4 Complex Numbers Dec 2019 (No Calculators)

3 pts **1.** Find the real number R, if $(6-4i)(R+2i)=17+6i$.

Ans. ____________________

4 pts **2.** If a and b are integers and $(3+ai)(b+5i)=32-i$, find the value of $a-b$.

Ans. ____________________

5 pts **3.** Express the expansion of $[1-\sqrt{3}\,i]^{10}$ in simplest a + bi form.

(7) Ans. ____________________ **5 pts**

Complex Numbers

1. $(6-4i)(R+2i)=17+6i \Rightarrow 6R+12i-4Ri-8i^2=17-6i$, $6R+8=17$, $R=3/2$. **Ans. 3/2**

2. $(3+ai)(b+5i)=32-i \Rightarrow 3b+15i+abi+5ai^2=32-i \Rightarrow 3b+15i+abi-5a=32-i$.

Thus (1) $3b-5a=32$ and $15+ab=-1$ or (2) $ab=-16$. In (2): $b=-16/a$. Therefore in (1):

$3(-16/a)-5a=32 \Rightarrow -48-5a^2=32a \Rightarrow 0=5a^2+5a+32=(5a+12)(a+4)$. So $a=-4$, since a and b are integers. In (2): $(-4)b=-16$, so $b=4$. $a-b=-4-4=-8$. **Ans. -8**

3. $[1-\sqrt{3}\,i]^2=1-2\sqrt{3}\,i+3i^2=-2-2\sqrt{3}\,i$. $[-2-2\sqrt{3}\,i]^2=4+8\sqrt{3}\,i-12=-8+8\sqrt{3}\,i$.

$[-8+8\sqrt{3}\,i]^2=64-128\sqrt{3}\,i-192=-128-128\sqrt{3}\,i$. This is the 8th power.

The tenth power is the 8th times the 2nd: $(-128-128\sqrt{3}\,i)(-2-2\sqrt{3}\,i)=256+512\sqrt{3}\,i-768=$

$-512+512\sqrt{3}\,i$. Using DeMoivre's Theorem: $[2(\cos 10(-60)+i\sin 10(-60)]^{10}=$

$2^{10}(\cos 120+i\sin 120)=1024(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)=-512+512\sqrt{3}\,i$. **Ans. $-512+512\sqrt{3}\,i$**

4 Complex Numbers Dec 2018 (No Calculators)

3 pts 1. Expand $(1 + i)^6$ **Ans.** ____________________

4 pts 2. Simplify $(8 + 4i)^2 - (8 - 4i)^2$. **Ans.** ____________________

5 pts 3. Find the following sum, $|a| + |b|$, if $(5 + 2i)(a + bi) = 51 - 26i$.

Ans. ____________________

Complex Numbers

1. $(1 + i)^2 = (1 + i)(1 + i) = 1 + 2i + i^2 = 2i$. $(1 + i)^6 = ((1 + i)^2)^3 = (2i)^3 = -8i$. **Ans. -8*i***

2. $a^2 - b^2 = (a + b)(a - b)$, so $(8 + 4i + 8 - 4i)(8 + 4i - 8 + 4i) = 16(8i) = 128i$. **Ans. 128*i***

3. $(5 + 2i)(a + bi) = 51 - 26i$ ➔ $5a + 5bi + 2ai - 2b = 51 - 26i$, so (1) $5a - 2b = 51$ and (2) $2a + 5b = -26$. $5(1) + 2(2)$ yields $29a = 203$, or $a = 7$. In (1): $5(7) - 2b = 51$. $-16 = 2b$, So $b = -8$. Thus $|7| + |-8| = 15$.

Ans. 15

4 Complex Numbers Dec 2017 (No Calculators)

3 pts 1. Simplify the following: $(3i + 4i^2)(2i^3 + 5i^4)$.

Ans. ____________________

4 pts 2. Solve the following: $ix^2 - 3x - 2i = 0$

Ans. ____________________

5 pts 3. Find the equation of the cubic with leading coefficient of 1, and whose solutions are $1 + i$, $1 - i$, and 2.

Ans. ____________________

Complex Numbers

1. $(3i + 4i^2)(2i^3 + 5i^4) = (-4 + 3i)(5 - 2i) = -20 + 23i + 6 = -14 + 23i$. **Ans. $-14 + 23i$**

2. Dividing $ix^2 - 3x - 2i$ by i yields $x^2 + 3ix - 2 = (x + 2i)(x + i) = 0$. $x = -2i$ or $-i$.

Ans. $-2i$ or $-i$

3. Since the roots are $1 + i$, $1 - i$, and 2, then (1) $x = 1 + i$, (2) $x = 1 - i$, and (3) $x = 2$. In (1) and (2) $x - 1 - i = 0$ and $x - 1 + i = 0$. Multiplying: $((x - 1) - i)((x - 1) + i)) = 0$ ➔ $x^2 - 2x + 1 + 1 = 0$ or $x^2 - 2x + 2 = 0$. In (2): $x - 2 = 0$. Multiplying: $(x^2 - 2x + 2)(x - 2)$ ➔ $x^3 - 2x^2 + 2x - 2x^2 + 4x - 4 = 0$ ➔ $x^3 - 4x^2 + 6x - 4 = 0$.

Ans. $x^3 - 4x^2 + 6x - 4 = 0$

4 Complex Numbers Dec 2016-17 (No Calculators)

3 pts 1. Find the value of the expression T - B, given that $T + Bi = (8 + 2i)(4 - 3i)$.

Ans. ____________________

4 pts 2. Evaluate $\left|\dfrac{i+3i^2-2i^3+4i^4-6i^5}{2i+1}\right|$.

Ans. ____________________

5 pts 3. Find the value of f(2), if a zero of f(x) = x^3+ax^2+bx+4 is 2i, where *a* and *b* are real numbers.

Ans. ____________________

Complex Numbers

1. $T + B\text{i} = (8 + 2\text{i})(4 - 3\text{i}) = 32 - 24\text{i} + 8\text{i} - 6\text{i}^2 = 38 - 16\text{i}$. $T - B = 38 - (-16) = 54$. **Ans. 54**

2. $\left|\dfrac{i+3i^2-2i^3+4i^4-6i^5}{2i+1}\right| = \left|\dfrac{i-3+2i+4-6i}{2i+1}\right| = \left|\dfrac{1-3i}{2i+1}\cdot\dfrac{2i-1}{2i-1}\right| = \left|\dfrac{2i-1-6i^2+3i}{4i^2-1}\right| = \left|\dfrac{5i+5}{-5}\right| = |-i-1| =$

$\sqrt{(-1)^2+(-1)^2} = \sqrt{2}$. **Ans. $\sqrt{2}$**

3. $f(2\text{i}) = (2\text{i})^3 + a(2\text{i})^2 + b(2\text{i}) + 4 = 0 \Rightarrow -8\text{i} - 4a + 2b\text{i} + 4 = 0$. Therefore $-8 + 2b = 0$ or $b = 4$ and $-4a + 4 = 0$ or $a = 1$. Thus f(x) = x^3+x^2+4x+4, and f(2) = $(2)^3+(2)^2+4(2)+4 = 8 + 4 + 8 + 4 = 24$. **Ans. 24**

4 Complex Numbers Dec 2015 (No Calculators)

3 pts 1. Find the cube of $2 - \sqrt{3}i$.

Ans. ____________________

4 pts 2. Find z, in simplest form for all a, b except for a = b = 0, if $\frac{a+bi}{z} = b - ai$.

Ans. ____________________

5 pts 3. Find all three roots of $y = 2x^3 + 3ix^2 - 4x^2 + 9x - 6ix - 18$.

Ans. ____________________

Complex Numbers

1. $(2 - \sqrt{3}i)(2 - \sqrt{3}i) = 4 - 4\sqrt{3}i + 3i^2 = 1 - 4\sqrt{3}i$. $(1 - 4\sqrt{3}i)(2 - \sqrt{3}i) = 2 - \sqrt{3}i - 8\sqrt{3}i + 12i^2 =$ $-10 - 9\sqrt{3}i$.

Ans. $-10 - 9\sqrt{3}i$

2. $\frac{a+bi}{z} = b - ai \Rightarrow z = \frac{a+bi}{b-ai}\cdot\frac{b+ai}{b+ai} = \frac{(a^2+b^2)i}{b^2+a^2} = i$.

Ans. i

3. $y = 2x^3 + 3ix^2 - 4x^2 + 9x - 6ix - 18 = (2x^3 - 4x^2) + (3ix^2 - 6ix) + (9x - 18) =$ $2x^2(x-2) + 3ix(x-2) + 9(x-2) \Rightarrow (x-2)(2x^2 + 3ix + 9) = 0$, 2 is one root. The other two through the quadratic formula: $x = \frac{-3i \pm \sqrt{-9-4(18)}}{4} = \frac{-3i \pm 9i}{4} = \frac{6i}{4}\, or\, \frac{-12i}{4} = \frac{3}{2}i$ or $-3i$.

Ans. 2, -3i, $\frac{3}{2}i$

4 Complex Numbers Dec 2014 (No Calculators)

3 pts 1. If $M = 1 + 3i$ and $N = 2 - i$, find $M(M + N)$.

Ans. ____________________

4 pts 2. Let $P = 1 + 3i$. Let $Q = 2 - i$. Find $\frac{P-Q}{Q^4}$.

Ans. ____________________

5 pts 3. Find all values of a such that $([a-i][a+i])^2 = 36$.

Ans. ____________________

Complex Numbers

1. $M + N = 3 + 2i$. $(3 + 2i)(1 + 3i) = 3 + 11i + 6i^2 = -3 + 11i$ **Ans.** $-3i + 11$ $-3 + 11i$

2. $\frac{1+3i-(2-i)}{(2-i)^4} = \frac{-1+4i}{(3-4i)^2} = \frac{(-1+4i)(-7+24i)}{(-7-24i)(-7+24i)} = \frac{7-24i-28i+96i^2}{49-576i^2} =$ **Ans.** $\frac{-89-52i}{625}$

3. $([a-i][a+i])^2 = 36 \Rightarrow (a^2+1)^2 = 36 \Rightarrow a^2 + 1 = \pm 6 \Rightarrow a^2 = -1 \pm 6$, therefore $a^2 = 5$ or $a^2 = -7$. $a = \pm\sqrt{5}$ or $a = \pm\sqrt{7}\,i$. **Ans.** $\pm\sqrt{5}, \pm\sqrt{7}\,i$

4 Complex Numbers Dec 2013 (No Calculators)

3 pts 1. Simplify: $(3-4i)(4-3i)-(3-4i)(3-4i)$

Ans. ______________________

4 pts 2. Solve for z: $(3+i)z+4-i=2i$.

Ans. ______________________

5 pts 3. Express the following in simplest form:

$$\left(1-\sqrt{3}i\right)^8$$

Ans. ______________________

Complex Numbers

1. $(3-4i)(4-3i)-(3-4i)(3-4i) = (3-4i)(1+i) = 3-4i+3i-4i^2 = 7-i$. **Ans. $7-i$**

2. $(3+i)z+4-i=2i \Rightarrow (3+i)z = -4+3i \Rightarrow z = \frac{-4+3i}{3+i}\cdot\frac{3-i}{3-i} = \frac{-12+4i+9i-3i^2}{9+1} = \frac{-9+13i}{10}$. **Ans.** $\frac{-9+13i}{10}$

3. $(1-\sqrt{3}i)^8 = (((1-\sqrt{3}i)^2)^2)^2 = ((-2\mp2\sqrt{3}i)^2)^2 = (-8-8\sqrt{3}i)^2 = -128-128\sqrt{3}$.
Ans. $-128-128\sqrt{3}$

4 Complex Numbers Dec 2012 (No calculators)

3 pts 1. Simplify: $(2 + 3i)^2(2 - 3i)^2$

Ans. ______________________

4 pts 2. Express the following as a single complex number in the form $a + bi$ where a and b are rational numbers:

$$\frac{(4+2i)(6+3i)}{(8-4i)(2-i)}$$

Ans. ______________________

5 pts 3. Find all values of x such that: $2ix = x^2 + 15$.

Ans. ______________________

Complex Numbers **Ans. 169**

$(2+3i)^2(2-3i)^2 = ((2+3i)(2-3i))^2 = (4-9i^2)^2 = 13^2 = 169.$

2. $\dfrac{(4+2i)(6+3i)}{(8-4i)(2-i)} = \dfrac{6(2+i)^2}{4(2-i)^2} = \dfrac{3(4+4i+i^2)}{2(4-4i+i^2)} = \dfrac{3(3+4i)(3+4i)}{2(3-4i)(3+4i)} = \dfrac{3(9+24i+16i^2)}{2(9+16)} =$

Ans. $-\dfrac{21}{50}+\dfrac{36}{25}i$

$\dfrac{3(-7+24i)}{50} = -\dfrac{21}{50}+\dfrac{36}{25}i.$

3. $2ix = x^2 + 15$ ➔ $x^2 - 2ix + i^2 = -15 + i^2$ ➔ $(x-i)^2 = -16$ ➔ $x - i = \pm 4i$ ➔
$x = i \pm 4i$. So $x = 5i$ or $-3i$

4 Complex Numbers Dec 2011 (No Calculators)

3 pts 1. Find the product: $(2+3i)(1-3i)(2-3i)$.

Ans. ____________________

4 pts 2. The equation $x^2 + 4ix - 9 = 0$ has two roots, one in quadrant 3 of the complex plane and the other in quadrant 4. Find the root in quadrant 4.

Ans. ____________________

5 pts 3. A cubic equation has the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c, and d are integers with a greatest common factor of 1. If two of the roots of the equation are 2 and $3-2i$, find the product $abcd$.

Ans. ____________________

Complex Numbers

1. $(2+3i)(1-3i)(2-3i) = (2+3i)(2-3i)(1-3i) = 13(1-3i) = 13-39i$. **Ans. $13-39i$**

2. Solving $x^2 + 4ix - 9 = 0$ using the quadratic formula: $x = \dfrac{-4i \pm \sqrt{-16+36}}{2} =$

$\dfrac{-4i \pm 2\sqrt{5}}{2} = -2i \pm \sqrt{5}$. The solution in quadrant 4 is $\sqrt{5} - 2i$. **Ans. $\sqrt{5} - 2i$**

3. If 2 is a root then $x-2$ is a factor. Since the coefficients are real, then if $3-2i$ is a root then $3+2i$ must also be a root. Thus $x^2 - 6x + 13$ is a factor. The only possible equations allowing a GCF of 1 are $\pm(x-2)(x^2 - 6x + 13) = 0$ or $\pm(x^3 - 8x^2 + 25x - 26) = 0$.
$abcd = 1(8)(25)(26) = 8(650) = 5200$. **Ans. 5200**

4 Complex Numbers (No Calculators) Dec 2010

3 pts 1. If $x = 3 - 2i$, and $y = 4 - i$, find the product of x and y.

Ans. ______________

4 pts 2. Find the solutions for $x^2 + 2x = -6$.

Ans. ______________

5 pts 3. If $\sqrt{-16-30i} = a + bi$, where a and b are real numbers, find the value of $(a + b)^2$.

Ans. ______________

Complex Numbers

1. $(3 - 2i)(4 - i) = 12 - 11i + 2i^2 = 12 - 11i - 2 = 10 - 11i$. **Ans. $10 - 11i$**

2. $x^2 + 2x = -6 \Rightarrow x^2 + 2x + 1 = -6 + 1 \Rightarrow (x + 1)^2 = -5 \Rightarrow x + 1 = \pm i\sqrt{5}$.

Ans. $-1 \pm i\sqrt{5}$

3. $\sqrt{-16-30i} = a + bi \Rightarrow -16 - 30i = a^2 + 2ab - b^2$. Since $2ab = -30$, then $a = -\frac{15}{b}$.
$a^2 - b^2 = -16$, so $\left(-\frac{15}{b}\right)^2 - b^2 = -16 \Rightarrow 225 - b^4 = -16b^2 \Rightarrow b^4 - 16b^2 - 225 = 0 \Rightarrow$
$(b^2 - 25)(b^2 + 9) = 0$. Thus $b = \pm 5$. If $b = 5$, $a = -3$. If $b = -5$, $a = 3$. $(a + b)^2 =$ **Ans. 4**

4 Complex Numbers Dec 09 (No Calculators)

3 pts 1. For what value(s) of k is $x - i$ a factor of $x^2 - 6x + k$.

Ans. ______________

4 pts 2. Solve over the set of complex numbers:

$$3z + iz + 4 - i = 2i$$

Ans. ______________

5 pts 3. The square of a complex number $a + bi$, where a and b are real numbers and i is $\sqrt{-1}$, is $2i$. What complex number(s) satisfy this?

Ans. ______________

Complex Numbers

1. Dividing synthetically:

i	1	-6	k
		i	$-1-6i$
	1	$i-6$	$k-1-6i$

Since $k - 1 - 6i = 0$ then $k = 1 + 6i$

Ans. 1 + 6i

2. $3z + iz + 4 - i = 2i$ ➔ $(3 + i)z = 3i - 4$. Thus $z = \dfrac{-4+3i}{3+i} \cdot \dfrac{3-i}{3-i} = \dfrac{-12+13i+3}{9+1} = \dfrac{-9+13i}{10}$ or $-\dfrac{9}{10} + \dfrac{13}{10}i$.

Ans. $-\dfrac{9}{10} + \dfrac{13}{10}i$

3. $(a+bi)^2 = 2i$ ➔ $a^2 + 2abi - b^2 = 0 + 2i$. Thus (1) $a^2 - b^2 = 0$ and (2) $2ab = 2$. In (1): $a = \pm b$. If $a = b$, then in (2): $2a^2 = 2$, so $a = \pm 1$. When $a = 1$, $b = 1$ and when $a = -1$, $b = -1$. Thus $1 + i$ and $-1 - i$ are the complex numbers. **Ans. $1 + i$ and $-1 - i$**