$. $z^2 = y^2 + (6-x)^2$. $w^2 = y^2 + (6+x)^2$.
We need $z^2 + w^2$, so $y^2 + (6-x)^2 + y^2 + (6+x)^2 =$
$2y^2 + 72 + 2x^2 = 72 + 50 = 122$. **Ans. 122**